Mathematics Grade Primary

Mental Math

Draft — November 2007

NOVA SCOTIA
Education
English Program Services

Acknowledgements

The Department of Education gratefully acknowledges the contributions of the following individuals to the preparation of the *Mental Math* booklets:

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The Department of Education would like to acknowledge the special contribution of **David McKillop,** Making Math Matter Inc. His vision and leadership have been a driving force behind this project since its inception.

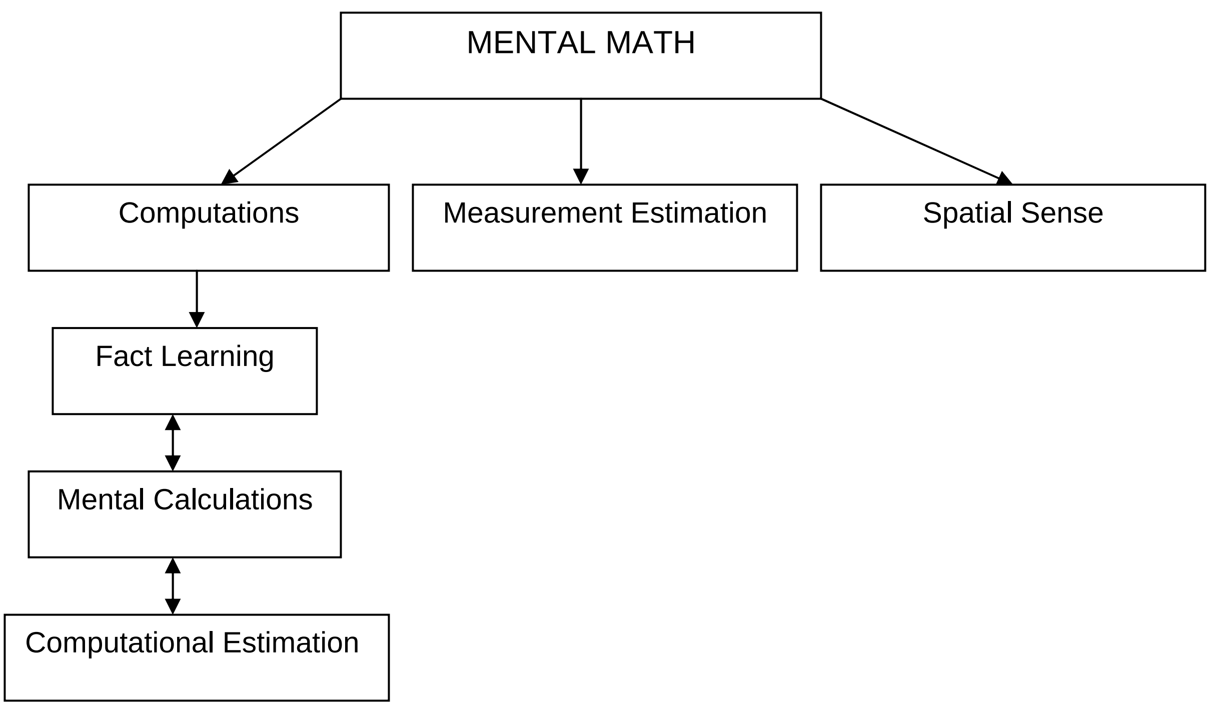
Contents

Introduction

Welcome to your grade-level mental math document. After the Department of Education released its *Time to Learn* document in which at least 5 minutes of mental math was required daily in every grade from 1 to 9, it became apparent we needed to clarify and outline expectations in each grade level. Therefore, grade-level documents were prepared for computational aspects of mental math and released in draft form in the 2006–2007 school year. Building on these drafts, the current documents describe the mental math expectations in computation, measurement, and geometry in each grade. These documents are supplements to the grade level documents of the Atlantic Canada mathematics curriculum. The expectations for your grade level are based on the full implementation of the expectations in the previous grades. Therefore, in the initial years of implementation, you may have to address some strategies from previous grades rather than all of those specified for your grade. It is critical that a school staff meets and plans the implementation of mental math until each grade-level's expectations can be addressed.

Definitions

For the purpose of these documents and to provide some uniformity in communication, it is important that some terms that are used are defined. Nova Scotia uses the term *mental math* to encompass the whole range of mental processing of information in all strands of the curriculum. This *mental math* is broken into three categories in the grade-level documents: *computations*, *measurement estimation*, and *spatial sense*. The *computations* are further broken down into *fact learning, mental calculations*, and *computational estimation*.



For the purpose of this booklet, *fact learning* will refer to the acquisition of the 100 number facts relating the single digits 0 to 9 for each of the four operations. When students know these facts, they can quickly retrieve them from memory (usually in 3 seconds or less). Ideally, through practice over time, students will achieve automaticity; that is, they will abandon the use of strategies and give instant recall. *Mental calculations* refer to using strategies to get exact answers by doing all the calculations in one's head, while *computational estimation* refers to using strategies to get approximate answers by doing calculations in one's head.

