Mental Math

In

Mathematics 6

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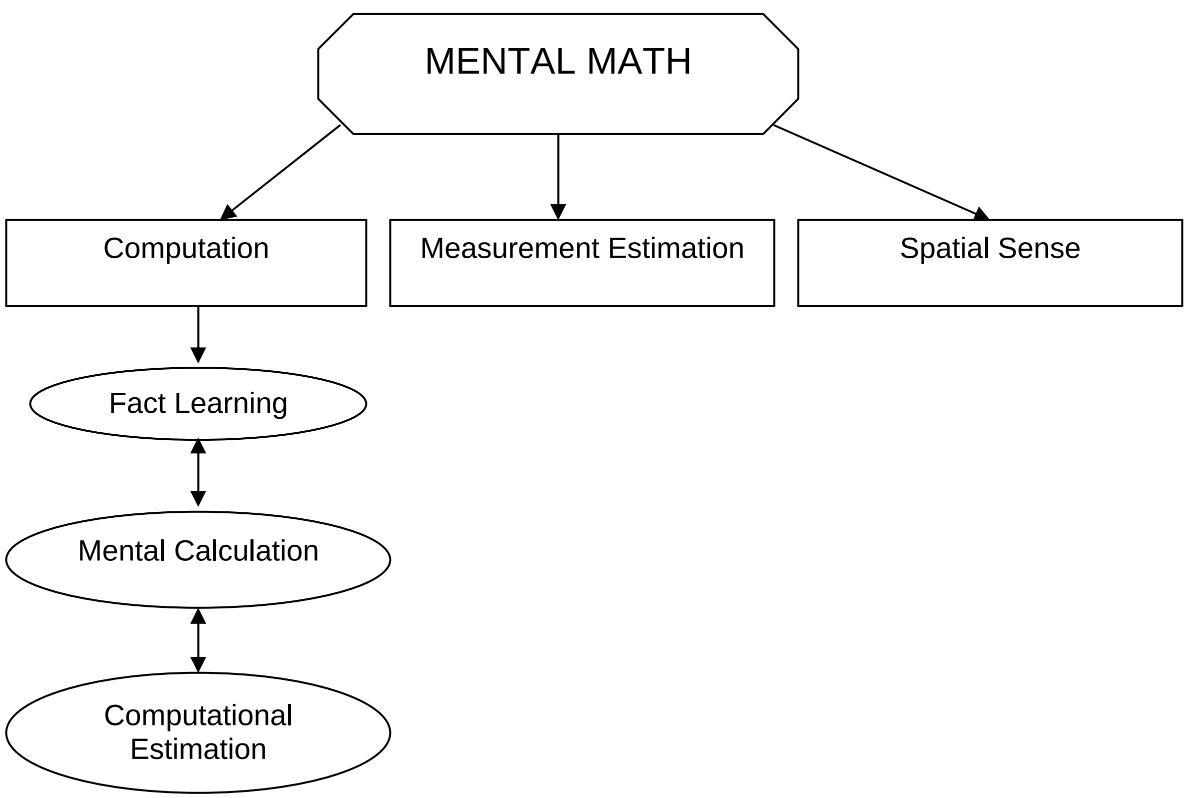
Introduction

Welcome to your grade-level mental math document. After the Department of Education released its *Time to Learn* document in which at least 5 minutes of mental math was required daily in every grade from 1–9, a need to clarify and outline expectations in each grade level became apparent. Therefore, grade-level documents were prepared for computational aspects of mental math and released in draft form in the 2006–2007 school year. Building on these drafts, the current documents describe the mental math expectations in computation, measurement, and geometry in each grade. These documents are supplements to the grade-level documents of the Atlantic Canada mathematics curriculum. The expectations for your grade level are based on the full implementation of the expectations in the previous grades. Therefore, in the initial years of implementation, you may have to address some strategies from previous grades rather than all of those specified for your grade. It is critical that a school staff meets and plans the implementation of mental math until the expectations at each grade-level can be addressed.

Definitions

For the purpose of these documents and to provide some uniformity in communication, it is important that some terms that are used are defined. Nova Scotia uses the term *mental math* to encompass the whole range of mental processing of information in all strands of the curriculum. This *mental math* is broken into three categories in the grade-level documents: *computations*, *measurement estimation*, and *spatial sense*. The *computations* are further broken down into *fact learning, mental calculations*, and *computational estimation*.



For the purpose of this booklet, *fact learning* will refer to the acquisition of the 100 number facts relating the single digits 0 to 9 for each of the four operations. When students know these facts, they can quickly retrieve them from memory (usually in 3 seconds or less). Ideally, through practice over time, students will achieve automaticity; that is, they will abandon the use of strategies and give instant recall. *Mental calculations* refer to using strategies to get exact answers by doing all the calculations in one's head, while *computational estimation* refers to using strategies to get approximate answers by doing calculations in one's head.

While each term in computations has been defined separately, this does not suggest that the three terms are totally separable. Initially, students develop and use strategies to get quick recall of the facts. These strategies and the facts themselves are the foundations for the development of other mental calculation strategies. When the facts are automatic, students are no longer employing strategies to retrieve them from memory. In turn, the facts and mental calculation strategies are the foundations for computational estimation strategies. In fact, attempts at computational estimation are often thwarted by the lack of knowledge of the related facts and mental calculation strategies.

Measurement estimation is the process of using internal and external visual (or tactile) information to get approximate measures or to make comparisons of measures without the use of measurement instruments.

Spatial sense is an intuition about shapes and their relationships, and an ability to manipulate shapes in one's mind. It includes being comfortable with geometric descriptions of shapes and positions.

Rationale for Mental Math

In modern society, the development of mental skills needs to be a major goal of any mathematical program for two major reasons. First of all, in their day-to-day activities, most people's computational, measurement, and spatial needs can be met by having well developed mental strategies. Secondly, while technology has replaced paper-and-pencil as the major tool for complex tasks, people need to have well developed mental strategies to be alert to the reasonableness of technological results.

PART 1

The Implementation of Mental Computations

General Approach

In general, a computational strategy should be introduced in isolation from other strategies, a variety of different reinforcement activities should be provided until it is mastered, the strategy should be assessed in a variety of ways, and then it should be combined with other previously learned strategies.

A. Introducing a Strategy

The approach to highlighting a computational strategy is to give the students an example of a computation for which the strategy would be useful to see if any of the students already can apply the strategy. If so, the student(s) can explain the strategy to the class with your help. If not, you could share the strategy yourself. The explanation of a strategy should include anything that will help students see the pattern and logic of the strategy, be that concrete materials, visuals, and/or contexts. The introduction should also include explicit modeling of the mental processes used to carry out the strategy, and explicit discussion of the situations for which the strategy is most appropriate and efficient. Discussion should also include situation for which the strategy would not be the most appropriate and efficient one. Most important is that the logic of the strategy should be well understood before it is reinforced; otherwise, its long-term retention will be very limited.

B. Reinforcement

Each strategy for building mental computational skills should be practised in isolation until students can give correct solutions in a reasonable time frame. Students must understand the logic of the strategy, recognize when it is appropriate, and explain the strategy. The amount of time spent on each strategy should be determined by the students' abilities and previous experiences.

The reinforcement activities for a strategy should be varied in type and should focus as much on the discussion of how students obtained their answers as on the answers themselves. The reinforcement activities should be structured to insure maximum participation. At first, time frames should be generous and then narrowed as students internalize the strategy. Student participation should be monitored and their progress assessed in a variety of ways to help determine how long should be spent on a strategy.

After you are confident that most of the students have internalized the strategy, you need to help them integrate it with other strategies they have developed. You can do this by providing activities that includes a mix of number expressions, for which this strategy and others would apply. You should have the students complete the activities and discuss the strategy/strategies that could be used; or you should have students match the number expressions included in the activity to a list of strategies, and discuss the attributes of the number expressions that prompted them to make the matches.

Language

Students should hear and see you use a variety of language associated with each operation, so they do not develop a single word-operation association. Through rich language usage students are able to quickly determine which operation and strategy they should employ. For example, when a student hears you say, "Six plus five", "Six and five", "The total of six and five", "The sum of six and five", or "Five more than six", they should be able to quickly determine that they must add 6 and 5, and that an appropriate strategy to do this is the *double-plus-one* strategy.

Context

You should present students with a variety of contexts for each operation in some of the reinforcement activities, so they are able to transfer the use of operations and strategies to situations found in their daily lives. By using contexts such as measurement, money, and food, the numbers become more real to the students. Contexts also provide you with opportunities to have students recall and apply other common knowledge that should be well known. For example, when a student hears you say, "How many days in two weeks?" they should be able to recall that there are seven days in a week and that double seven is14 days.

Number Patterns

You can also use the recognition and extension of number patterns can to reinforce strategy development. For example, when a student is asked to extend the pattern "30, 60, 120, …,", one possible extension is to double the previous term to get 240, 480, 960. Another possible extension, found by adding multiples of 30, would be 210, 330, 480. Both possibilities require students to mentally calculate numbers using a variety of strategies. This may also include open-frame questions as appropriate.

Examples of Reinforcement Activities

Reinforcement activities will be included with each strategy in order to provide you with a variety of examples. These are not intended to be exhaustive; rather, they are meant to clarify how language, contexts, common knowledge, and number patterns can provide novelty and variety as students engage in strategy development.

C. Assessment

Your assessments of computational strategies should take a variety of forms. In addition to the traditional quizzes that involve students recording answers to questions that you give one-at-a-time in a certain time frame, you should also record any observations you make during the reinforcements, ask the students for oral responses and explanations, and have them explain strategies in writing. Individual interviews can provide you with many insights into a student's thinking, especially in situations where pencil-and-paper responses are weak.

Assessments, regardless of their form, should shed light on students' abilities to compute efficiently and accurately, to select appropriate strategies, and to explain their thinking.

Response Time

Response time is an effective way for you to see if students can use the computational strategies efficiently and to determine if students have automaticity of their facts.

For the facts, your goal is to get a response in 3-seconds or less. You would certainly give students more time than this in the initial strategy reinforcement activities, and reduce the time as the students become more proficient applying the strategy until the 3-second goal is reached. In subsequent grades, when the facts are extended to 10s, 100s and 1000s, you should also ultimately expect a 3-second response.

In the early grades, the 3-second response goal is a guideline for you and does not need to be shared with your students if it will cause undue anxiety.

With mental calculation strategies and computational estimation, you should allow 5 to 10 seconds, depending upon the complexity of the mental activity required. Again, in the initial application of these strategies, you would allow as much time as needed to insure success and gradually decrease the wait time until students attain solutions in a reasonable time frame.

A. Addition — Mental Calculation

Addition Facts Applied to Multiples of Powers of 10 (Extension)

Knowledge of all single-digit addition facts within a 3-second response time was an expectation in mental math in grade 2. These facts were applied to 10s and 100s in grade 3 and to 1000s in grade 4. In grade 5, these facts and applications should have been reviewed and extended to tens of thousands and to tenths. In grade 6, these facts will be further applied to tenth, extended to hundredths, and extended to very large numbers such as 2 million or 0.8 billion.

The strategies for 88 of the 100 facts involving single-digit addends are:

a) Doubles Facts

b) Plus-One Facts

c) Near-Doubles (1-Aparts) Facts

d) Plus-Two Facts

e) Plus Zero Facts

f) Make-10 Facts

There are a variety of strategies that can be used for the last 12 facts. Further information about the fact learning strategies can be found in the mental math documents for grades 2 or 3.

Examples

For 40 + 60, think: If 10 from the 60 is given to the 40, the question becomes 50 + 50, or 10.