While each term in computations has been defined separately, this does not suggest that the three terms are totally separable. Initially, students develop and use strategies to get quick recall of the facts. These strategies and the facts themselves are the foundations for the development of other mental calculation strategies. When the facts are automatic, students are no longer employing strategies to retrieve them from memory. In turn, the facts and mental calculation strategies are the foundations for computational estimation strategies. In fact, attempts at computational estimation are often thwarted by the lack of knowledge of the related facts and mental calculation strategies.

Measurement estimation is the process of using internal and external visual (or tactile) information to get approximate measures or to make comparisons of measures without the use of measurement instruments.

Spatial sense is an intuition about shapes and their relationships, and an ability to manipulate shapes in one's mind. It includes being comfortable with geometric descriptions of shapes and positions.

Rationale for Mental Math

In modern society, the development of mental skills needs to be a major goal of any mathematical program for two major reasons. First of all, in their day-to-day activities, most people's computational, measurement, and spatial needs can be met by having well developed mental strategies. Secondly, while technology has replaced paper-and-pencil as the major tool for complex tasks, people need to have well developed mental strategies to be alert to the reasonableness of technological results.

The Implementation of Mental Computations

General Approach

In general, a computational strategy should be introduced in isolation from other strategies, a variety of different reinforcement activities should be provided until it is mastered, the strategy should be assessed in a variety of ways, and then it should be combined with other previously learned strategies.

A. Introducing a Strategy

The approach to highlighting a computational strategy is to give the students an example of a computation for which the strategy would be useful to see if any of the students already can apply the strategy. If so, the student(s) can explain the strategy to the class with your help. If not, you could share the strategy yourself. The explanation of a strategy should include anything that will help students see the pattern and logic of the strategy, be that concrete materials, visuals, and/or contexts. The introduction should also include explicit modeling of the mental processes used to carry out the strategy, and explicit discussion of the situations for which the strategy is most appropriate and efficient. Discussion should also include situation for which the strategy would not be the most appropriate and efficient one. Most important is that the logic of the strategy should be well understood before it is reinforced; otherwise, it's long-term retention will be very limited.

B. Reinforcement

Each strategy for building mental computational skills should be practised in isolation until students can give correct solutions in a reasonable time frame. Students must understand the logic of the strategy, recognize when it is appropriate, and explain the strategy. The amount of time spent on each strategy should be determined by the students' abilities and previous experiences.

The reinforcement activities for a strategy should be varied in type and should focus as much on the discussion of how students obtained their answers as on the answers themselves. The reinforcement activities should be structured to insure maximum participation. At first, time frames should be generous and then narrowed as students internalize the strategy. Student participation should be monitored and their progress assessed in a variety of ways to help determine how long should be spent on a strategy.

After you are confident that most of the students have internalized the strategy, you need to help them integrate it with other strategies they have developed. You can do this by providing activities that includes a mix of number expressions, for which this strategy and others would apply. You should have the students complete the activities and discuss the strategy/strategies that could be used; or you should have students match the number expressions included in the activity to a list of strategies, and discuss the attributes of the number expressions that prompted them to make the matches.

Language

Students should hear and see you use a variety of language associated with each operation, so they do not develop a single word-operation association. Through rich language usage students are able to quickly determine which operation and strategy they should employ. For example, when a student hears you say, "Six plus five", "Six and five", "The total of six and five", "The sum of six and five", or "Five more than six", they should be able to quickly determine that they must add 6 and 5, and that an appropriate strategy to do this is the *double-plus-one* strategy.

Context

You should present students with a variety of contexts for each operation in some of the reinforcement activities, so they are able to tranfer the use of operations and strategies to situations found in their daily lives. By using contexts such as measurement, money, and food, the numbers become more real to the students. Contexts also provide you with opportunities to have students recall and apply other common knowledge that should be well known . For example, when a student hears you say, "How many days in two weeks?" they should be able to recall that there are seven days in a week and that double seven is 14 days.

Number Patterns

You can also use the recognition and extension of number patterns can to reinforce strategy development. For example, when a student is asked to extend the pattern "30, 60, 120, …,", one possible extension is to double the previous term to get 240, 480, 960. Another possible extension, found by adding multiples of 30, would be 210, 330, 480. Both possibilities require students to mentally calculate numbers using a variety of strategies.

Examples of Reinforcement Activities

Reinforcement activities will be included with each strategy in order to provide you with a variety of examples. These are not intended to be exhaustive; rather, they are meant to clarify how language, contexts, common knowledge, and number patterns can provide novelty and variety as students engage in strategy development.

C. Assessment

Your assessments of computational strategies should take a variety of forms. In addition to the traditional quizzes that involve students recording answers to questions that you give one-at-a-time in a certain time frame, you should also record any observations you make during the reinforcements, ask the students for oral responses and explanations, and have them explain strategies in writing. Individual interviews can provide you with many insights into a student's thinking, especially in situations where pencil-and-paper responses are weak.

Assessments, regardless of their form, should shed light on students' abilities to compute efficiently and accurately, to select appropriate strategies, and to explain their thinking.

Response Time

Response time is an effective way for you to see if students can use the computational strategies efficiently and to determine if students have automaticity of their facts.

For the facts, your goal is to get a response in 3-seconds or less. You would certainly give students more time than this in the initial strategy reinforcement activities, and reduce the time as the students become more proficient applying the strategy until the 3-second goal is reached. In subsequent grades, when the facts are extended to 10s, 100s and 1000s, you should also ultimately expect a 3-second response.

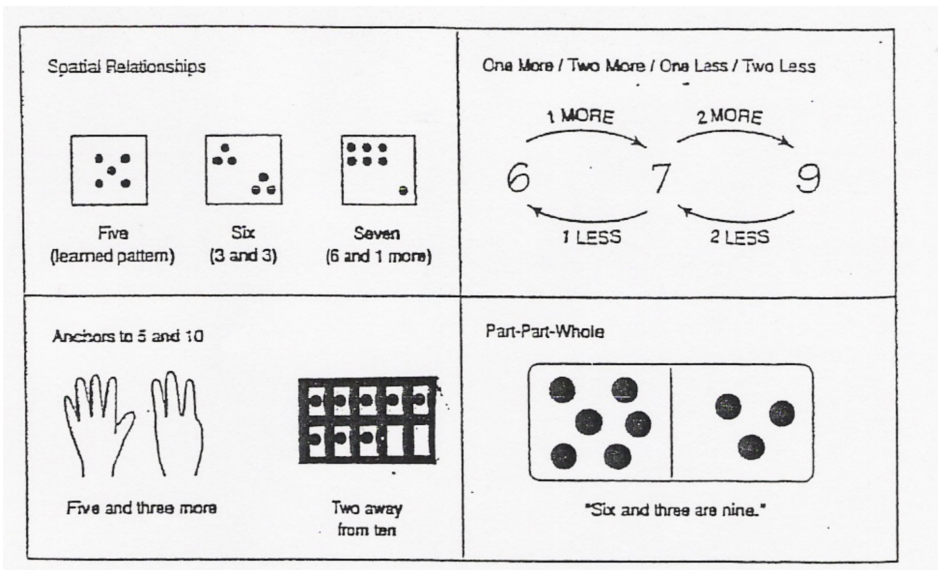
In the early grades, the 3-second response goal is a guideline for you and does not need to be shared with your students if it will cause undue anxiety.

With mental calculation strategies and computational estimation, you should allow 5 to 10 seconds, depending upon the complexity of the mental activity required. Again, in the initial application of these strategies, you would allow as much time as needed to insure success, and gradually decrease the wait time until students attain solutions in a reasonable time frame.

Grade Primary

While there is no mandated time allotted for mental math in grade primary, children need to develop some important concepts about number to prepare them for mental math learning in grade one. These concepts include:

- Counting
- Representing numbers
- Spatial relationships
- One more / Two more / One less / Two less
- Anchors to 5 and 10
- Part-part-whole



Throughout the year, students should be working toward developing these very important concepts using flash cards, games, die, ten frames, etc.

Every child learns differently and some concepts may take longer to develop than others. Students need to review previously learned concepts on a regular basis.

Part 1

A. Counting

Description

Being able to count involves an understanding of the following principles:

- One number is said for each item in the group
- Counting begins with the number 1
- No item is counted twice
- The arrangement of objects is irrelevant
- The number in the set is the last number said

Activities

Observe students as they count:

- do they touch each object as they count?
- do they set aside items/line them up as they count them?
- do they show confidence in their count or do they feel the need to check?
- do they check their counting in the same order as the first count or in a different order?
- need to start at the beginning to count additional objects?

Students will learn how to count forward from 1 and backward from 10. Some students may be able to count onward from a number i.e. 4 (5…6…7).

B. Representing Numbers

Description

Students need to be able to represent numbers. Students can practice making their numbers while performing meaningful counting or mathematical tasks. For example, students may be asked to roll a die and record the number of dots.

Activities

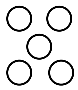
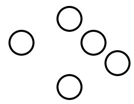
They may practice writing their phone number or record the number of counters when counting collections of objects.

		3			
1	2	3	4	5	6
1	2	3	4	5	6

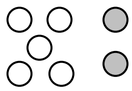
C. Spatial Relationships

Description

Students should recognize that there are many ways to arrange a set of objects, and that some arrangements are easier to recognize than others. Observe whether students are able to immediately say how many objects are displayed in familiar arrangements without doing a 1-to-1 count.

Ex: 5  **vs** 

For most numbers, there are common patterns (i.e. the ones found on dominoes and dice). Patterns for larger numbers can be made up of two or more easier patterns for smaller numbers.