For 100, 300, 500, 700, ______ , think: Each number is 200 more than the number before, so the next number is 700 + 200 = 900.

For 4000 + 5000, think: 4000 and 4000 is 8000, so 1000 more is 9000; or think: 4 and 5 is 9, but these are thousands, so the answer is 9000.

For 0.07 + 0.05, think: If 1-hundredth from 0.07 is moved to 0.05, the question becomes 0.06 + 0.06, or 0.12; or think: 7-hundredths plus 5-hundredths is 12-hundredths which is 12 hundredths (0.12).

Examples of Some Practice Items

a) Some practice items for numbers in the 10s, 100s, and 1000s:

- 90 + 60
- 80 increased by 30
- 600 girls and 600 boys. How many children?
- $5 000 + $9 000

b) Some practice items for numbers in the 10 000s:

- 10 000, 40 000, 70 000, ________
- 20 000 + 30 000

c) Some practice items for numbers in the tenths and hundredths:

- 0.6 + 0.3
- 0.5 kg plus 0.7 kg
- 0.04 m increased by 0.08 m

The sum of 0.09 and 0.06

Examples of Some Practice Items (Patterns)

Students can apply patterns with addition as well as other operations in the form of Function Tables.

Input	Output
0	4
1	7
2	10
3	

(Add 4)
(Add 6)
(Add 8)

Front End Addition (Extension)

This strategy is applied to questions that involve two combinations of non-zero digits, one combination of which may require regrouping. The strategy involves first adding the digits in the highest place-value position, then adding the non-zero digits in another place-value position, and making any needed regrouping. After a review of this strategy applied to 2-digit and 3-digit whole numbers, it should be extended in grade 5 to 4-digit numbers including numbers in tens of thousands , and large numbers such as millions or billions. In grade 6, this strategy should also be extended to tenths and hundredths.

Examples

For 26 + 37, think: 20 plus 30 is 50, 6 plus 7 is 13, and 50 plus 13 is 63.

For 307 + 206, think: 300 plus 200 is 500, 7 plus 6 is 13, and 500 plus 13 is 513.

For 3 600 + 2 500, think: 3 thousand plus 2 thousand is 5 thousand, 6 hundred and 5 hundred is 11 hundred, and 5 thousand and 11 hundred is 6 100.

For 25 000 + 38 000, think: 20 thousand plus 30 thousand is 50 thousand, 5 thousand plus 8 thousand is 13 thousand, and 50 thousand plus 13 thousand is 63 thousand (63 000).

For 7.2 + 2.6, think: 7 plus 2 is 9 and 2-tenths plus 6-tenths is 8-tenths, so the answer is 9 and 8-tenths (9.8).

For 5.06 + 3.09, think: 5 and 3 is 8, 6-hundredths and 9 hundredths is 15-hundredths, and 8 and 15-hundredths is 8.15.

For 5.8 million + 2.5 million, think: 5 and 2 is 7, 8-tenths and 5-tenths is 13-tenths, and 7 and 13-tenths is 8 and 3-tenths million (8.3 million).

Examples of Some Practice Items

a) Some practice items for numbers in the 10s and 100s:

- 45 + 36
- 18 kg more than 56 kg
- 102 more than 567
- $660 + $270

b) Some practice items for numbers in the 1000s and 10 000s:

- 3 400 km and 5 800 km
- The sum of 2 040 and 6 090
- 56 000 females and 47 000 males. What is the total?
- $60 080 increased by $10 090

c) Some practice items for numbers involving tenths and hundredths:

- 3.5 m and 2.4 m
- 4.3 kg more than 7.8 kg
- 7.5 km increased by 2.9 km
- The sum of $0.12 and $0.09
-

Quick Addition — No Regrouping (Extension)

This strategy is actually the *Front-End* strategy applied to questions that involve more than two combinations with no regrouping. The questions are always presented visually and students quickly record their answers on paper. While it could be argued that this is a pencil-and-paper strategy because answers will always be recorded on paper before answers are read, it is included here as a mental math strategy because most students will do all the combinations in their heads starting at the front end.

This strategy requires students to holistically examine each question to confirm there will be no regrouping. This habit of holistically examining each question as a first step in determining the most efficient strategy needs to pervade all mental math lessons. (One suggestion for an activity during the discussion of this strategy is to present students with a list of 20 questions, some of which do require regrouping, and direct students to apply quick addition to the appropriate questions and leave out the other ones.) It is important to present examples of these addition questions in both horizontal and vertical formats. Students should have applied this strategy to 3-digit, 4-digit, and 5-digit numbers up to the end of grade 5, so in grade 6 they should apply it to large numbers and to tenths and hundredths as well.

Most likely, students will add the digits in corresponding place values of the two addends without consciously thinking about the names of the place values. Therefore, in the discussion of the questions, you should encourage students to read the numbers correctly and to use place-value names. This will reinforce place value concepts at the same time as addition.

Examples

For 543 + 256, think and record each resultant digit: 5 and 2 is 7, 4 and 5 is 9, and 3 and 6 is 9, so the answer is 799 (seven hundred ninety-nine); or think: 500 and 200 is 700, 40 and 50 is **9**0, 3 and 6 is **9** to get 799.

For 2 341 increased by 3 415, think and record each resultant digit: 2 and 3 is 5, 3 and 4 is 7, 4 and 1 is 5, and 1 and 5 is 6, so the answer is 5 756 (five thousand, seven hundred fifty-six); or think: 2 000 and 3 000 is **5** 000, 300 and 400 is 700, 40 and 10 is **5**0, 1 and 5 is **6** to get 5 756.

For $23 451 + $41 426, think and record each resultant digit: 2 and 4 is 6, 3 and 1 is 4, 4 and 4 is 8, 5 and 2 is 7, and 1 and 6 is 7, so the answer is $64 877 (sixty-four thousand, eight hundred seventy-seven); or think: 20 000 and 40 000 is **6**0 000, 3 000 and 1 000 is **4** 000, 400 and 400 is **8**00, 50 and 20 is 70, 1 and 6 is **7** to get $64 877.

For 34.32 + 23.57, think and record each resultant digit: 3 and 2 is 5, 4 and 3 is 7, 3 and 5 is 8, and 2 and 7 is 9, so the answer is 57.89 (fifty-seven and eighty-nine hundredths); or think: 30 and 20 is **5**0, 4 and 3 is **7**, 3-tenths and 5-tenths is **8**-tenths, 2-hundredths and 7-hunredths is **9**-hundredths to get 57.89.

Examples of Some Practice Items

a) Some practice items for numbers in the 100s and 1000s:

- The sum of 291 and 703
- $\begin{array}{r} 537 \\ \underline{+\ 341} \end{array}$
- There were 333 girls and 144 boys at the concert. What was the total attendance?

- $4 532 + $2 367
- 8107 people in town. 1742 people on the outskirts. What is the total population?
- 372 more than 5 116

b) Some practice items for numbers in the 10 000s and large numbers:

- 10 357
  <u>+ 42 111</u>
- 34 680 + 21 318
- The sum of $12 045 and $36 920
- Population of 2.4 billion increased by 3.5 billion.

c) Some practice items for numbers involving tenths and hundredths:

- 3.5 m and 2.7 m
- The sum of 4.6 and 3.9
- Fred counted $0.75 in one pocket and $0.18 in the other. How much money does Fred have in his pockets?
- 5.05 km more than 7.09 km
- 45.5 km + 12.3 km
- 235.6 m increased by 22.2 m
- $456.17
  <u>+ $502.62</u>
- 23.08 more than 534.71

Finding Compatibles (Extension)

This strategy for addition involves looking for pairs of numbers that combine easily to make a sum that is a power of ten that will be easy to work with. In grade 5, this should involve searching for pairs of numbers that add to 10 000, as well as the other powers of ten (10, 100 and 1000) that were the focus in previous grades. Some examples of common compatible numbers are 1 000 and 9 000, 4 000 and 6 000, 3 000 and 7 000, and 7 500 and 2 500. (In some resources, these compatible numbers are referred to as *friendly* numbers or *nice* numbers.) You should be sure that students are convinced that the numbers in an addition expression can be combined in any order (the associative property of addition).

Example:

For 1 + 7 + 9 + 8 + 3, think: 1 + 9 is 10 and 7 + 3 is 10, so 10 + 10 + 8 is 28.

For 30 + 75 + 70 + 25, think: 30 + 70 is 100 and 75 + 25 is 100, so 100 + 100 is 200.

For 300 + 800 + 700 + 600 + 200, think: 300 + 700 is 1000, 800 + 200 is 1000, so 1000 + 1000 + 600 is 2600.

For 250 + 470 + 750, think: 250 and 750 is 1000, so 1000 and 470 is 1470.

For 4 000 + 5 000 + 6 000, think: 4 000 and 6 000 is 10 000, so 10 000 and 5 000 is 15 000.

For 9 500 + 2 200 + 500, think: 9 500 and 500 is 10 000, so 10 000 plus 2 200 is 12 200.

For 0.4 + 0.3 + 0.6, think: 4-tenths and 6-tenths is 1, so 1 and 3-tenths is 1.3.

Examples of Some Practice Items

a) Some practice items for numbers in the 10s and 100s:

- 60 + 30 + 40 + 70
- The total of three items costing $75, $95, and $25.
- The sum of 200, 700, 500, 800, and 300.
- The total of three deposits: $50, $460, $950.

b) Some practice items for numbers in the 1000s:

- 5 000 + 3 000 + 5 000 + 7 000
- $2 500 and $3 500 and $7 500.
- 8 000 km + 4 000 km + 6 000 + 7 000 km + 2 000 km
- Total of three items: $1 000, $5 000, $9 000.

c) Some practice items for numbers involving tenths and hundredths:

- 0.2 + 0.4 + 0.3 + 0.8 + 0.6
- 6-tenths + 9-tenths + 4-tenths + 1-tenth
- The sum of three lengths: 0.09 m, 0.13 m, 0.01 m
- Gina has $0.50 and Joe has $0.75. Jill has $0.25 more than their total. How much money does Jill have?
-

Break Up and Bridge (Extension)

This strategy involves starting with the first number in its entirety and adding the place values of the second number, one-at-a-time, starting with the largest value. In grade 5, the practice items should include numbers in the thousands, in the tenths, and in the hundredths, as well as numbers in the tens, hundreds, and thousands that were the goals of previous grades. Remember that the practice items should only include questions that require two combinations with one regrouping.

Examples

For 45 + 36, think: 45 and 30 (from the 36) is 75, and 75 plus 6 (the rest of the 36) is 81. In symbols: 45 + 36 = (45 + 30) = 6 = 75 + 6 = 81.

For 537 + 208, think: 537 and 200 is 737, and 737 plus 8 is 745. In symbols: 537 + 208 = (537 + 200) + 8 = 737 + 8 = 745.

For 5 300 + 2 800, think: 5 300 and 2 000 (from the 2 800) is 7 300 and 7 300 plus 800 (the rest of 2 800) is 8 100. In symbols: 5 300 + 2 800 = (5 300 + 2 000) + 800 = 7 300 + 800 = 8 100.

For 34 000 + 27 000, think: 34 000 plus 20 000 is 54 000, and 54 000 plus 7 000 is 61 000. In symbols: 34 000 + 27 000 = (34 000 + 20 000) = 7 0000 = 54 000 + 7 000 = 61 000.

For two items costing $3.60 and $5.70, think: $3.60 and $5 (from the $5.70) is $8.60, and $8.60 plus $0.70 (the rest of $5.70) is $9.30. In symbols: $3.60 + $5.70 = ($3.60 + $5.00) + $0.70 = $8.60 + $0.70 = $9.30.