7 

Activities

Learning Patterns with Dot Cards

To introduce patterns, provide each student with about 10 counters and a piece of construction paper as a mat. Hold up a dot card for about 3 seconds. Ask, "How many dots did you see? How did you see them? Make the pattern you saw using the counters on the mat". Spend some time discussing the configuration of the pattern and how many dots. Do this with a few new patterns each day

Dot Card/Plate Flash

Hold up a dot card for only 1 to 3 seconds. Ask, "How many? How did you see it?" Children like to see how quickly they can recognize and say how many dots. Include lots of easy patterns and a few with more dots as you build their confidence. Students can also flash the dot plates to each other as a workstation activity.

Dot Cards and Number Cards

Give each student a set of number cards (0-10). Hold up a dot card and have the students hold up the corresponding number card.

Dot Card Challenge

Two players each turn over a card from a stack of cards. The winner is the one with the larger total number and gets to take the cards (or whatever you wish to make as a rule). Children should be encouraged to determine who is the winner just by looking rather than counting.

Dot Card Differences

Students each have a pile of dot cards. There should also be a pile of about 50 counters. On each play, the players turn over their cards as usual. The player with the greater number of dots wins as many counters from the pile as the difference between the two cards. The players keep their cards. The game is over when the counter pile runs out. The player with the most counters wins the games.

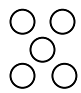
D. One More / Two More / One Less / Two Less

Description

Students should learn how to count on and count back from a number without starting at the beginning. This would include counting on one more and two more and counting back one less and two less. In order to do this, students must understand the concepts of more and less as well as the counting sequence.

Ex: you could show students one of the following:

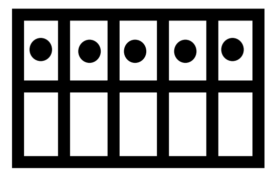
an arrangement of dots



number

5

ten frame



Ask, "what is one more?"
"what is two more?
"what is one less?"
"what is two less?"

You would only work on one of these at a time.

Encourage students not to start at the beginning but count on (or back) from the number being presented.

Activities

Make a One-More / One-Less Than Set (Two-More / Two-Less)

Hold up a dot card and have the students construct a set of counters that is one more than the set.
-…one less than the set
-…two more than the set
-…two less than the set

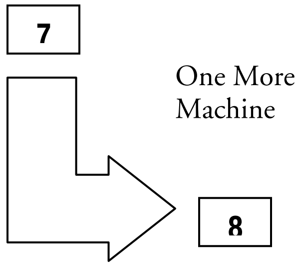
Dot Card: One-More / One-Less (Two-More / Two-Less)

Similar to the Dot Cards and Number Cards only ask the students to hold up the number card that is one more
-…hold up the number card that in one less
-…hold up the number card that is two more
-…hold up the number card that is two less

Number Machine

Draw a number machine on the board. It requires an input hopper and an output chute. Tell the children what the machine does; for example, "This is a magic one-more-than machine. It takes in a number up here and spits out a number that is one more."

Put in a



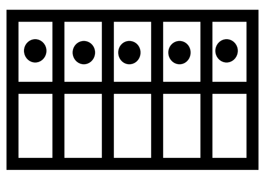
Dot Card Trains

Make a long row of dot cards from 0 up to 9, then go back again to 1, then up, and so on. Alternatively, begin with 0 or 1 and make a two-more/two-less train.

Ten-Frame flash (One-More/Two-More)

Flash a ten-frame card and ask the students to say one more or two more than the number of dots shown on the card.

Ex: Show

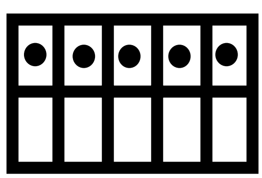


Students would say, "6"

Ten-Frame flash (One-Less/Two-Less)

Flash a ten-frame card and ask the students to say one less or two less than the number of dots shown on the card.

Ex: Show



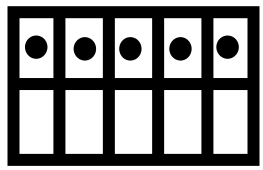
Students would say, "4"

E. Anchors to 5 and 10

Description

Ten plays an important role in our number system and since five and five make ten, it is important for students to develop and understand the relationships for the numbers 1 to 10 to the important anchors of 5 and 10.

Students will use the ten-frame to help develop and learn about these anchors.



Activities

Five-And game

The teacher calls out numbers between 5 and 10. The children respond "Five and ___" using the appropriate number. For example, if you say, "Eight!" the children respond, "Five and three."

Make-Ten game

Call out numbers between 0 and 10. The children respond by saying how many more are needed to make 10. This is most effective with numbers between 5 and 10.

Five Game

Hold up a ten frame and the children respond by stating the relationship to five. For example, show the ten frame with three dots. The children would respond, "Three is two less than five". Or holding up a ten frame with eight dots, the children would respond, "Eight is three more than five."

Ten Game

Hold up a ten frame and the children respond by stating the relationship to ten. For example, show the ten frame with three dots. The children would respond, "Three is seven less than ten". (These will always be less than statements except when ten is held up-"ten is the same as ten")

F. Part-Part-Whole

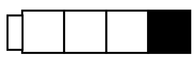
Description

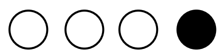
Children need to think about numbers as being made up of other numbers.

Ex: 6 is made up of 1 and 5, 2 and 4, 3 and 3, and 0 and 6.

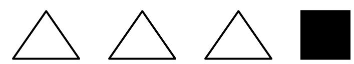
There is a clear connection between part-part-whole concepts and addition and subtraction. Children should be encouraged to make sets for numbers using a variety of materials.

Ex: cubes, counters, numbers, shapes   Show the number **4**



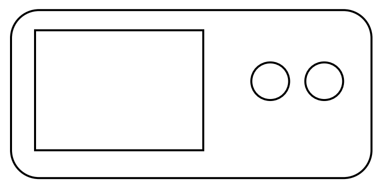


3 and 1



Related to part-part-whole is the missing-part concept where one of the two parts can be missing when the whole is known. This concept is necessary for understanding subtraction.

To show six in a missing-part activity, you might ask, "how many counters are hidden under the paper on the mat?" (the answer would be 4)



Part-Part-Whole Activities

Part-Part-Whole Mat

Each child has a part-part-whole mat and counters. Call out a number and the children place the counters on their mat in two parts to match the number. Ask for different configurations for the number. Eventually the children should be able to name the parts without the mats and counters.

Two-Part Dot Plates

Make a set of dot plates with solid and outline dots in various combinations to make numbers

Two-Part Dot Cards

Flash dot cards (with dots arranged in two groupings and have students say the two parts and the total)

Two out of Three

Make lists of three numbers, two of which total the whole that children are focusing on. Here is an example list for the number 5:

2-3-4
5-0-1
1-3-2
3-1-4
2-2-3
4-3-1

With the list on the board or overhead, children can take turns selecting the two numbers that make the whole. As with all problem-solving activities, children should be challenged to justify their answers. The same activity can be used in a worksheet format, but the real value lies in the discussion and justification.

Show with your Hands

Call out a number and have children show configurations with their fingers for that number

Missing-Part Activities

I Wish I Had

Hold up a ten frame or dot card/plate showing 7 or less. Say, "I wish I had seven." The children respond with the part that is needed to make 7. Counting on can be used to check. The game can focus on a single whole, or the "I Wish I Had" number can change each time.

Number Machine (see One more/Two more/One less/Two less activities)

Draw a funny looking machine on the board. It requires an input hopper and an output chute. Tell the children what the machine does; for example, "This is a magic parts of 8 machine. It takes in a number up here and spits out a number that is one more." If 3 goes in a parts-of-8 machine, a 5 comes out. Do not forget the discussion concerning how the students decided on the outcome.

Covered Parts

A set of counters equal to the target amount is counted out and the rest put aside. Put the counters in a cup or tub so the students can't see. Take some out of the cup and show the students. (This amount could be none, all, or any amount in between.) For example, if 6 is the whole and 4 are showing, the students should say, "Four and *two* is six". If there is hesitation or if the hidden part is unknown, the hidden part is immediately shown. The focus is on learning and thinking, not on testing and anxiety.

PART 2

The Implementation of Measurement Estimation Strategies

General Approach

Rather than being part of mental math time, it is expected that a measurement estimation strategy would be introduced as part of the general development of measurement concepts, at the appropriate grade level. The strategy would be explored, modeled, and reinforced through a variety of class activities that promote understanding, the use of estimation language, and the refinement of the strategy. As well, the assessment of the strategy during the instructional and practice stages would be ongoing and varied. Then, for the most part, a measurement estimation strategy would be reinforced and assessed during mental math time in the grades following the initial introduction with the goal of increasing a student's competency. Competency means that a student reaches a reasonable estimate using an appropriate strategy in a reasonable time frame. While what is considered a reasonable estimate will vary with the size of the unit and the size of the object, a general rule-of-thumb would be to aim for an estimate that is within 10% of the actual measure. A reasonable time frame will vary with the strategy being used; for example, using the *benchmark strategy* might take 5 to 10 seconds but using the *chunking strategy* might take 10 to 30 seconds depending upon the complexity of the task.

A. Introducing a Strategy in Regular Classroom Time

Measurement estimation strategies should be introduced when appropriate as part of a rich and carefully sequenced development of measurement concepts. For example, in Grade 3, the distance from the floor to most door handles is employed as a benchmark for a metre so students can use a *benchmark strategy* to estimate lengths in metres. This has followed many other experiences with linear measurement in earlier grade: in grade primary, students compared and ordered lengths of objects concretely and visually; in Grade 1, students estimated lengths of objects using non-standard units such as paper clips; in Grade 2, students developed the standard unit of a metre and were merely introduced to the idea of a benchmark for a metre.