Examples of Some Practice Items

a) Some practice items for numbers in the 10s and 100s:

- 46 + 36
- 17 more than 64
- The sum of $370 and $440
- 365 increased by 109

b) Some practice items for numbers in the 1000s:

- 2 500 + 3 700
- The sum of 16 800 km and 1 300 km
- The total of 4 070 girls and 3 080 boys
- 7 009 increased by 2 008

c) Some practice items for numbers in the 10 000s:

- 46 000 + 37 000
- The total of $66 000 and $15 000
- 56 000 increased by 24 000
- 17 000 km more than 28 000 km
-

d) Some practice items for numbers involving tenths and hundredths:

- 4.7 m + 3.5 m
- The total of 15.6 km and 10.7 km
- 12.5 kg increased by 5.6 kg
- $1.65 + $2.20
- The sum of 4.06 m and 3.07 m
- 4.56 kg more than 4.40 kg
-

Compensation (Extension)

This strategy involves changing one number in the addition question to a nearby multiple of a power of ten, carrying out the addition using that multiple of a power of ten, and adjusting the answer to compensate for the original change. Students should understand that the number is changed to make it more compatible, and that they have to hold in their memories the amount of the change. In the last step, it is helpful if they remind themselves that they added too much so they will have to take away that amount.

This strategy is perhaps most effective when one of the addends has an 8 or 9 in its lowest place value, although some students are comfortable using it with a 7 as well. In grade 5, the practice items should include numbers in the thousands and tens of thousands.

Examples

For 52 + 39, think: 40 is easier to work with than 39. Then 52 plus 40 is 92, but I added 1 too many; so, to compensate I subtract one from my answer, 92, to get 91.

For 345 + 198, think: 200 is easier to work with than 198. Then 345 + 200 is 545, but I added 2 too many; so, I subtract 2 from 545 to get 543.

For 4500 + 1900, think: 2000 is easier to work with than 1900. Then 4500 + 2000 is 6500, but I added 100 too many; so, I subtract 100 from 6500 to get 6400.

For 34 000 + 9 900, think: 10 000 is easier to work with than 9 900. Then 34 000 plus 10 000 is 44 000, but I added 100 too many; so, I subtract 100 to get 33 900.

For 59 000 + 25 000, think: 60 000 plus 25 000 is 85 000, but I added 1000 too many; so, I subtract 1000 to get 84 000.

For 4.6 + 1.8, think: 4.6 plus 2 is 6.6, but I added 2-tenths too much; so, I subtract 2-tenths from 6.6 to get 6.4 (six and 4-tenths).

For 0.54 plus 0.29, think: 54-hundredths + 30-hundredths is 84-hundredths, but I added 1-hundredth too much; so, I subtract 1-hundredth from 84-hundredths to compensate to get 83-hundredths, or 0.83.

Examples of Some Practice Items

a) Some practice items for numbers in the 10s and 100s:

- 58 + 9
- 49 + 38
- 265 + 399
- $198 more than $465

b) Some practice items for numbers in the 1000s and 10 000s:

- 3 456 km increased by 999 km
- The sum of 2 998 and 3 525

- 16 000 + 39 000
- The sum of 28 000 and 65 000
- The total of $38 000 and $9 900

74 000 km increased by 18 000

c) Some practice items for numbers involving tenths and hundredths:

- 3.9 m + 2.5 m
- 3.5 km more than 4.8 km
- $0.36 + $0.39
- $2.47 more than $4.99

Make Multiples of Powers of Ten (Extension)

In previous grades, students would have been introduced to this strategy as *Make 10*, *Make 10s*, and *Make 10s, 100s, 1000s,* and *10 000s*. In grade 6, this strategy is extended to *Make 1* and *Make 1s*.

Like the *Compensation* strategy, this strategy is best applied when one of the addends has an 8 or 9 in its lowest place value, and it makes use of the compatibility of multiples of powers of ten in addition. This strategy, however, involves getting the amount needed to make one addend a multiple of a power of ten from the other addend, thus changing both addends to numbers that are easier to combine. A common error is for students to forget that both addends have changed; this means that more has to be kept in their short-term memories. Therefore, questions used for reinforcement should not involve too many non-zero digits.

The *Compensation* and the *Make-Multiples-of-Powers-of-Ten* strategies should be compared so students are clear about how they are alike and how they are different because both strategies are appropriately applied to the same questions.

Examples

For 92 + 69, think: If 1 is taken from 92 and given to 69, the question becomes 91 + 70, which is easier to add to get 161.

For 298 + 345, think: If 2 is taken from 345 and given to 298, the question becomes 300 + 343, which is easier to add to get 643.

For 650 + 190, think: If 10 is taken from 650 and given to 190, the question becomes 640 + 200, which is easier to add to get 840.

For 34 000 + 28 000, think: If 2 000 is taken from the first addend and given to the second addend, the question becomes 32 000 + 30 000, which is easier to add to get 62 000.

For 56 700 + 3 900, think: If 100 is taken from the first addend and given to the second addend, the question becomes 56 600 + 4 000, which is easier to add to get 60 600.

For 1.3 + 0.9, think: If 1-tenth is taken from the first addend and given to the second addend, the question becomes 1.2 + 1, which is easier to add to get 2.2.

For 1.4 + 2.9, think: If 1-tenth is taken from the first addend and given to the second addend, the question becomes 1.3 + 3, which is easier to add to get 4.3.

For 3.98 + 4.24, think: If 2-hundredths is taken from the second addend and given to the first addend, the question becomes 4 + 4.22, which is easier to add to get 8.22.

Examples of Some Practice Items

a) Some practice items for numbers in the 10s and 100s:

- 45 + 29
- $298 more than $465

b) Some practice items for numbers in the 1000s and 10 000s:

- 6 476 increased by 999
- The sum of 18 000 and 46 000
- The total of 78 200 km and 9 900 km
- $56 000 increased by $18 000

c) Some practice items for numbers involving tenths and hundredths:

- 1.9 m + 2.6 m
- 4.5 km more than 5.8 km
- $0.25 + $0.59
- $3.56 more than $2.99

B. Subtraction — Mental Calculation

Applying Subtraction Facts to Multiples of Powers of 10 (Extension)

This strategy applies to calculations involving the subtraction of two numbers with the same place values and with only one non-zero digit. Students applied this strategy in previous grades to numbers in the tens, hundreds, and thousands. In grade 5, the application of this strategy should be extended to numbers in tens of thousands and in tenths. The strategy involves subtracting the single non-zero digits as if they were the single-digit subtraction facts and then attaching the appropriate place-value name and symbols. For tens of thousands, however, some students may prefer to subtract the tens parts of the tens of thousands (see example 4 below). This strategy should be reviewed and modeled with base-10 blocks so students understand that 7 blocks subtract 3 blocks will be 4 blocks whether those blocks are small cubes, rods, flats, or large cubes.

Since this strategy rests on students' knowledge of subtraction facts, the facts should be reviewed and consolidated. If some students need remediation on the subtraction facts, information can be found in the document for mental math in grade 3. The principal strategy advocated for these facts is the *Think-Addition* strategy, although the *Back-Through-10* strategy and the *Up-Through-10* strategy are also helpful when the minuends are greater than 10.

Examples

For 80 – 30, think: 8 tens subtract 3 tens is 5 tens, or 50; or think: 8 subtract 3 is 5, but this is 5 tens, so the answer is 50.

For 1500 – 600, think: 15 hundreds subtract 6 hundreds is 9 hundreds, or 900; or think: 15 subtract 6 is 9, but this is 9 hundreds, so the answer is 900.

For 6 000 – 2 000, think: 6 thousands subtract 2 thousands is 4 thousands, or 4 000; or think: 6 subtract is 4, but this is 4 thousands, so the answer is 4 000.

For 90 000 – 40 000, think: 9 subtract 4 is 5, but this is tens of thousands, so the answer is 50 000; or think: 90 thousand subtract 40 thousand is 50 thousand or 50 000.

For 0.8 – 0.5, think: 8-tenths subtract 5-tenths is 3-tenths, or 0.3; or think: 8 subtract 5 is 3, but this is tenths, so the answer is 0.3.

For 1.4 – 0.7, think: 14-tenths – 7-tenths is 7-tenths, or 0.7; or think: 14 subtract 7 is 7, but this is tenths, so the answer is 0.7.

For 0.17 – 0.09, think: 17-hundredths subtract 9-hundredths is 8-hundredths, or 0.08; or think: 17 subtract 9 is 8, but this is hundredths, so the answer is 0.08.

Examples of Some Practice Items

a) Some practice items for numbers in the 10s, 100s, and 1000s:

- 120 – 70
- $20 less than $90
- 700 kg decreased by 300 kg
- The difference between 1100 km and 400 km
- 6000 minus 1000
- $13 000 less $6000

b) Some practice items for numbers in the 10 000s:

- 40 000 – 10 000
- 80 000 minus 20 000
- The difference between $90 000 and $50 000
- 120 000 km decreased by 30 000

c) Some practice items for numbers involving tenths and hundredths:

- 0.7 kg – 0.2 kg
- The difference between 1.5 km and 0.6 km
- 0.5 m less than 0.8 m
- 1.6 kg decreased by 0.9 kg
- 0.05 m less than 0.08 m
- 0.16 kg decreased by 0.09 kg
-

Quick Subtraction (Extension)

This strategy is actually the *Front-End* strategy applied to subtraction questions that involve no regrouping. If questions only require two subtractions to get an answer, students should be able to do them mentally. However, questions involving three, or more, subtractions should be presented visually with students quickly recording their answers on paper. While it could be argued that this is a pencil-and-paper strategy for these questions because answers will always be recorded on paper before answers are read, it is included here as a mental math strategy because most students will do all the subtractions in their heads starting at the front end.

This strategy requires students to holistically examine each question to confirm there will be no regrouping. This habit of holistically examining each question as a first step in determining the most efficient strategy needs to pervade all mental math lessons. (One suggestion for an activity during the discussion of this strategy is to present students with a list of 20 questions, some of which do require regrouping, and direct students to apply quick subtraction to the appropriate questions and leave out the other ones.) It is important to present examples of these subtraction questions in both horizontal and vertical formats. Students should have applied this strategy to 3-digit and 4-digit numbers up to the end of grade 4, so in grade 5 they should apply it to 5-digit numbers as well. The numbers should include decimal examples as well as whole number examples.

Most likely, students will subtract the digits in corresponding place values of the minuend and subtrahend without consciously thinking about the names of the place values. Therefore, in the discussion of the questions, you should encourage students to read the numbers correctly and to use place-value names. This will reinforce place value concepts at the same time as subtraction is reinforced.

Examples

For 560 – 120, think: 500 – 100 is 400 and 60 – 20 is 40, so the answer is 440. (Record the answer if required.)

For 568 – 135, think and record each difference: Subtract 100 from 500, 30 from 60, and 5 from 8 to get 433; or think and record each resultant digit: 5 – 1 = 4, 6 – 3 = 3, 8 – 5 = 3, so the answer is 433 (four hundred thirty-three).

For 4 070 – 3 030, think: 4 000 – 3000 is 1 000 and 70 – 30 is 40, so the answer is 1 040. (Record answer if required.)

For 4568 – 1135, think and record each difference: Subtract 1000 from 4000, 100 from 500, 30 from 60, and 5 from 8 to get 3433; or think and record each resultant digit: 4 – 1 = 3, 5 – 1 = 4, 6 – 3 = 3, 8 – 5 = 3, so the answer is 3433 (three thousand, thirty-three).