The introduction of a measurement estimation strategy should a variety of activities that use concrete materials, visuals, and meaningful contexts, in order to have students thoroughly understand the strategy. For example, a meaningful context for the *chunking strategy* might be to estimate the area available for bookshelves in the reading corner. Explicit modeling of the mental processes used to carry out a strategy should also be part of this introduction. For example, when demonstrating the *subdivision strategy* to get an estimate for the area of a wall, you would orally describe the process of dividing the wall into fractional parts, the process of determining the area of one part in square metres, and the multiplication process to get the estimate of the area of the wall in square metres. During this introductory phase, you should also lead discussions of situations for which a particular strategy is most appropriate and efficient, and of situations for which it would not be either appropriate or efficient.

B. Reinforcement in Mental Math Time

Each strategy for building measurement estimation skills should be practised in isolation until students can give reasonable estimates in reasonable time frames. Students must understand the logic of the strategy, recognize when it is appropriate, and be able to explain the strategy. The amount of time spent on each strategy should be determined by the students' abilities and previous experiences.

The reinforcement activities for a strategy should be varied in type and should focus as much on the discussion of how students obtained their answers as on the answers themselves. The reinforcement activities should be structured to insure maximum participation. At first, time frames should be generous and then narrowed as students internalize the strategy. Student participation should be monitored and their progress assessed in a variety of ways to help determine how long should be spent on a strategy.

After you are confident that most students have internalized the strategy, you need to help them integrate it with other strategies that have been developed. You should do this by providing activities that include a mix of attributes to be estimated in different units using a variety of contexts. You should have the students complete the activities and discuss the strategy/strategies that were, or could have been, used. You should also have students match measurement estimates required to a list of strategies, and have them discuss the reasons behind their strategy matches.

C. Assessment

Your assessments of measurement estimation strategies should take a variety of forms. In addition to the traditional quizzes that involve students recording answers to estimation questions that you give one-at-a-time in a certain time frame, you should also record any observations you make during the reinforcements, ask the students for oral responses and explanations, and have them explain strategies in writing. Individual interviews can provide you with many insights into a student's thinking, especially in situations where pencil-and-paper responses are weak.

Assessments, regardless of their form, should shed light on students' abilities to estimate measurements efficiently and accurately, to select appropriate strategies, and to explain their thinking.

Measurement Estimation Strategies

The Benchmark Strategy
This strategy involves mentally matching the attribute of the object being estimated to a known measurement. For example, to estimate the width of a white board, a student might mentally compare this width to the distance of the doorknob from the floor. This distance that is known to be 1 metre is the benchmark. When the student mentally matches the width of the white board to this benchmark, she may estimate that the width is about twice as much; therefore, her estimate would be 2 metres. In mathematics education literature you will often see remarks on *personal referents.* These are benchmarks that individuals establish using their own bodies; for example, the width of their little fingers might be 1 cm, their hand spans might be 20 cm, and their hand widths might be 1 dm. These benchmarks have the advantage of being portable and always present should an estimate be needed.

The Chunking Strategy
This strategy involves dividing an object into a number of chunks, estimating each of these chunks, and adding the estimates of all the chucks to get the total estimate. For example, to estimate the length of a wall, a student might estimate the distance from one corner to the door, estimate the width of the door, estimate the distance from the door to the bookcase, and estimate the length of the bookcase which goes right to the other corner. All these individual estimates would be added to get the estimate of the length of the wall.

The Subdivision Strategy
This strategy involves mentally dividing an object repeatedly in halves until a more manageable part of the object can be estimated and this estimate is multiplied by the appropriate factor to get an estimate for the whole object.
For example, to estimate the area of a table top, a student might mentally divide the table top into fourths, estimate the area of one-fourth in square decimeters, and then multiply that estimate by four to get the estimate of the whole table top.

The Unitizing Strategy

This strategy involves establishing an object (that is smaller than the object being estimated) as a unit, estimating the size of this unit, and multiplying by the number of these units it would take to make the object being measured. For example, a student might get an estimate for the length of a wall by noticing it is about five bulletin boards long. The bulletin board becomes the unit, the length of which the student estimates, and then the student multiplies this estimate by 5 to get the length of the wall.

Measurement Estimation

While there is no mandated time allotted for mental math in primary, children need to develop some important concepts and skills related to number, measurement, geometry and patterning to prepare them for mental math learning in grade one.

In grade primary, the emphasis is on what it means to measure, rather than how to measure with non-standard or standard units. Activities focus on language and concept development related to exploring length, capacity, mass and sequencing events. A variety of early measurement activities help students' mental math skills, such as using visual perception and the ability to estimate and predict.

G. Measurement Estimation — Length

Linear measurement refers to the length of an object and to the distance between two objects. It also refers to height, width and thickness.

Initially, students should compare lengths informally by simply viewing the length of 2 or more objects, describing them as longer or shorter, or by ordering them visually from longest to shortest.

Examples of Some Practice Items

- Show the class three different length "trains" made from interlocking cubes. Ask students to order them from shortest to longest or longest to shortest.
- Show the class a "giant's foot" or a "troll's foot" drawn on a piece of paper. Ask students to estimate whether it is longer that their own foot.
- Using a pattern block, ask students to estimate whether their glue bottle is longer or shorter than the pattern block.
- Using a paperclip chain, ask students to compare the length and width of their crayon box, the whiteboard brush or overhead marker, and their pencil to the chain.

H. Measurement Estimation — Capacity

Capacity is often used to refer to the amount of space that can be filled. Students in Primary and grade 1 require a great deal of hands on experience with a variety of containers, of different sizes and shapes, prior to mental math activities. Remember, the focus is on comparison, rather than on describing the capacities of individual containers.

Examples of Some Practice Items

- Display 3 different sized containers. Ask students to estimate which container holds the most / least.
- Display several containers, Ask students to estimate which 2 containers hold about the same amount.
- Display a "target" container, such as a drinking glass, along with a collection of containers. Asks students to estimate which "holds more", "holds less", or "holds about the same amount".

I. Measurement Estimation — Mass

Mass
Mass is the amount of matter contained in an object. In primary the ideas of "heavy" and "light" are developed as the students hold a variety of objects. In many cases the masses of two objects cannot be compared by sight; they must be held to determine which is heavier or lighter.

Examples of Some Practice Items
Ask students to estimate which of two objects is heavier/lighter

- A wooden block or two interlocking cubes
- 3 paper clips or 3 pattern blocks
- a glue stick or a small book

J. Measurement Estimation — Time

Time
Time is used to specify when and event occurred or will occur and also to describehow long an event lasted.

Sequencing
Primary students focus on sequencing events, not in measuring duration of time.

Questions such as the following help children develop concepts and vocabulary relating to the sequencing of events.

- When you get dressed, which do you put on first, your shoes or your socks?
- Name some things we do in class before/after lunch.
- List in order 3 to 5 things you do before going to bed.

Comparison
Early comparison tasks rely on memory. Two events are described and the student is asked:

Which takes longer or more time to complete?

- Eating breakfast or putting on your shoes

Which takes the least amount of time to complete?

- Lunch time or recess

PART 3

The Development of Spatial Sense

What is spatial sense?

Spatial sense is an intuition about shapes and their relationships, and an ability to manipulate shapes in one's mind. It includes being comfortable with geometric descriptions of shapes and positions.

There are seven abilities in spatial sense that need to be developed in the classroom:

- **Eye-motor co-ordination**. This is the ability to co-ordinate vision with body movement. In the mathematics classroom, this is seen in children learning to write and draw using pencils, crayons, and markers; to cut with scissors; and to handle materials.
- **Visual memory**. This is the ability to recall objects no longer in view.
- **Position-in-space perception**. This is the ability to determine the relationship between one object and another, and an object's relationship to the observer. This includes the development of associated language, such as over, under, beside, on top of, right, and left; and the transformations of translations, reflections, and rotations that change an object's position.
- **Visual discrimination**. This is the ability to identify the similarities and differences between, or among, objects.
- **Figure-ground perception.** This is the ability to focus on a specific object within a picture or within a group of objects, while treating the rest of the picture or the objects as background.
- **Perceptual constancy**. This is the ability to recognize a shape when it is seen from a different viewpoint or from a different distance (enlargement/reduction). This perception is connected to prior experiences to enable the brain to "see" what it expects to see when it interprets visual information it receives.
- **Perception of spatial relationships.** This is the ability to see the relationship between two or more objects.

These abilities develop naturally through many experiences in life; therefore, most students come to school with varying degrees of ability in each of these seven aspects. However, students who have been identified as having a learning disability are likely to have deficits in one or more of these abilities. Classroom activities should be planned to allow most students to further develop these perceptions. Most spatial sense activities involve many, if not most, of the spatial abilities, although often one of them is highlighted.

Classroom Context

Spatial abilities are essential to the development of geometric concepts, and the development of geometric concepts can provide the opportunity for further development of spatial abilities through strategic planning of rich experiences with shapes and spatial relationships consistently over time. Classroom experiences should also include activities specifically designed for the development of spatial abilities. These activities should focus on shapes new to the grade as well as shapes from previous grades. As the shapes become more complex, students' spatial sense should be further developed through activities aimed at the discovery of relationships between and among properties of these shapes. Ultimately, students should be able to visualize shapes and their transformations.