For 87 000 – 32 000, think: 80 000 – 30 000 is 50 000 and 7 000 – 2 000 is 5 000 so the answer is 55 000. (Record if required.)

For 25 786 – 12 125, think and record each subtraction: Subtract 10 000 from 20 000, 2000 from 5000, 100 from 700, 20 from 80, and 5 from 6 to get 13 661; or think and record each difference: 2 – 1 = **1**, 5 – 2 = **3**, 7 – 1 = **6**, 8 – 2 = **6**, and 6 – 5 = **1**, so the answer is 13 661(thirteen thousand, six hundred sixty-six).

For 345.84 – 112.42, think and record each subtraction: Subtract 100 from 300, 10 from 40, 2 from 5, 4-tenths from 8 tenths, and 2-hundredths from 4-hundredths to get 233.42; or think and record each digit: 3 – 1 = **2**, 4 – 1 = **3**, 5 – 2 = **3**, 8 – 4 = **4**, and 4 – 2 = **2**, so the answer is 233.42 (two hundred thirty-three and forty-two hundredths).

Examples of Some Practice Items

a) Some practice items that can be done mentally with recording an option:

- 56 – 21

- 604 – 203
- 590 – 230
- 6 700 – 1 100
- 4 080 – 1 020
- 14 000 – 2 000
- 38 000 – 1 500

b) Some practice items for numbers in the 100s, and 1000s:

- $\begin{array}{r} 537 \\ \underline{-101} \end{array}$
- 304 fewer people than 8605 people
- \$3245 less than \$7366
- The difference between 1225 km and 3575 km
- Subtract 575 from 3889

c) Some practice items for numbers in the 10 000s:

- 45 678 – 21 543
- $\begin{array}{r} 83\ 419 \\ \underline{-\ 21\ 417} \end{array}$
- The difference between \$96 475 and \$5 125
- 75 575 km decreased by 31 235

d) Some practice items for numbers involving tenths and hundredths:

- 213.7 kg – 101.2 kg
- The difference between 456.9 km and 45.6 km
- 45.12 m less than 57.75 m
- $\begin{array}{r} 575.86 \\ \underline{-\ 125.36} \end{array}$

Back Through a Multiple of a Power of Ten (Extension)

This strategy involves subtracting a part of the subtrahend to get to the nearest multiple of a power of ten, and then subtracting the rest of the subtrahend. This strategy is most effective when the subtrahend is relatively small compared to the minuend.

In previous grades, students would have been introduced to this strategy as *Back Through 10* and *Back Through 10s/100s*. In grade 5, the strategy is extended to going back through thousands and tens of thousands and the strategy name reflects the more generalized application.

Example

For 35 – 8, think: 35 subtract 5 (one part of the 8) is 30, and 30 subtract 3 (the other part of the 8) is 27.

For 530 – 70, think: 530 subtract 30 (one part of the 70) is 500, and 500 subtract 40 (the other part of the 70) is 460.

For example: For 8 600 – 700, think: 8 600 subtract 600 (one part of the 700) is 8 000 and 8 000 subtract 100 (the rest of the 700) is 7 900.

For 74 000 – 9 000, think: 74 000 subtract 4 000 (one part of the 9 000) is 70 000, and 70 000 subtract 5 000 (the rest of the 9 000) is 65 000.

For 4.5 – 0.9, think: 4.5 – 0.5 (one part of 0.9) is 4, and 4 subtract 0.4 (the other part of 0.9) is 3.6.

For 1.63 – 0.07, think: 1.63 subtract 0.03 (one part of 0.07) is 1.6, and 1.6 subtract 0.04 (the other part of 0.07) is 1.56.

Examples of Some Practice Items

a) Some practice items for numbers in the 10s, 100s, and 1000s:

- $$\begin{array}{r} 57 \\ -\underline{\ \ 8} \end{array}$$
- 9 fewer people than 92 people
- $40 less than $210
- The difference between 630 km and 80 km
- Subtract 600 from 2 300
- 7 500 less 700

b) Some practice items for numbers in the 10 000s:

- 45 000 – 8 000
- 83 400 minus 600
- The difference between $42 000 and $7 000
- 33 000 km decreased by 5 000 km

c) Some practice items for numbers involving tenths and hundredths:

- 13.2 kg – 0.7 kg
- The difference between 23.5 km and 0.8 km
- 0.06 m less than 1.21 m
- $2.53 – $0.07
-

Up Through a Multiple of a Power of Ten (Extension)

This strategy involves finding the difference between the two numbers in two steps starting from the smaller: first, find the difference between the subtrahend and the next multiple of a power of ten, then find the difference between that multiple of a power of ten and the minuend, and finally add these two differences to get the total difference. This strategy is particularly effective when the two numbers involved are quite close together, although in making change in money situations, this is the principal strategy that traditionally has been used, regardless of the difference. For example, to get the change from a $20-bill for an item that costs $6.95, you select a nickel to get to $7, a $1 coin and a $2 coin to get to $10, and a $10-bill to get to $20.

In previous grades, students would have been introduced to this strategy as *Up Through 10* and *Up Through 10s/100s*. In grade 5, because the strategy is extended to going up through thousands, tens of thousands, ones, and tenths, the strategy name reflects the more generalized application.

Examples

For example: For 84 – 77, think: It is 3 from 77 to 80 and 4 from 80 to 84, so the total difference is 3 plus 4, or 7.

For 613 – 594, think: It is 6 from 594 to 600 and 13 from 600 to 613, so the total difference is 6 plus 13, or 19.

For 2 310 – 1 800, think: It is 200 from 1 800 to 2 000 and 310 from 2 000 to 2 310, so the total difference is 200 plus 310, or 510.

For 57 000 – 49 000, think: It is 1 000 from 49 000 to 50 000 and 7 000 from 50 000 to 57 000, so the total difference is 1 000 + 7 000, or 8 000.

For 12.4 – 11.8, think: It is 2-tenths from 11.8 to 12 and 4-tenths from 12 to 12.4, so the total difference is 2-tenths plus 4-tenths, or 0.6.

For 6.12 – 5.99, think: It is 1-hundredth from 5.99 to 6.00 and 12-hundredths from 6.00 to 6.12, so the total difference is 1-hundredth plus 12-hundredths, or 0.13.

For 12.54 – 12.48, think: It is 2-hundredths from 12.48 to 12.5 and 4-hundredths from 12.5 to 12.54, so the total difference is 2-hundredths + 4-hundredths, or 0.06.

Examples of Some Practice Items

a) Some practice items for numbers in the 10s, 100s, and 1000s:

- $\begin{array}{r} 57 \\ -\underline{\ 48} \end{array}$
- 92 – 86
- $140 less than $210
- The difference between 630 km and 580 km
- 2 400 minus 1 700
- 8 500 decreased by 7 800

b) Some practice items for numbers in the 10 000s:

- 45 000 – 38 000
- 83 000 less 79 000
- The difference between $42 000 and $35 000
- 35 000 km subtract 26 000 km

c) Some practice items for numbers involving tenths and hundredths:

- 13.2 kg – 12.7 kg
- The difference between 23.5 km and 22.8 km
- 1.99 m less than 2.21 m
- $2.53 – $2.45

Break Up and Bridge

This strategy involves starting with the minuend in its entirety and subtracting the values in the place values of the subtrahend, one-at-a-time, starting with the largest. If students were modelling subtraction on a number line, they would probably naturally use this strategy.

Examples

For 92 – 26, think: Start with 92 and subtract 20 (the tens place of 26) to get 72, and then subtract 6 (the ones place in 26) from 72 to get 66.

For 745 – 207, think: Start with 745 and subtract 200 (the hundreds place in 207) to get 545, and then subtract 7 (the ones place in 207) from 545 to get 538.

For 860 – 370, think: Start with 860 and subtract 300 (the hundreds place in 370) to get 560, and then subtract 70 (the tens place in 370) from 560 to get 490. (*Likely a* Back-Through-100s *strategy in the last step.)*

For 8 300 – 2 400, think: Start with 8 300 and subtract 2 000 to get 6 300, and then subtract 400 from 6 300 to get 5 900. (*Likely a* Back-Through-1000s *strategy in the last step.)*

For 5 750 – 680, think: Start with 5 750 and subtract 600 to get 5 150, and then subtract 80 from 5 150 to get 5 070. (*Likely a* Back-Through-100 *strategy in the last step.)*

For 47 000 – 28 000, think: Start with 47 000 and subtract 20 000 to get 27 000, and then subtract 8 000 from 27 000 to get 19 000. (*Likely a* Back-Through-10 000s *strategy in the last step.)*

For 24 500 – 2 700, think: Start with 24 500 and subtract 2 000 to get 22 500, and then subtract 700 from 22 500 to get 21 800. (*Likely a* Back-Through-100s *strategy in the last step.)*

Examples of Some Practice Items

a) Some practice items for numbers in the 10s and 100s:

- $\begin{array}{r} 74 \\ -\underline{\ 36} \end{array}$
- 53 – 25
- $306 less than $870
- The difference between 640 km and 170 km
- 750 minus 260
- 803 decreased by 306

b) Some practice items for numbers in the 1000s:

- 5 400 – 1 500

- 7 100 less 2 600
- The difference between $8 020 and $3 050
- $\begin{array}{r} 6\ 425 \\ -\ \underline{\ \ 307} \end{array}$

c) Some practice items for numbers in the 10 000s:

- 63 000 – 25 000
- The difference between 66 500 km and 18 000 km
- $75 500 – $4 900
- 10 600 less than 32 100

Compensation (Extension)

This strategy for subtraction involves changing the subtrahend to the next multiple of a power of ten, carrying out the subtraction, and then adjusting the answer to compensate for the difference between the original subtrahend and the multiple of a power of ten that was used. Students should understand that the subtrahend is changed to make it more compatible, and that they have to hold in their memories the amount of that change. In the last step, it is helpful if they remind themselves that they subtracted too much, so they will have to add that amount back on. This strategy is most effective when the digit in the lowest non-zero place value is an 8 or a 9.

In grade 4, students applied this strategy to numbers in the tens and hundreds. In grade 5, this strategy should be extended to numbers in the thousands and tens of thousands.

Examples

` For 36 – 8, think: 36 – 10 = 26, but I subtracted 2 too many; so, I add 2 to 26 and get 28.

For 85 – 29, think: 85 – 30 = 55, but I subtracted 1 too many; so, I add 1 to 55 to get 56.

For 145 – 99, think: 145 – 100 = 45, but I subtracted 1 too many; so, I add 1 to 45 to get 46.

For 750 – 190, think: 750 – 200 = 550, but I subtracted 10 too many; so, I add 10 to 550 to get 560.

For 5 700 – 997, think: 5 700 – 1000 is 4 700, but I subtracted 3 too many; so, I add 3 to 4 700 to get 4 703.

For 3 600 – 990, think: 3 600 – 1000 is 2 600, but I subtracted 10 too many; so, I add 10 to 2 600 to get 2 610.

For 24 000 – 995, think: 24 000 – 1000 is 23 000, but I subtracted 5 too many; so, I add 5 to 23 000 to get 23 005.

For 56 000 – 980, think: 56 000 – 1000 is 55 000, but I subtracted 20 too many; so, I add 20 to 55 000 to get 55 020.

For 47 000 – 19 000, think: 47 000 – 20 000 is 27 000, but I subtracted 1000 too many; so, I add 1000 to 27 000 to get 28 000.

Examples of Some Practice Items

a) Some practice items for numbers in the 10s and 100s:

- $\begin{array}{r} 57 \\ -\underline{\ 29} \end{array}$
- 92 less 38
- $399 less than $875
- The difference between 630 km and 298 km
- 450 minus 190
- 830 decreased by 380

b) Some practice items for numbers in the 1000s:

- 5 700 – 997
- 4 500 less 1 990
- The difference between $7 500 and $2 900
- 6 500 km subtract 1 980 km

c) Some practice items for numbers in the 10 000s:

- 23 000 – 1 997
- The difference between 33 000 km and 2 980 km
- $64 000 – $9 900
- Subtract 29 000 from 92 000

Balancing For a Constant Difference (Extension)

In subtraction questions that require regrouping, this strategy can be used most effectively. By adding the same amount to both numbers in order to get the subtrahend to a ten, hundred, thousand, or ten thousand, any regrouping is eliminated, so the subtraction is much easier to do. This strategy needs to be carefully introduced because students need to be convinced it actually works! They need to understand that by adding the same amount to both numbers, the two new numbers have the same difference as the original two numbers. Examining possible numbers on a metre stick that are a fixed distance apart can help students with the logic of this strategy. (For example, place a highlighter that is more than 10 cm long against a metre stick so that its bottom end is at the 18-cm mark, note where its top end is located, and write the subtraction sentence that gives the length of the highlighter. Repeat by placing the bottom end of the highlighter at the 20-cm mark. Ask, Is the length of the highlighter the same in both number sentences? Which subtraction would be easier to do?)

This strategy was introduced in grade 4 and applied to tens and hundreds. In grade 5, the strategy should be extended to thousands and ten of thousands.

Note: Because both numbers change in carrying out this strategy, many students may need to record the changed minuend to keep track, especially for numbers greater than 2-digit. This strategy should be compared to the *Compensation* strategy so students see how it is alike and how it is different.

This strategy can lead to a very effective pencil-and-paper strategy for questions in which the minuends are multiples of powers of ten. These questions traditionally required subtracting with regrouping from one, or more, zeros; however, if 1 is subtracted from both numbers, the questions will require no regrouping. For example, for 4 000 – 3 467, if 1 is subtracted from both the minuend and the subtrahend, the question becomes 3 999 – 3 466 which is then much easier to subtract by *Quick Subtraction*. This strategy would not be part of mental math time; rather, it could be a discussion in math class time.

Examples

For 87 – 19, think: If 1 is added to both numbers, the question becomes 88 – 20 which is easy to subtract to get 68.

For 345 – 198, think: If 2 is added to both numbers, the question becomes 347 – 200 which is easy to subtract to get 147.

For 5 600 – 1 990, think: If 10 is added to both numbers, the question becomes 5 610 – 2000 which is easy to subtract to get 3 610.

For 7 800 – 3 998, think: If 2 is added to both numbers, the questions becomes 7 802 – 4 000 which is easy to subtract to get 3 802.

For 45 000 – 19 000, think: If 1 000 is added to both numbers, the question becomes 46 000 – 20 000 which is easy to subtract to get 26 000.

For 67 000 – 29 999, think: If 1 is added to both numbers, the question becomes 67 001 – 30 000 which is easy to subtract to get 37 001.

For 52 000 – 9 800, think: If 200 is added to both numbers, the question becomes 52 200 – 10 000 which is easy to subtract to get 42 200.

Examples of Some Practice Items

a) Some practice items for numbers in the 10s and 100s:

- $\begin{array}{r} 77 \\ -\underline{39} \end{array}$
- 53 – 28
- $399 less than $875
- The difference between 640 km and 198 km
- 750 minus 290
- 830 decreased by 380

b) Some practice items for numbers in the 1000s:

- 5 400 – 997
- 7 500 less 2 990
- The difference between $8 500 and $3 900
- $\begin{array}{r} 6\ 500 \\ -\underline{1\ 980} \end{array}$

c) Some practice items for numbers in the 10 000s:

- 43 000 – 2 997

- The difference between 66 000 km and 4 980 km
- $75 000 – $9 900
- Subtract 38 000 from 92 000

C. Multiplication and Division — Mental Calculation

Some of the following material is review from grade 5, but it is necessary to include it here to consolidate the understanding of multiplying by tenths, hundredths and thousandths; the related division by tens, hundreds and thousands; the reverse of multiplying by tens, hundreds and thousands; the related division by tenths, hundredths and thousandths.

Quick Multiplication – No Regrouping

Note: This pencil-and-paper strategy is used when there is no regrouping and the questions are presented visually instead of orally. It is included here as a mental math strategy because students will do all the combinations in their heads starting at the front end.

Examples

For example: For 52 × 3, simply record, starting at the front end, 150 + 6 = 156.

For example: Foe 423 × 2, simply record, starting at the front end, 800 + 40 + 6 = 846.

Some examples of practice items

Here are some practice items.

43 × 2 =

1.42 × 2 =

The perimeter of a square with a side of 4.2 cm

3 groups of 12.3

Quick Division – No Regrouping

Note: This pencil-and-paper strategy is used when there is no regrouping and the questions are presented visually instead of orally. It is included here as a mental math strategy because students will do all of the combinations in their heads starting at the front end.

Examples

For 640 ÷ 2, simply record, starting at the front end, 300 + 20 = 320.

For 1 290 ÷ 3, simply record, starting at the front end, 400 + 30 = 430.

Some examples of practice items

Here are some practice items.

360 ÷ 3 =

How many groups of 8 are there in 7280?

The length of a side on a square with a perimeter of 32.8 cm

Multiplying & Dividing by 10, 100 and 1000

Multiplication: This strategy involves keeping track of how the place values have changed.

Multiplying by 10 increases all of the place values of a number by one place. For 10 × 67, think: the 6 tens will increase to 6 hundreds and the 7 ones will increase to 7 tens; therefore, the answer is 670.

Multiplying by 100 increases all of the place values of a number by two places. For 100 × 86, think: the 8 tens will increase to 8 thousands and the 6 ones will increase to 6 hundreds; therefore, the answer is 8 600. It is necessary that students use the correct language when orally answering questions where they multiply by 100. For example, the answer to 100 × 86 should be read as 86 hundred and not 8 thousand 6 hundred.

Multiplying by1000 increases all the place values of a number by three places. For 1000 × 45, think: the 4 tens will increase to 40 thousands and the 5 ones will increase to 5 thousands; therefore, the answer is 45 000. It is necessary that students use the correct language when orally answering questions where they multiply by 1000. For example the answer to 1000 × 45 should be read as 45 thousand and not 4 ten thousands and 5 thousand.

Some examples of practice items

Some mixed practice items are:

$73 × 1 000 =

5m = ___ cm

4.5 × 10 =

4 tenths times one hundred

2.3, 23, 230, _____, _____, ______

Dividing by tenths (0.1), hundredths (0.01) and thousandths (0.001)

When students fully understand decimal tenths and hundredths, they will be able to use this knowledge in understanding multiplication and division by tenths and hundredths in mental math situations.

Multiplying by 10s, 100s and 1 000s, is similar to dividing by tenths, hundredths and thousandths.

1) Dividing by tenths increases all the place values of a number by one place.
2) Dividing by hundredths increases all the place values of a number by two places.

Examples

1) For 3 ÷ 0.1, think: the 3 ones will increase to 3 tens, therefore the answer is 30.
2) For 3 ÷ 0.01, think: the 3 ones will increase to 3 hundreds, therefore the answer is 300.

   For 0.4 ÷ 0.01, think: the 4 tenths will increase to 4 tens, therefore the answer is 40.

Some examples of practice items

1) Here are some practice items:

   5 ÷ 0.1 =

   42, 420, 4200, _____, _____, _____

   How many decimetres are there in 4 meters?

   Twenty three divided by one tenth

2) Here are some practice items:

   4 ÷ 0.01 =

   How many hundredths are there in 12?

   How many centimetres are there in 23 meters?

Dividing by Ten, Hundred and Thousand

Division: This strategy involves keeping track of how the place values have changed.

Dividing by 10 decreases all the place values of a number by one place.

Dividing by 100 decreases all the place values of a number by two places.

Dividing by 1000 decreases all the place values of a number by three places.

Examples

For 340 ÷ 10, think: the 3 hundreds will decrease to 3 tens and the 4 tens will decrease to 4 ones; therefore, the answer is 34.

For, 7 500 ÷ 100; think: the 7 thousands will decrease to 7 tens and the 5 hundreds will decrease to 5 ones; therefore, the answer is 75.

For, 63 000 ÷ 1000; think: the 6 ten thousands will decrease to 6 tens and the 3 thousands will decrease to 3 ones; therefore, the answer is 63.

Some examples of practice items

Here are some mixed practice items:

80 ÷ 10 =

72 000, 7200, 720, ____, ____, ____

Twenty two thousand divided by one hundred

How many kilograms are there in forty six thousand grams?

Division when the divisor is a multiple of 10 and the dividend is a multiple of the divisor.

Division by a power of ten should be understood to result in a uniform "shrinking" of hundreds, tens and units which could be demonstrated and visualized with base -10 blocks.

Example

For 400 ÷ 20, think: 400 shrinks to 40 and 40 divided by 2 is 20.

Some examples of practice items

Here are some practice items:

500 ÷ 50=

What is the length of a rectangle with an area of 600 cm^2 and a width of 20 cm?

Find the quotient when 690 000 is divided by thirty.

Division using the Think Multiplication strategy

This is a convenient strategy to use when dividing mentally. For example, when dividing 60 by 12, think: "What times 12 is 60?"

This could be used in combination with other strategies.

Example

For 920 ÷ 40, think: "20 groups of 40 would be 800, leaving 120, which is 3 more groups of 40 for a total of 23 groups.

Some examples of practice items

Some practice items are:

880 ÷ 40 =

How many groups of 70 are there in 1470?

Find the length of a rectangle with area of 240 cm^2 and a side width of 40 cm.

Multiplication and Division of tenths, hundredths and thousandths

Multiplying by tenths, hundredths and thousandths is similar to dividing by tens, hundreds and thousands.

This strategy involves keeping track of how the place values have changed.

1) Multiplying by 0.1 decreases all the place values of a number by one place.
2) Multiplying by 0.01 decreases all the place values of a number by two places.
3) Dividing by 100 decreases all the place values of a number by two places.
4) Multiplying by 0.001 decreases all the place values of a number by three places.
5) Dividing by 1000 decreases all the place values of a number by three places.

Example

1) For 5 × 0.1, think: the 5 ones will decrease to 5 tenths; therefore, the answer is 0.5.

   For, 0.4 × 0.1, Think: the 4 tenths will decrease to 4 hundredths, therefore the answer is 0.04.

2) For 5 × 0.01, think: the 5 ones will decrease to 5 hundredths, therefore the answer is 0.05.

   For, 0.4 × 0.01, think: the 4 tenths will decrease to 4 thousandths, therefore the answer is 0.004.

3) For, 7 500 ÷ 100; think: the 7 thousands will decrease to 7 tens and the 5 hundreds will decrease to 5 ones; therefore, the answer is 75. This is an opportunity to show the relationship between multiplying by one hundredth and dividing by 100.

4) For 5 × 0.001, think: the 5 ones will decrease to 5 thousandths; therefore, the answer is 0.005.

   For, 8 × 0.001, think: the 8 ones will decrease to 8 thousandths; therefore, the answer is 0.008.

5) For, 75 000 ÷ 1000; think: the 7 ten thousands will decrease to 7 tens and the 5 thousands will decrease to 5 ones; therefore, the answer is 75. This is an opportunity to show the relationship between multiplying by one thousandth and dividing by 1000.

Some examples of practice items

6 × 0.1 =

7 600 ÷ 1000 =

One hundredth of 12

Ninety eight thousand divided by one thousand

The product of one thousandth and three hundred twenty five

How many dollars are 23 000 pennies?

9 mm = _______ m

Compensation

This strategy for multiplication involves changing one of the factors to a ten, hundred or thousand; carrying out the multiplication; and then adjusting the answer to compensate for the change that was made. This strategy could be carried out when one of the factors is near ten, hundred or thousand.

Examples

For 6 × $4.98, think: 6 times 5 dollars less 6 × 2 cents, therefore $30 subtract $0.12 which is $29.88. The same strategy applies to decimals. This strategy works well with 8s and 9s.

For example: For 3.99 × 4, think: 4 × 4 is 16 subtract 4 × 0.01(0.04) which is 15.96.

Some examples of practice items

Here are some practice items:

How much will five CD's for $19.98 each cost?

$9.99 × 8 =

Find the perimeter of a square with each side measuring 4.99 cm each.

Halving and Doubling

This strategy involves halving one factor and doubling the other factor in order to get two new factors that are easier to calculate. While the factors have changed, the product is equivalent, because multiplying by one-half and then by 2 is equivalent to multiplying by 1, which is the multiplicative identity. Halving and doubling is a situation where students may need to record some sub-steps.

Examples

For example: For 42 × 50, think: one-half of 42 is 21 and 50 doubled is 100; therefore, 21 × 100 is 2 100.

For example: For 500 × 88, think: double 500 to get 1000 and one-half of 88 is 44; therefore, 1 000 × 44 is 44 000.

For example: For 12 × 2.5, think: one-half of 12 is 6 and double 2.5 is 5; therefore, 6 × 5 is 30.

For example: For 4.5 × 2.2, think: double 4.5 to get 9 and one-half of 2.2 is 1.1; therefore, 9 × 1.1 is 9.9.

For example: For 140 × 35, think: one-half of 140 is 70 and double 35 is 70; therefore 70 × 70 is 4 900.

Some examples of practice items

Here are some practice items:

86 × 50 =

18 × 2.5 =

The product of 140 and 5

Five tenths of one hundred twenty

The area of a rectangular garden with dimensions 32 m and 2.5 m

How many hours in 5 days?

Front End Multiplication or the Distributive Principle in 10s, 100s and 1000s

Note: This strategy involves finding the product of the single-digit factor and the digit in the highest place value of the second number, and adding to this product a second sub-product. This strategy is also known as the distributive principle.

Examples

1) For, 62 × 3, think: 3 times 6 tens is 18 tens, or 180; and 3 times 2 is 6; so 180 plus 6 is 186.
2) For, 2 × 706, think: 2 times 7 hundreds is 14 hundreds, or 1 400; and 2 times 6 is 12; so 1 400 plus 12 is 1412.
3) For, 5 × 6 100, think: 5 times 6 thousands is 30 thousands, or 30 000; and 5 times 100 is 500; so 30 000 plus 500 is 30 500.

Some examples of practice items

62 × 4 =

Four glasses of milk each with 250 ml.

4 × 2 100 =

A pair of factors of ______ are 6 and 3100

The area of a bathroom tile measuring 75 mm by 8 mm

Finding Compatible Factors

This strategy for multiplication involves looking for pairs of factors whose product is a power of ten and re-associating the factors to make the overall calculation easier. This is possible because of the associative property of multiplication.

Examples

1) For 25 × 63 × 4, think: 4 times 25 is 100, and 100 times 63 is 6 300.

   For 2 × 78 × 500, think: 2 times 500 is 1000, and 1 000 times 78 is 78 000.

   For 5 × 450 × 2, think: 2 times 5 is 10, and 10 times 450 is 4 500.

2) Sometimes this strategy involves factoring one of the factors to get a compatible.

   For 25 × 28, think: 28(7 × 4) has 4 as a factor, so 4 times 25 is 100, and 100 times 7 is 700.

   For 68 × 500, think: 68 (34 x2) has 2 as a factor, so 500 times 2 is 1 000, and 1 000 times 34 is 34 000

Some examples of practice items

1) Here are some practice items:

   4 × 38 × 25 =

   250 × 16 =

   Find the product of 2, 12, and 50

   One box contains 50 bags of peppermints. Each bag contains 81 peppermints. How many peppermints are in 2 boxes?

Using Division Facts for Tens, Hundreds and Thousands

This strategy applies to dividends of tens, hundreds and thousands divided by a single digit divisor. There would be only one non-zero digit in the quotient.

Example

60 ÷ 3, think: 6 ÷ 3 is 2 and therefore 60 ÷ 3 is 20.

Some examples of practice items

90 ÷ 3 =

35 000 ÷ 5 =

Divide five hundred sixty by eight

Find the side length of an equilateral triangle with a perimeter of 33 000 mm

Partitioning the Dividend

This strategy involves partitioning the dividend into two parts, both of which are easily divided by the given divisor. Students should look for ten, hundred or thousand that is an easy multiple of the divisor and that is close to, but less than, the given dividend.

Examples

For 372 ÷ 6, think: (360 + 12) ÷ 6, so 60 + 2 is 62.

For 3150 ÷ 5, think: (3 000 + 150) ÷ 5, so 600 + 30 is 630.

Some examples of practice items

Here are some practice items:

248 ÷ 4 =

8 280 divided by 9

The quotient of 344 divided by 8

One fifth of a year in days

Compensation

This strategy for division involves increasing the dividend to an easy multiple of ten, hundred or thousand to get the quotient for that dividend, and then adjusting the quotient to compensate for the increase.

Example

For 348 ÷ 6, think: 348 is about 360 and 360 ÷ 6 is 60 but that is 12 too much; so each of the 6 groups will need to be reduced by 2, so the quotient is 58.

Some examples of practice items

Here are some practice items:

304 ÷ 8 =

$1393 shared between seven people will give each person $________

Three times _________ is 264

Balancing For a Constant Quotient

This strategy involves changing a given division question to an equivalent question that will have the same quotient by multiplying both the divisor and the dividend by the same amount. This is done to make the actual dividing process simpler.

Example

For 125 ÷ 5, think: I could multiply both 5 and 125 by 2 to get 250 ÷ 10, which is easy to do. The quotient is 25.

For 120 ÷ 2.5, think: I could multiply both 2.5 and 120 by 4 to get 480 ÷ 10, which is easy to do. The quotient is 48.

For 23.5 ÷ 0.5, think: I could multiply both 23.5 and 0.5 by 2 to get 47 ÷ 1, so the quotient is obviously 47.

Some examples of practice items

Here are some practice items:

140 ÷ 5 =

32.3 ÷ 0.5 =

How many groups of five tenths are there in 125?

You have 250 L of oil and each can holds 2.5 L of oil. How many cans are needed?

D. Estimation for Addition, Subtraction, Multiplication and Division

Computational Estimation

It is essential that estimation strategies are used by students before attempting pencil/paper or calculator computation to help them find “ball park” or reasonable answers.

When teaching estimation strategies, it is necessary to use the language of estimation with your students. Some of the common words and phrases are: about, just about, between, a little more than, a little less than, close, close to and near.

Rounding:

Examples

1) Here are some examples of rounding multiplication questions with a double digit factor by a triple digit factor.

   To round 688 × 79, think: 688 rounds to 700 and 79 rounds to 80, and 700 times 80 is 56 000.

2) Here are some examples of rounding multiplication questions when there are two of the following kinds of factors, one a 3-digit number with a 5or larger in the tens, and the other a 2-digit number with a 5or greater in the ones. Consider rounding the smaller factor up and the larger factor down to give a more accurate estimate. For example, 653 × 45 done with a conventional rounding rule would be 700 × 50 = 35 000, which would not be close to the actual product of 29 385.Using the rounding strategy above, the 45 would round to 50 and the 653 would round to 600, giving an estimate of 30 000, much closer to the actual product. (When both numbers would normally round up, the above rule does not hold true.)

   To round 763 × 36, round 763 (the larger number, down to 700) and round 36 (the smaller number, up to 40) which equals 700 × 40 = 28 000. This produces a closer estimate than rounding to 800 × 40 = 32 000, when the actual product is 27 468.

3) Some examples of rounding division questions with a double digit divisor and a triple digit dividend are:

   For example, to round 789 ÷ 89, round 89 to 90 and think: “90 multiplied by what number would give an answer close to 800(789 rounded)? Since 9 × 9 = 81, therefore 800 ÷ 90 is about 9.

4) Here are some examples of rounding division questions with a 2-digit divisor where you might convert the question to have a single digit divisor.

For example, 7 843 ÷ 30, think of it as 750 tens ÷ 3 tens to get 250.

Some examples of practice items

1) Here are some practice items:

384 × 68 =

7 011 × 39 =

The product of 708 and 49

Find the cost of 31 students to pay for a year of university with a cost of $6950 each.

2) Here are some practice items:

87 × 371 =

48 rows of 562

About how many hours in a year?

3) Here are some practice items:

411 360 ÷ 71 =

810.3 ÷ 89 =

The quotient of two hundred thirty three divided by twenty nine

4) Here are some practice items:

2 689 ÷ 90 =

Forty divided into 3989

About how many sixties are there in 3494

Front End Addition, Subtraction and Multiplication

Note: This strategy involves combining only the values in the highest place value to get a "ball- park" figure. Such estimates are adequate in many circumstances. Although estimating to tenths and hundredths is included here, it is most important to estimate to the nearest whole number.

Estimate

1) To estimate 0. 093 + 4.236, think: 0.1 + 4.2 = 4.3(to the nearest tenth).

To estimate 0.491 + 0.321, think: 0.4 + 0.3 = 0.7(to nearest tenth).

To estimate 3.871 + 0.124, think: 3 + 0 = 3 (to nearest whole number).

2) To estimate 5.711 – 3.421, think: 5.7 – 3.4 = 2.3(to nearest tenth).

To estimate 3.871 – 0.901, think: 4 – 1 = 3(to nearest whole number).

3) To estimate 3 125 × 6, think: 3 000 × 6 is 6 groups of 18 thousands, or 18 000.

To estimate 42 175 × 4, think: 40 000 × 4 is 4 groups of 4 ten thousands, or 160 000.

4) To estimate 3 × 4.952, think: 4 × 5 or 20.

To estimate 63.141 × 8, think: 60 × 8 or 480.

To estimate 5 × 0.897, think: 5 × 1, or 5.

Some examples of practice items

1) Some practice items for estimating addition of decimal numbers to tenths and whole numbers are:

Estimate to the nearest tenth:

0.701 + 0.001 =

10.673 + 20.241 =

0.615 increased by 0.013

2) Some practice items for estimating subtraction of decimal numbers to tenths and whole numbers are:

0.512 – 0.111 =

The difference of 15.3 and 10.1

0.09 less than 0.81

15.3 g – 10.1 g =

3) Some practice items for estimating multiplication of numbers in the 1 000s are:

7 200 × 3 =

Find the total of three tanks with 8112 litres of oil in each tank.

The product of 6 and 41 296

4) Some practice items for estimating multiplication of numbers in the thousandths by a single digit whole number are:

5 times 3.171 is about ________

Estimate the mass of nine containers of hockey pucks with a mass 7.921 kg each

Estimate 202.273 × 8

Front End Division:

Note: This strategy involves rounding the dividend to a number related to a factor of the divisor and then determining in which place value the first digit of the quotient belongs, to get a "ball- park" answer. Such estimates are adequate in many circumstances.

Example

For 425 ÷ 8, round the 425 to 400, we know that the first digit in the quotient is a

5 (5 × 8 = 40) and it is in the tens place, therefore, the quotient is 50.

Some examples of practice items

Here are some practice items:

191 ÷ 3 =

$276.50 shared equally with nine people

Adjusted Front End or Front End with Clustering

Examples

1) Here are some practice examples for estimating multiplication of double digit factors by double and triple digit factors. Here students may use paper and pencil to record part of the answer.

   To estimate 93 × 41, think: 90 × 40 is 40 groups of 9 tens, or 3 600; and 3 × 40 is 40 groups of 3, or 120; 3 600 plus 120 is 3 720.

   To estimate 241 × 29, think: 200 × 30 is 200 groups of 3 tens, or 6 000, and 30 groups of 40, or 1 200, and 6 000 plus 1 200 is 7 200.

2) Some practice examples for estimating multiplication of numbers in tenths and hundredths by double digit numbers are:

   To estimate 6.1 × 23.4, think: 6 times 20 (120) plus 6 × 3 (18) is 138 and a little more is 140. Or, think of 23.4 as 25 and 25 × 6 is 150.

Some examples of practice items

1) Here are some practice items:

   86 × 39 =

   352 rows of 61

2) Here are some practice items:

   38.2 × 5.9 = (30 × 6) + (8 × 6) = 180 + 48 = 228 plus a little more = 230.

   91.2 × 1.9 =

Doubling for Division

Doubling for division involves rounding and doubling both the dividend and divisor. This does not change the solution but can produce “friendlier” divisors.

Example

For 2 223 ÷ 5 can be thought of as 4 448 ÷ 10, or about 445. It is important that the students understand why this works. See math guide 5- 42.

For 1 333.97 ÷ 5 can be thought of as 2 668 ÷ 10, or about 266.

Some examples of practice items

Here are some practice items:

243 ÷ 5 =

$3 212.11 shared between 5 people

Find the length of one side of a regular pentagon with a perimeter of 235 cm.

PART 2

Measurement Estimation

The Implementation of Measurement Estimation

General Approach

For the most part, a measurement estimation strategy would be reinforced and assessed during mental math time in the grades following its initial introduction. The goal in mental math is to increase a student's *competency* with the strategy. It is expected that measurement estimation strategies would be introduced as part of the general development of measurement concepts at the appropriate grade levels. Each strategy would be explored, modeled, and reinforced through a variety of class activities that promote understanding, the use of estimation language, and the refinement of the strategy. As well, the assessment of the strategy during the instructional and practice stages would be ongoing and varied.

Competency means that a student reaches a reasonable estimate using an appropriate strategy in a reasonable time frame. While what is considered a reasonable estimate will vary with the size of the unit and the size of the object, a general rule would be to aim for an estimate that is within 10% of the actual measure. A reasonable time frame will vary with the strategy being used; for example, using the *benchmark strategy* to get an estimate in metres might take 5 to 10 seconds, while using the *chunking strategy* might take 10 to 30 seconds, depending upon the complexity of the task.

A. Introducing a Strategy in Regular Classroom Time

Measurement estimation strategies should be introduced when appropriate as part of a rich and carefully sequenced development of measurement concepts. For example, in grade 3, the distance from the floor to most door handles is employed as a *benchmark* for a metre so students can use a *benchmark strategy* to estimate lengths of objects in metres. This has followed many other experiences with linear measurement in earlier grades: in grade primary, students compared and ordered lengths of objects concretely and visually; in grade 1, students estimated lengths of objects using non-standard units such as paper clips; in grade 2, students developed the standard unit of a metre and were merely introduced to the idea of a *benchmark* for a metre.

The introduction of a measurement estimation strategy should include a variety of activities that use concrete materials, visuals, and meaningful contexts, in order to have students thoroughly understand the strategy. For example, a meaningful context for the *chunking strategy* might be to estimate the area available for bookshelves in the classroom. Explicit modeling of the mental processes used to carry out a strategy should also be part of this introduction. For example, when demonstrating the *subdivision strategy* to get an estimate for the area of a wall, you would orally describe the process of dividing the wall into fractional parts, the process of determining the area of one part in square metres, and the multiplication process to get the estimate of the area of the entire wall in square metres. During this introductory phase, you should also lead discussions of situations for which a particular strategy is most appropriate and efficient, and of situations for which it would not be appropriate or efficient.

B. Reinforcement in Mental Math Time

Each strategy for building measurement estimation skills should be practised in isolation until students can give reasonable estimates in reasonable time frames. Students must understand the logic of the strategy, recognize when it is appropriate, and be able to explain the strategy. The amount of time spent on each strategy should be determined by the students' abilities, progress, and previous experiences.

The reinforcement activities for a strategy should be varied in type and should focus as much on the discussion of how students obtained their answers, as on the answers themselves. The reinforcement activities should be structured to ensure maximum participation. At first, time frames should be generous and then narrowed as students internalize the strategy and become more efficient. Student participation should be monitored and their progress assessed in a variety of ways. This will help determine the length of time that should be spent on a strategy.

During the reinforcement activities, the actual measures should not be determined every time an estimate is made. You do not want your students to think that an estimate is always followed by measurement with an instrument: there are many instances where an estimate is all that is required. When students are first introduced to an activity, it is helpful to follow their first few estimates with a determination of the actual measurement in order to help them refine their estimation abilities. Afterwards, however, you should just confirm the reasonable estimates, having determined them in advance

Most of the reinforcement activities in measurement will require the availability of many objects and materials because students will be using some objects and materials as benchmarks and will be estimating the attributes of others. To do this, they must see and/or touch those objects and materials.

After you are confident that most students have achieved a reasonable competency with the strategy, you need to help them integrate it with other strategies that have been developed. You should do this by providing activities that include a mix of attributes to be estimated in different units using a variety of contexts. You should have the students complete the activities and discuss the strategy/strategies that were, or could have been, used. You should also have students match measurement estimation tasks to a list of strategies, and have them discuss the reasoning for their matches.

C. Assessment

Your assessments of measurement estimation strategies should take a variety of forms. Assessment opportunities include making and noting observations during the reinforcements, as well as students' oral and written responses and explanations. Individual interviews can provide you with many insights into a student's thinking about measurement tasks. As well, traditional quizzes that involve students recording answers to estimation questions that you give one-at-a-time in a certain time frame can be used.

Assessments, regardless of their form, should shed light on students' abilities to estimate measurements efficiently and accurately, to select appropriate strategies, and to explain their thinking.

Measurement Estimation Strategies

The Benchmark Strategy
This strategy involves mentally matching the attribute of the object being estimated to a known measurement. For example, to estimate the width of a large white board, students might mentally compare its width to the distance from the doorknob to the floor. This distance that is known to be about 1 metre is a *benchmark*. When students mentally match the width of the white board to this benchmark, they may estimate that the width would be about two of these benchmarks; therefore, their estimate would be 2 metres. In mathematics education literature you will often see reference made to *personal referents.* These are benchmarks that individuals establish using their own bodies; for example, the width of a little finger might be a personal referent for 1 cm, a hand span a referent for 20 cm, and a hand width a referent for 1 dm. These benchmarks have the advantage of being portable and always present whenever and wherever an estimate is needed.

The Chunking Strategy (Starting in Grade 5)
This strategy involves mentally dividing the attribute of an object into a number of chunks, estimating each of these chunks, and adding the estimates of all the chunks to get the total estimate. For example, to estimate the length of a wall, a student might estimate the distance from one corner to the door, estimate the width of the door, estimate the distance from the door to the bookcase, and estimate the length of the bookcase which extends to the other corner. All these individual estimates would be added to get the estimate of the length of the wall.

The Unitizing Strategy (Starting in Grade 5)
This strategy involves establishing an object (that is smaller than the object being estimated) as a unit, estimating the size of the attribute of this unit, and multiplying by the number of these units it would take to make the object being measured. For example, students might get an estimate for the length of a wall by noticing it is about five bulletin boards long. The bulletin board becomes the unit. Students estimate the length of the bulletin board and multiply this estimate by five to get an estimate of the length of the wall.

The Subdivision Strategy (Starting in Grade 6)
This strategy involves mentally dividing an object repeatedly in half until a more manageable part of the object can be estimated and then this estimate is multiplied by the appropriate factor to get an estimate for the whole object. For example, to estimate the area of a table top, students might mentally divide the table top into fourths, estimate the area of that one-fourth in square decimeters, and then multiply that estimate by four to get the estimate of the area of the entire table top.

E. Measurement Estimate — Length/Perimetre/Area

In grade 4, students developed understanding of the benchmark for the millimetre and its relationship to other linear measurement units.

In grades 4 and 5, students developed understanding of measuring area and perimeter using standard units.

Note: They also developed formulas for calculating area and perimeter, estimating perimeter and area using formulas is done in the Addition and multiplying sections of Part 1 as they are a context for the addition or multiplication.

They are now ready to use this understanding to develop the following estimation competencies.

Benchmark — Millimetre

Student should gain confidence and accuracy with estimating lengths of objects as 'greater than' or 'less than' 1000 mm. Students could use the width of a dime as a benchmark for 1 dm, or a personal referent such as the distance between their figure and thumb nails, just before the fingers touch while forming an "O".

Examples of Some Practice Items
Estimate as greater than or less than 1000 mm

- Will the height of the teacher's desk be 'more than' 1000mm or 'less than' 1000mm? (900mm approx.)
- Will the height of the filing cabinet be 'more than,' 'less than,' or 'about' 1000mm. (1320mm)

- If the height of a waste bin is about 38cm, will this be 'more than' or 'less than' 4 dm?
- The length of a tissue box is about 24cm. About how long is it in dm?
- The height of a tissue box is about 120mm. What would this measurement be in cm?

Benchmarks — Decimetre / Metre

Student should gain confidence and accuracy with estimating lengths of objects that are between 1 dm and 10 dm and rename decimetre estimates as tenths of metres. Students could use the length of the rod in the base-10 blocks as a benchmark for 1 dm, or a personal referent such as their hand width. For a metre, students can use the length of 10 rods placed end to end, the height of the door knob, or the length of a metre stick as a benchmark for a metre. They should be very comfortable with 1 dm = 0.1 of a metre equivalence.

Examples

- the length of a book is about 3 dm or about 0.3 m
- the width of a chart is about 9 dm or 0.9 m

Examples of Some Practice Items

- Provide students with various objects, such as a length of a piece of ribbon. Ask them to estimate its length in decimetres and in metres is about 38 dm.
- Give students a length and have them name objects that are approximately that length. Name some items with a length about: 2.5 dm, 7 dm, 0.6 m, 8 metres, 1.5 metres, 13 meters, or 45 metres.
- Have students measure the distance of 10 walking steps. They can then calculate the approximate length of one step. They can then use this information to estimate the distance of several things, such as, distance around the perimeter of the gym, the length of the hallway outside of your classroom, and the distance you walk to your bus each day from home.

Benchmarks — Kilometre

Student should gain confidence and accuracy with estimating lengths of objects that are between 1 km and 100 km. Students should be encouraged to find a personal referent for kilometre, e.g. the distance from their home or school to a certain landmark.

Examples

- The distance from school to the park is about 5km.
- The length of 10 soccer fields end to end would be approximately 1 km.

Examples of Some Practice Items

- Give possible distances in km from the school to nearby landmarks.
- Estimate the distance it is from your home to the nearest video store.
- About how far is it from your community to the next community?
-

Chunking — Length/ Perimetre/Area

This strategy involves mentally dividing the attribute of an object into a number of chunks, estimating each of these chunks, using the benchmarks from earlier of previous years, and adding the estimates of all the chunks to get the total estimate. It is important that lengths students are asked to estimate lend themselves to chunking. There must be divisions along the way that are not uniform.

Example

- estimate the length of a wall, a student might estimate the distance from one corner to the door, estimate the width of the door, estimate the distance from the door to the bookcase, and estimate the length of the bookcase which extends to the other corner. All these individual estimates would be added to get the estimate of the length of the wall.

Unitizing — Length/ Perimetre/Area

This strategy involves establishing an object (that is smaller than the object being estimated) as a unit, estimating the size of the attribute of this unit, and multiplying by the number of these units it would take to make the object being measured.

Example

- students might get an estimate for the length of a wall by noticing it is about five bulletin boards long. The bulletin board becomes the unit. Students estimate the length of the bulletin board and multiply this estimate by five to get an estimate of the length of the wall.

Subdivision — Length/ Perimetre/Area

This strategy involves mentally dividing an object repeatedly in half until a more manageable part of the object can be estimated and then this estimate is multiplied by the appropriate factor to get an estimate for the whole object. For example, to estimate the area of a table top, students might mentally divide the table top into fourths, estimate the area of that one-fourth in square decimeters, and then multiply that estimate by four to get the estimate of the area of the entire table top.

Example

- estimate the area of a table top, students might mentally divide the table top into fourths, estimate the area of that one-fourth in square decimeters, and then multiply that estimate by four to get the estimate of the area of the entire table top.

F. Measurement Estimate — Volume and Capacity

In Grades 4 and 5, students developed an understanding of estimating and measuring volume of rectangular prisms using non standard and standard units ($cm^{3,}$ dm^3, m^3). They also developed an understanding of measuring and estimating capacity using millilitres and Litres. In Grade 5 they investigate the relationship between volume and capacity to discover 1cm^3 holds exactly 1 ml of liquid. They are now ready to use this understanding to develop the following estimation competency.

Benchmark — Volume

Student should gain confidence and accuracy with estimating the volume of various containers. Student could use a small cube from the base-10 unit blocks as a benchmark for one centimetre cubed (1 cm^3), a large cube from the base ten is one cubic decimetre (1 dm^3), and one cubic metre (m^3)can be created using meter sticks.

Examples

- Estimate the volume of your desk using a cm^3 as the unit of measurement.
- About how many cube a links do you think would fit into a shoe box?
- Estimate the volume of your classroom using m^3.
- Name four rectangular prisms that would have a volume between 2000 cm^3 and 3000 cm^3.
- Estimate the volume of our classroom using the unitizing method.
- What might be a good unit to use in order to find the volume of a school?
- Estimate the volume of your bedroom.
- Find three rectangular prisms in your classroom and estimate the volume of each.

Benchmark — Capacity

Student should gain confidence and accuracy with estimating the capacity of various containers. Student could use a small cube from the base-10 unit blocks as a benchmark for 1 ml, a rod as a benchmark for 10 mL, a flat as a benchmark for 100 mL, and a large cube from the base ten holds 1 L of liquid.

Examples:

- List three containers with a capacity between 300 ml and 500 ml.
- Estimate the capacity of a water bottle and then check it.
- If you could fill your desk with water, how much do you think it would take?

G. Measurement Estimation – Angles

In Grade 5, students developed understanding of 0 degree, 90 degree, and 180 degree angles. They are now ready to develop the following estimation competencies:

Benchmark — Degrees

Student should gain confidence and accuracy with estimating the size of angles as being nearest in size to 0°, 45°, 90°, 135°, or 180° if they are within 10 degrees of their actual size.

Examples

- have students estimate the size of the following angles shown to them with the Geo Strips: close to 90, 45, 180, 0, and 135 degrees

Examples of Some Practice Items

- Have students use their hands to show an angle close to 90 degrees, a little more than 90 degrees and a little less than 90 degrees.
- Have students use two pencils to show an angle about 135 degrees.
- Display a variety of angles on an overhead, representing angles close to 0, 45, 90, 135 and 180 degrees and have students estimate the angles.
- Have students make a right angle with their feet.
- Have students make a 180 degree angle with two pencils.
- Have students make a 90 degree angle with their feet.
- Ask the students what time it could be if the hands on a clock make a right angle? An obtuse angle? An acute angle?

Estimate

- Using two pencils show an acute angle that is less than one-half the size of a right angle.
- Using two pencils show an acute angle that is more than one-half of a right angle but less than a right angle.
- Using two pencils, or geo-strips, or pipe-cleaners, show an obtuse angle of 135 degrees.
- Using two pencils, or geo-strips, or pipe-cleaners show an obtuse angle more than 135 degrees but less than 180 degrees.
- The size of each angle to within 5–10 degrees of their actual size

PART 3

Spatial Sense

The Development of Spatial Sense

What is spatial sense?

Spatial sense is an intuition about shapes and their relationships, and an ability to manipulate shapes in one's mind. It includes being comfortable with geometric descriptions of shapes and positions.

There are seven abilities in spatial sense that need to be addressed in the classroom:

- **Eye-motor co-ordination**. This is the ability to co-ordinate vision with body movement. In the mathematics classroom, this involves children learning to write and draw using pencils, crayons, and markers; to cut with scissors; and to handle materials.
- **Visual memory**. This is the ability to recall objects no longer in view.
- **Position-in-space perception**. This is the ability to determine the relationship between one object and another, and an object's relationship to the observer. This includes the development of associated language (over, under, beside, on top of, right, left, etc.) and the transformations (translations, reflections, and rotations) that change an object's position.
- **Visual discrimination**. This is the ability to identify the similarities and differences between, or among, objects.
- **Figure-ground perception.** This is the ability to focus on a specific object within a picture or within a group of objects, while treating the rest of the picture or the objects as background.
- **Perceptual constancy**. This is the ability to recognize a shape when it is seen from a different viewpoint, or from a different distance. This is the perception at play when students recognize similar shapes (enlargements/reductions), and when they perceive as squares and rectangles, the rhombi and parallelograms in isometric drawings.
- **Perception of spatial relationships.** This is the ability to see the relationship between/among two or more objects. This perception is central when students assemble materials to create an object or when they solve puzzles, such as tangram, pattern block, and jigsaw puzzles.

These abilities develop naturally through many experiences in life; therefore, most students come to school with varying degrees of ability in each of these seven aspects. However, students who have been identified as having a learning disability are likely to have deficits in one or more of these abilities. Classroom activities should be planned to allow most students to further develop these perceptions. Most spatial sense activities involve many, if not most, of the spatial abilities, although often one of them is highlighted.

Classroom Context

Spatial abilities are essential to the development of geometric concepts, and the development of geometric concepts provides the opportunity for further development of spatial abilities. This mutually supportive development can be achieved through consistent and ongoing strategic planning of rich experiences with shapes and spatial relationships. Classroom experiences should also include activities specifically designed for the development of spatial abilities. These activities should focus on shapes new to the grade, as well as shapes from previous grades. As the shapes become more complex, students' spatial senses should be further developed through activities aimed at the discovery of relationships between and among properties of these shapes. Ultimately, students should be able to visualize shapes and their various transformations, as well as sub-divisions and composites of these shapes.

Spatial Sense in Mental Math Time

Following the exploration and development of spatial abilities during geometry instruction, mental math time can provide the opportunity to revisit and reinforce spatial abilities periodically throughout the school year.

Assessment

Assessment of spatial sense development should take a variety of forms. The focus in this aspect of mental math is on individual growth and development in spatial sense, rather than on an arbitrary level of competency to be achieved. You should record any observations of growth students make during the reinforcements, as well as noting students' oral and written responses and explanations. For spatial sense, traditional quizzes that involve students recording answers to questions that you give one-at-a-time in a certain time frame should play a very minor role.

H: Spatial Sense in 2-D Geometry

In grade 6, students apply generalizations about the sum of angles in triangles and quadrilaterals. They sort the members of the quadrilateral “family” under property headings. They make generalizations about the rotational symmetry property of all members of the quadrilateral “family” and of regular polygons. They recognize and represent dilatation images of 2-D figures and make connections to similar figures.

Examples of Spatial Sense Activities

1. Show a rectangle drawn on square dot paper on the overhead. Ask students to draw the same rectangle in the same orientation on square dot paper. Then ask them to draw a half-turn rotation of this rectangle. After a reasonable amount of time, have them show their work. Show the half-turn rotation on the overhead and have students compare their drawings with this. Repeat this activity using other members of the quadrilateral “family,” asking for various rotations.
2. Ask students to find the size of the missing angles in each of the following triangles and quadrilaterals and to record their answers on their whiteboards or in their mental math notebooks. Discuss responses.
   - Triangles

   a) two of the angles are 70 degrees and 45 degrees (65 degrees)
   b) two of the angles are each 75 degrees (30 degrees)
   c) it is a right angle with a 60 degree angle (30 degrees)
   d) it is an isosceles triangle with an angle of 102 degrees (39
   e) degrees, 39 degrees)
   - Quadrilaterals

   a) A diagonal divides the square into two right triangles. What is the size of the two equal angles in each triangle? (45 degrees)
   b) A diagonal divides the rhombus into two congruent isosceles triangles. In each triangle, the two equal angles add up to 116 degrees. What is the size of the third angle in each triangle? (64 degrees)
3. Show students a group of ten quadrilaterals, labeled A to J, on the overhead. Give students a sheet with a Venn diagram to sort the quadrilaterals into those that have 1 or more 90 degree angles and those that have 2 pairs of parallel sides. Have students compare responses with a partner before checking responses together as a class.

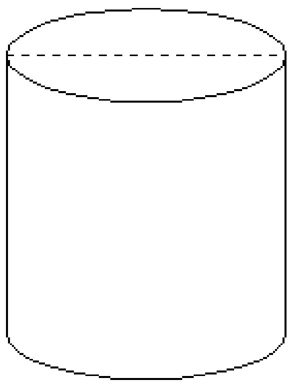
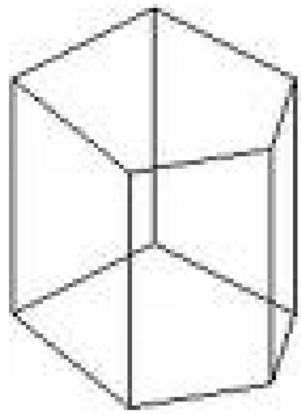
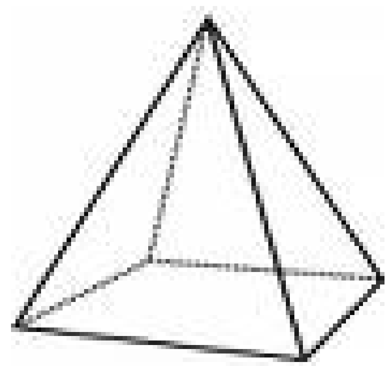
H: Spatial Sense in 3-D Geometry

In grade 6, students describe and represent the various cross-sections of cones, cylinders, pyramids, and prisms. They make generalizations about the planes of symmetry of 3-D shapes. They make and interpret isometric drawings of shapes made from cubes, building on the work done in previous grades.

Examples of Spatial Sense Activities

1. Show students a net of a solid on the overhead, with one part of the net missing. Ask them to draw the complete net on their individual whiteboards, putting an ‘x’ on the part of the net that was missing. (*See Blackline Masters)* Ask students to show their completed nets when directed. Choose a student to complete the overhead net and discuss results.

2. Show students pictures, one at a time, of 3-D shapes on the overhead –a cylinder, a square pyramid, and a pentagonal prism. Ask them to draw on their whiteboards the cross-sections produced when a cut is made, first horizontally (parallel to the base), and then vertically

3. Show students a shape made from cubes. Ask them to build the same shape. After agreeing which side of the shape will be the "front", have them draw the front, right, back, left, and top views on dot paper. (This activity could be done initially as a matching activity, matching the words front, right, back, left and, top with the drawn views.)