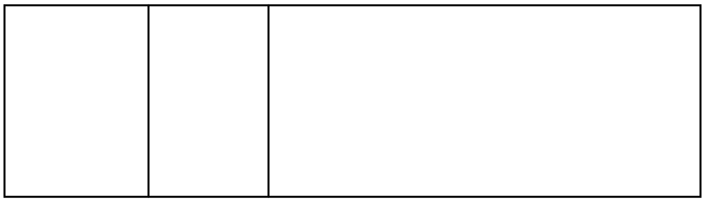
Spatial Sense in Mental Math Time

Following the exploration and development of spatial abilities during geometry instruction, mental math time can provide the opportunity to revisit and reinforce spatial abilities throughout the year.

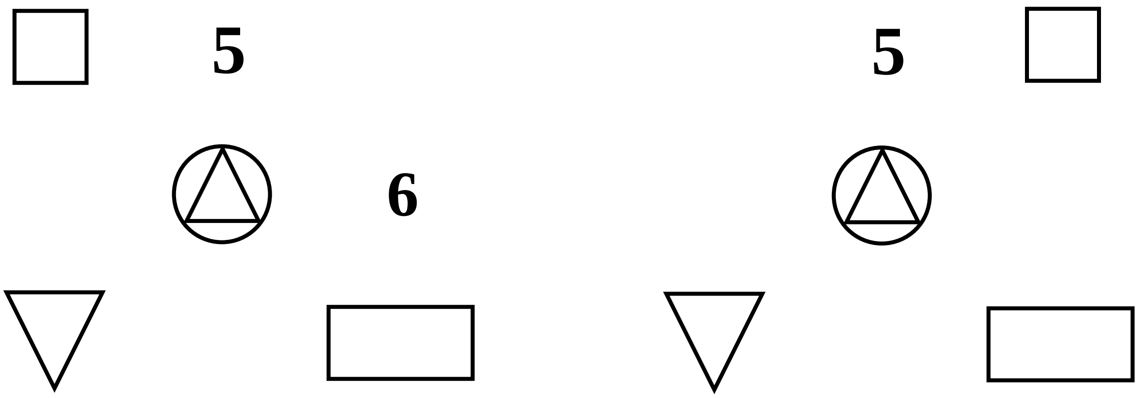
Assessment

Assessment of spatial sense development should take a variety of forms. In addition to the traditional quizzes that involve students' recording answers to questions that you give during mental math time, you should also record any observation you make during lessons, ask the students for oral responses and explanations, and have them explain their thinking in writing. Individual interviews can provide you with many insights into a student's spatial sense development, especially in situations where pencil-and-paper responses are weak and/or you are uncertain about specific students from your observations of them in the large group.

How many rectangles can you find?



What is alike and what is different?



PART 4

Patterning

Activities in grade primary help students to develop important patterning concepts.
These activities include:

- Copying and extending patterns
- Representing the same pattern in multiple ways

Copying and Extending Pattern

It is important to help students identify patterns from many contexts, e.g., art, music, nature, and to recognize the mathematical aspects of these patterns. Before they can create patterns, students need a great deal of experience with a variety of given patterns, e.g., a stand-sit pattern, action patterns (clap, snap, touch toes), patterns in chants and songs and patterns in predictable books.

Have students reproduce and extend patterns that focus on shape, size, colour, position, and number auditory patterns.

It is important to remember that students recognize that there is not a pattern until there is a repetition, e.g., A B C is not a pattern, but A B C A B C A B C is a pattern.

Number

Extend:

- 2, 1, 3, 2, 1, 3, 2, 1, 3, __________
  (dot cards) Show the above pattern

- What's my pattern rule? 3, 1, 2, 3, 1, 2, 3, 1, 2, __________
  (dot cards) Show the above pattern

Shape

Extend:
(Use shapes)

- circle, square, circle, square….
- Circle, triangle, square, circle, triangle, square….

(Use pattern blocks)

- Hexagon, trapezoid, hexagon, trapezoid

What's my patterning rule?

Size

Extend:

- Large circle, little circle, large circle, little circle….
- Little square, large square, little square, large square….

What comes next?

Position

Extend: (use pattern blocks)

What comes next?

Auditory

Extend:

- oink, oink, moo, oink, oink, moo, oink, oink, moo…
- beep, clap, clap, beep, clap, clap, beep, clap, clap…

Whisper **SHOUT**

- one, two, three, four…
- one, two, three, four, five, six…
- one, two, three, four, five, six, seven, eight…
- one, two, three, four, five, six, seven, eight, nine, ten
- snap, snap, snap, snap, clap, snap, snap, snap, snap, clap…

Representing Patterns

In addition to developing the abilities to recognize, describe and continue patterns, grade primary students should also be learning to translate from one representation of a pattern to another representation and to represent the same pattern in a variety of ways. For example, represent snap, clap, snap with red and white unifix cubes or pencils and crayons. These are more complex tasks.

Represent a simple pattern such as sit, stand, sit, stand, sit, stand as:

- clap, snap, clap, snap, clap, snap
- a,b,a,b,a,b,
- circle, square, circle, square, circle, square
- one, two, three, four, five, six
- Position: