Mental Math

In

Mathematics 5

NOVA SCOTIA
Education
English Program Services

Acknowledgements

The Department of Education gratefully acknowledges the contributions of the following individuals to the preparation of the *Mental Math* booklets:

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Lois Boudreau—Annapolis Valley Regional School Board
Sharon Boudreau—Cape Breton-Victoria Regional School Board
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Martha Stewart—Annapolis Valley Regional School Board
Sherene Sharpe—South Shore Regional School Board
Brad Pemberton—Annapolis Valley Regional School Board
Angela West—Halifax Regional School Board
Susan Wilkie—Halifax Regional School Board

The Department of Education would like to acknowledge the special contribution of **David McKillop**, Making Math Matter Inc. His vision and leadership have been a driving force behind this project since its inception.

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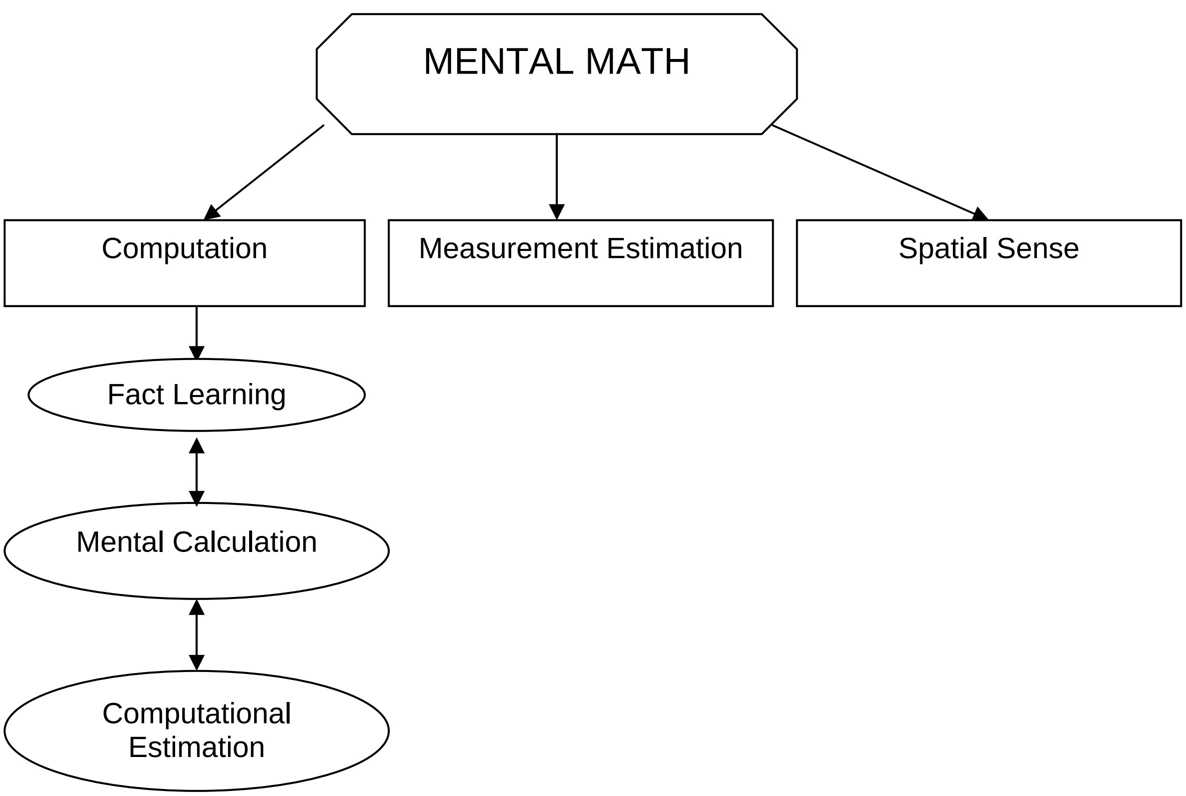
Introduction

Welcome to your grade-level mental math booklet. After the Department of Education released the *Time to Learn* document in which at least five minutes of mental math was required daily in grades 1 – 9, it was decided to support teachers by clarifying and outlining the specific mental math expectations at each grade. Therefore, grade-level booklets for computational aspects of mental math were prepared and released in draft form in the 2006–2007 school year. Building on these drafts, the current booklets describe the mental math expectations in computation, measurement, and geometry at each grade. These resources are supplements to, not replacements of, the Atlantic Canada mathematics curriculum. You should understand that the expectations for your grade are based on the full implementation of the expectations in the previous grades. Therefore, in the initial years of implementation, you may have to address some strategies from previous grades rather than all of those specified for your grade. It is critical that a school staff meets and plans on an on-going basis to ensure the complete implementation of mental math.

Definitions

In the mathematics education literature, there is not consensus on the usage of some of the words and expressions in mental math. In order to provide uniformity in communication in these booklets, it is important that some of these terms be defined. For example, the Department of Education in Nova Scotia uses the term *mental math* to encompass the whole range of mental processing in all strands of the mathematics curriculum. *Mental math* is broken into three categories in the grade-level booklets: *mental computation*, *measurement estimation*, and *spatial sense*. *Mental computation* is further broken down into *fact learning, mental calculation*, and *computational estimation*.



Fact learning refers to the acquisition of the 100 number facts related to the single digits 0 to 9 for each of the four operations. When students know these facts, they can quickly retrieve them from memory (usually in 3 seconds or less). Ideally, through practice over time, students will achieve automaticity; that is, they will have instant recall without using strategies. *Mental calculation* refers to getting exact answers by using strategies to do the calculations in one's head, while *computational estimation* refers to getting approximate answers by using strategies to do calculations in one's head.

While each category in computations has been defined separately, this does not suggest that the three categories are entirely separable. Initially, students develop and use strategies to get quick recall of the facts. These strategies and the facts themselves are the foundations for the development of other mental calculation strategies. When the facts are automatic, students are no longer employing strategies to retrieve them from memory. In turn, the facts and mental calculation strategies are the foundations for computational estimation strategies. Actually, attempts at computational estimation are often thwarted by the lack of knowledge of the related facts and mental calculation strategies.

Measurement estimation is the process of using internal and external visual (or tactile) information to get approximate measures, or to make comparisons of measures, without the use of measurement instruments.

Spatial sense is an intuition about shapes and their relationships, and an ability to manipulate shapes in one's mind. It includes being comfortable with geometric descriptions of shapes and positions.

Rationale for Mental Math

In modern society, the development of mental skills needs to be a major goal of any mathematics program for two major reasons. First, in their day-to-day activities, most people's computational, measurement, and spatial needs can be met by having well developed mental strategies. Secondly, because technology has replaced paper-and-pencil as the major tool for complex tasks, people need to have well developed mental strategies to be alert to the reasonableness of the results generated by this technology.

PART 1

Computation

The Implementation of Mental Computations

General Approach

In general, a computational strategy should be introduced in isolation from other strategies, a variety of different reinforcement activities should be provided until it is mastered, the strategy should be assessed in a variety of ways, and then it should be combined with other previously learned strategies.

A. Introducing a Strategy

The approach to highlighting a computational strategy is to give the students an example of a computation for which the strategy would be useful to see if any of the students already can apply the strategy. If so, the student(s) can explain the strategy to the class with your help. If not, you could share the strategy yourself. The explanation of a strategy should include anything that will help students see the pattern and logic of the strategy, be that concrete materials, visuals, and/or contexts. The introduction should also include explicit modeling of the mental processes used to carry out the strategy, and explicit discussion of the situations for which the strategy is most appropriate and efficient. Discussion should also include situation for which the strategy would not be the most appropriate and efficient one. Most important is that the logic of the strategy should be well understood before it is reinforced; otherwise, it's long-term retention will be very limited.

B. Reinforcement

Each strategy for building mental computational skills should be practised in isolation until students can give correct solutions in a reasonable time frame. Students must understand the logic of the strategy, recognize when it is appropriate, and explain the strategy. The amount of time spent on each strategy should be determined by the students' abilities and previous experiences.

The reinforcement activities for a strategy should be varied in type and should focus as much on the discussion of how students obtained their answers as on the answers themselves. The reinforcement activities should be structured to insure maximum participation. At first, time frames should be generous and then narrowed as students internalize the strategy. Student participation should be monitored and their progress assessed in a variety of ways to help determine how long should be spent on a strategy.

After you are confident that most of the students have internalized the strategy, you need to help them integrate it with other strategies they have developed. You can do this by providing activities that includes a mix of number expressions, for which this strategy and others would apply. You should have the students complete the activities and discuss the strategy/strategies that could be used; or you should have students match the number expressions included in the activity to a list of strategies, and discuss the attributes of the number expressions that prompted them to make the matches.

Language

Students should hear and see you use a variety of language associated with each operation, so they do not develop a single word-operation association. Through rich language usage students are able to quickly determine which operation and strategy they should employ. For example, when a student hears you say, "Six plus five", "Six and five", "The total of six and five", "The sum of six and five", or "Five more than six", they should be able to quickly determine that they must add 6 and 5, and that an appropriate strategy to do this is the *double-plus-one* strategy.

Context

You should present students with a variety of contexts for each operation in some of the reinforcement activities, so they are able to transfer the use of operations and strategies to situations found in their daily lives. By using contexts such as measurement, money, and food, the numbers become more real to the students. Contexts also provide you with opportunities to have students recall and apply other common knowledge that should be well known. For example, when a student hears you say, “How many days in two weeks?” they should be able to recall that there are seven days in a week and that double seven is 14 days.

Number Patterns

You can also use the recognition and extension of number patterns can to reinforce strategy development. For example, when a student is asked to extend the pattern “30, 60, 120, …,”, one possible extension is to double the previous term to get 240, 480, 960. Another possible extension, found by adding multiples of 30, would be 210, 330, 480. Both possibilities require students to mentally calculate numbers using a variety of strategies.

Examples of Reinforcement Activities

Reinforcement activities will be included with each strategy in order to provide you with a variety of examples. These are not intended to be exhaustive; rather, they are meant to clarify how language, contexts, common knowledge, and number patterns can provide novelty and variety as students engage in strategy development.

C. Assessment

Your assessments of computational strategies should take a variety of forms. In addition to the traditional quizzes that involve students recording answers to questions that you give one-at-a-time in a certain time frame, you should also record any observations you make during the reinforcements, ask the students for oral responses and explanations, and have them explain strategies in writing. Individual interviews can provide you with many insights into a student’s thinking, especially in situations where pencil-and-paper responses are weak.

Assessments, regardless of their form, should shed light on students’ abilities to compute efficiently and accurately, to select appropriate strategies, and to explain their thinking.

Response Time

Response time is an effective way for you to see if students can use the computational strategies efficiently and to determine if students have automaticity of their facts.

For the facts, your goal is to get a response in 3-seconds or less. You would certainly give students more time than this in the initial strategy reinforcement activities, and reduce the time as the students become more proficient applying the strategy until the 3-second goal is reached. In subsequent grades, when the facts are extended to 10s, 100s and 1000s, you should also ultimately expect a 3-second response.

In the early grades, the 3-second response goal is a guideline for you and does not need to be shared with your students if it will cause undue anxiety.

With mental calculation strategies and computational estimation, you should allow 5 to 10 seconds, depending upon the complexity of the mental activity required. Again, in the initial application of these strategies, you would allow as much time as needed to insure success, and gradually decrease the wait time until students attain solutions in a reasonable time frame.

Integration of Strategies

After students have achieved competency using one strategy, you should provide opportunities for them to integrate it with other strategies they have learned. The ultimate goal is for students to have a network of mental strategies that they can flexibly and efficiently apply whenever a computational situation arises. This integration can be aided in a variety of ways, some of which are described below.

- You should give them a variety of questions, some of which could be done just as efficiently by two or more different strategies and some of which are most efficiently done by one specific strategy. It is important to have a follow-up discussion of the strategies and the reasons for the selection of specific strategies.
- You should take every opportunity that arises in regular math class time to reinforce the strategies learned in mental math time.
- You should include written questions in regular math time. This could be as journal entry, a quiz/test question, part of a portfolio, or other assessment for which students will get individual feedback. You might ask students to explain how they could mentally compute a given question in one or more ways, to comment on a student response that has an error in thinking, or to generate sample questions that would be efficiently done by a specified strategy.

A. Addition — Mental Calculation

Addition Facts Applied to Multiples of Powers of 10 (Extension)

Knowledge of all single-digit addition facts within a 3-second response time was an expectation in mental math in grade 2. These facts were applied to 10s and 100s in grade 3 and to 1000s in grade 4. In grade 5, these facts and applications should be reviewed and extended to tens of thousands and to tenths.

The strategies for 88 of the 100 facts involving single-digit addends are:

a) Doubles Facts
b) Plus-One Facts
c) Near-Doubles (1-Aparts) Facts
d) Plus-Two Facts
e) Plus Zero Facts
f) Make-10 Facts

There are a variety of strategies that can be used for the last 12 facts.

Further information about the fact learning strategies can be found in the mental math documents for grades 2 or 3.

Examples

For 80 + 60, think: If 20 from the 60 is given to the 80, the question becomes 100 + 40, or 140.

For 100, 300, 500, 700, ______ , think: Each number is 200 more than the number before, so the next number is 700 + 200 = 900.

For 700 + 800, think: 700 and 700 is 1400, so 100 more is 1500; or think: 7 and 8 is 15, but these are hundreds, so the answer is 1500.

For 3000 + 5000, think: 3000 and 3000 is 6000, so 2000 more is 8000; or think: 3 and 5 is 8, but these are thousands, so the answer is 8000.

For 90 000 + 30 000, think: If 10 000 from 30 000 is moved to 90 000, the question becomes 100 000 + 20 000, or 120 000.

For 0.7 + 0.5, think: If 1-tenth from 0.7 is moved to 0.5, the question becomes 0.6 + 0.6, or 1.2; or think: 7-tenths plus 5-tenths is 12-tenths which is 1 and 2-tenths (1.2).

Examples of Some Practice Items

a) Some practice items for numbers in the 10s, 100s, and 1000s:

- 90 + 80
- 70 increased by 20
- $300 more than $600
- 400 girls and 400 boys. How many children?
- 4 000 and 1 000
- $8 000 + $6 000
- $\square$ – 300 = 800

b) Some practice items for numbers in the 10 000s:

- 10 000, 40 000, 70 000, _______
- 20 000 + 30 000
- 50 000 more than 30 000
- I had 70 000 dollars and earned $40 000. How much do I have now?
- ☐ – 40 000 = 10 000

c) Some practice items for numbers in the tenths:

- 0.6 + 0.3
- 0.5 kg plus 0.7 kg
- 0.4 m increased by 0.8 m
- The sum of 0.9 and 0.6
- ☐ – 0.5 = 0.8

Front End Addition (Extension)

This strategy is applied to questions that involve two combinations of non-zero digits, one combination of which may require regrouping. The strategy involves first adding the digits in the highest place-value position, then adding the non-zero digits in another place-value position, and doing any needed regrouping. After a review of this strategy applied to 2-digit and 3-digit whole numbers, it should be extended in grade 5 to 4-digit numbers including numbers in tens of thousands and in tenths.

After students have achieved competency using this strategy, you should provide opportunities for them to integrate it with other strategies.

Examples

For 26 + 37, think: 20 plus 30 is 50, 6 plus 7 is 13, and 50 plus 13 is 63.

For 307 + 206, think: 300 plus 200 is 500, 7 plus 6 is 13, and 500 plus 13 is 513.

For 450 plus 380 think: 400 plus 300 is 700, 50 plus 80 is 130, and 700 plus 130 is 830.

For 3 600 + 2 500, think: 3 thousand plus 2 thousand is 5 thousand, 6 hundred and 5 hundred is 11 hundred, and 5 thousand and 11 hundred is 6 100.

For 43 000 + 54 000, think: 40 thousand plus 50 thousand is 90 thousand, 3 thousand plus 4 thousand is 7 thousand, and 90 thousand plus 7 thousand is 97 000.

For 25 000 + 38 000, think: 20 thousand plus 30 thousand is 50 thousand, 5 thousand plus 8 thousand is 13 thousand, and 50 thousand plus 13 thousand is 63 thousand (63 000).

For 7.2 + 2.6, think: 7 plus 2 is 9 and 2-tenths plus 6-tenths is 8-tenths, so the answer is 9 and 8-tenths (9.8).

For 5.8 + 2.5, think: 5 and 2 is 7, 8-tenths and 5-tenths is 13-tenths, and 7 and 13-tenths is 8 and 3-tenths (8.3).

Examples of Some Practice Items

a) Some practice items for numbers in the 10s and 100s:

- 45 + 36
- 18 kg more than 56 kg
- 102 more than 567
- $660 + $270
- $\square$ – 120 = 290

b) Some practice items for numbers in the 1000s and 10 000s:

- 3 400 km and 5 800 km
- The sum of 2 040 and 6 090
- 45 000 + 23 000
- 30 700 more than 40 500
- 56 000 females and 47 000 males. What is the total?
- $60 080 increased by $10 090
- $\square$ – 45 000 = 16 000

c) Some practice items for numbers in the 10ths:

- 3.5 m and 2.4 m
- The sum of 4.6 and 3.9
- 4.3 kg more than 7.8 kg
- 7.5 km increased by 2.9 km

Quick Addition (Extension)

This strategy is actually the *Front-End* strategy applied to questions that involve more than two combinations and with no regrouping needed. The questions are always presented visually and students quickly record their answers on paper. While this is a pencil-and-paper strategy because answers will always be recorded on paper before answers are read, it is included here as a mental math strategy because most students will do all the combinations in their heads starting at the front end.

This strategy requires students to holistically examine each question to confirm there will be no regrouping. This habit of holistically examining each question as a first step in determining the most efficient strategy needs to pervade all mental math lessons. (One suggestion for an activity during the discussion of this strategy is to present students with a list of 20 questions, some of which do require regrouping, and direct students to apply quick addition to the appropriate questions and leave out the other ones.) It is important to present examples of these addition questions in both horizontal and vertical formats. Students should have applied this strategy to 3-digit and 4-digit numbers up to the end of grade 4, so in grade 5 they should apply it to 5-digit numbers as well. The numbers should include decimal examples as well as whole number examples.

Most likely, students will add the digits in corresponding place values of the two addends without consciously thinking about the names of the place values. Therefore, in the discussion of the questions, you should encourage students to read the numbers correctly and to use place-value names. This will reinforce place value concepts at the same time as addition is reinforced.

After students have achieved competency using this strategy, you should provide opportunities for them to integrate it with other strategies.

Examples

For 543 + 256, think and record each resultant digit: 5 and 2 is 7, 4 and 5 is 9, and 3 and 6 is 9, so the answer is 799 (seven hundred ninety-nine); or think: 500 and 200 is 700, 40 and 50 is **9**0, 3 and 6 is **9** to get 799.

For 2 341 increased by 3 415, think and record each resultant digit: 2 and 3 is 5, 3 and 4 is 7, 4 and 1 is 5, and 1 and 5 is 6, so the answer is 5 756 (five thousand, seven hundred fifty-six); or think: 2 000 and 3 000 is **5** 000, 300 and 400 is 700, 40 and 10 is **5**0, 1 and 5 is **6** to get 5 756.

For $23 451 + $41 426, think and record each resultant digit: 2 and 4 is 6, 3 and 1 is 4, 4 and 4 is 8, 5 and 2 is 7, and 1 and 6 is 7, so the answer is $64 877(sixty-four thousand, eight hundred seventy-seven); or think: 20 000 and 40 000 is **6**0 000, 3 000 and 1 000 is **4** 000, 400 and 400 is **8**00, 50 and 20 is 70, 1 and 6 is **7** to get $64 877.

For 34.3 + 23.5, think and record each resultant digit: 3 and 2 is 5, 4 and 3 is 7, and 3 and 5 is 8, so the answer is 57.8 (fifty-seven and eight-tenths); or think: 30 and 20 is **5**0, 4 and 3 is **7**, and 3-tenths and 5-tenths is **8**-tenths to get 57.8.

Examples of Some Practice Items

a) Some practice items for numbers in the 100s and 1000s:

- The sum of 291 and 703
- $\begin{array}{r} 537 \\ \underline{+\ 341} \end{array}$
- There were 333 girls and 144 boys at the concert. What was the total attendance?
- $4 532 + $2 367
- 8107 people in town. 1742 people on the outskirts. What is the total population?
- 372 more than 5 116
- $\square$ – 2 342 = 5 146

b) Some practice items for numbers in the 10 000s:

- $\begin{array}{r} 10\ 357 \\ \underline{+\ 42\ 111} \end{array}$
- 34 680 + 21 318
- The sum of $12 045 and $36 920
- Population of 67 306 increased by 2 560

c) Some practice items for numbers in the 10ths and 100ths:

- 45.5 km + 12.3 km
- 235.6 m increased by 22.2 m
- 456.1
+ 502.6
- 23.8 more than 534.

Finding Compatibles (Extension)

This strategy for addition involves looking for pairs of numbers that combine easily to make a sum that is a power of ten that will be easy to work with. In grade 5, this should involve searching for pairs of numbers that add to 10 000, as well as the other powers of ten (10, 100 and 1000) that were the focus in previous grades. Some examples of common compatible numbers are 1 000 and 9 000, 4 000 and 6 000, 3 000 and 7 000, and 7 500 and 2 500. (In some resources, these compatible numbers are referred to as *friendly* numbers or *nice* numbers.) You should be sure that students are convinced that the numbers in an addition expression can be combined in any order (the associative property of addition).

After students have achieved competency using this strategy, you should provide opportunities for them to integrate it with other strategies.

Example:

For 1 + 7 + 9 + 8 + 3, think: 1 + 9 is 10 and 7 + 3 is 10, so 10 + 10 + 8 is 28.

For 30 + 75 + 70 + 25, think: 30 + 70 is 100 and 75 + 25 is 100, so 100 + 100 is 200.

For 300 + 800 + 700 + 600 + 200, think: 300 + 700 is 1000, 800 + 200 is 1000, so 1000 + 1000 + 600 is 2600.

For 250 + 470 + 750, think: 250 and 750 is 1000, so 1000 and 470 is 1470.

For 4 000 + 5 000 + 6 000, think: 4 000 and 6 000 is 10 000, so 10 000 and 5 000 is 15 000.

For 9 500 + 2 200 + 500, think: 9 500 and 500 is 10 000, so 10 000 plus 2 200 is 12 200.

Examples of Some Practice Items

a) Some practice items for numbers in the 10s and 100s:

- 60 + 30 + 40 + 70
- The total of three items costing $75, $95, and $25.
- The sum of 200, 700, 500, 800, and 300.
- The total of three deposits: $50, $460, $950.

b) Some practice items for numbers in the 1000s:

- 5 000 + 3 000 + 5 000 + 7 000
- $2 500 and $3 500 and $7 500.
- 8 000 km + 4 000 km + 6 000 + 7 000 km + 2 000 km
- Total of three items: $1 000, $5 000, $9 000.

Break Up and Bridge (Extension)

This strategy involves starting with the first number in its entirety and adding the place values of the second number, one-at-a-time, starting with the largest value. In grade 5, the practice items should include numbers in the thousands and tens of thousands, as well as numbers in the tens, and hundreds, that were the goals of previous grades. Remember that the practice items should only include questions that require two combinations with one regrouping.

After students have achieved competency using this strategy, you should provide opportunities for them to integrate it with other strategies.

Examples

For 45 + 36, think: 45 and 30 (from the 36) is 75, and 75 plus 6 (the rest of the 36) is 81.
In symbols: 45 + 36 = (45 + 30) = 6 = 75 + 6 = 81.

For 537 + 208, think: 537 and 200 is 737, and 737 plus 8 is 745.
In symbols: 537 + 208 = (537 + 200) + 8 = 737 + 8 = 745.

For 5 300 + 2 800, think: 5 300 and 2 000 (from the 2 800) is 7 300 and 7 300 plus 800 (the rest of 2 800) is 8 100.
In symbols: 5 300 + 2 800 = (5 300 + 2 000) + 800 = 7 300 + 800 = 8 100.

For 34 000 + 27 000, think: 34 000 plus 20 000 is 54 000, and 54 000 plus 7 000 is 61 000.
In symbols: 34 000 + 27 000 = (34 000 + 20 000) = 7 0000 = 54 000 + 7 000 = 61 000.

Examples of Some Practice Items

a) Some practice items for numbers in the 10s and 100s:

- 46 + 36
- 17 more than 64
- The sum of $370 and $440
- 365 increased by 109
- $\square$ – 308 = 409

b) Some practice items for numbers in the 1000s:

- 2 500 + 3 700
- The sum of 16 800 km and 1 300 km
- The total of 4 070 girls and 3 080 boys
- 7 009 increased by 2 008
- $\square$ – 5 600 = 3 900

c) Some practice items for numbers in the 10 000s:

- 46 000 + 37 000
- The total of $66 000 and $15 000
- 56 000 increased by 24 000
- 17 000 km more than 28 000 km
- $\square$ 000 – 47 000 = 35 000

Compensation (Extension)

This strategy involves changing one number in the addition question to a nearby multiple of a power of ten, carrying out the addition using that multiple of a power of ten, and adjusting the answer to compensate for the original change. Students should understand that the number is changed to make it more compatible, and that they have to hold in their memories the amount of the change. In the last step, it is helpful if they remind themselves that they added too much so they will have to take away that amount.

This strategy is perhaps most effective when one of the addends has an 8 or 9 in its lowest place value, although some students are comfortable using it with a 7 as well. In grade 5, the practice items should include numbers in the thousands and tens of thousandsas well as tens and hhundreds that were the goals of previous grades.

After students have achieved competency using this strategy, you should provide opportunities for them to integrate it with other strategies.

Examples

For 52 + 39, think: 40 is easier to work with than 39. Then 52 plus 40 is 92, but I added 1 too many; so, to compensate I subtract one from my answer, 92, to get 91.

For 345 + 198, think: 200 is easier to work with than 198. Then 345 + 200 is 545, but I added 2 too many; so, I subtract 2 from 545 to get 543.

For 4500 + 1900, think: 2000 is easier to work with than 1900. Then 4500 + 2000 is 6500, but I added 100 too many; so, I subtract 100 from 6500 to get 6400.

For 34 000 + 9 900, think: 10 000 is easier to work with than 9 900. Then 34 000 plus 10 000 is 44 000, but I added 100 too many; so, I subtract 100 to get 33 900.

For 59 000 + 25 000, think: 60 000 plus 25 000 is 85 000, but I added 1000 too many; so, I subtract 1000 to get 84 000.

Examples of Some Practice Items

a) Some practice items for numbers in the 10s and 100s:

- 58 + 9
- 49 + 38
- 265 + 399
- $198 more than $465
- □ – 298 = 375

b) Some practice items for numbers in the 1000s and 10 000s:

- 3 456 km increased by 999 km
- The sum of 2 998 and 3 525
- 16 000 + 39 000
- The sum of 28 000 and 65 000
- The total of $38 000 and $9 900
- 74 000 km increased by 18 000
- □ – 29 000 = 54 000

Make Multiples of Powers of 10 (Extension)

In previous grades, students would have been introduced to this strategy as *Make 10*, *Make 10s*, and *Make 100s*. In grade 5, this strategy is extended to thousands and tens of thousands and the strategy name reflects this more generalized application.

Like the *Compensation* strategy, this strategy is best applied when one of the addends has an 8 or 9 in its lowest place value, and it makes use of the compatibility of multiples of powers of ten in addition. This strategy, however, involves getting the amount needed to make one addend a multiple of a power of ten from the other addend, thus changing both addends to numbers that are easier to combine. A common error is for students to forget that both addends have changed; this means that more has to be kept in their short-term memories. Therefore, questions used for reinforcement should not involve too many non-zero digits.

The *Compensation* and the *Make-Multiples-of-Powers-of-10* strategies should be compared so students are clear about how they are alike and how they are different because both strategies are appropriately applied to the same questions.

After students have achieved competency using this strategy, you should provide opportunities for them to integrate it with other strategies.

Examples

For 92 + 69, think: If 1 is taken from 92 and given to 69, the question becomes 91 + 70, which is easier to add to get 161.

For 298 + 345, think: If 2 is taken from 345 and given to 298, the question becomes 300 + 343, which is easier to add to get 643.

For 650 + 190, think: If 10 is taken from 650 and given to 190, the question becomes 640 + 200, which is easier to add to get 840.

For 4 700 + 1 900, think: If 100 is taken from the first addend and given to 1 900, the question becomes 4 600 + 2 000 which is easier to add to get 6 600.

For 34 000 + 28 000, think: If 2 000 is taken from the first addend and given to the second addend, the question becomes 32 000 + 30 000, which is easier to add to get 62 000.

For 56 700 + 3 900, think: If 100 is taken from the first addend and given to the second addend, the question becomes 56 600 + 4 000, which is easier to add to get 60 600.

Examples of Some Practice Items

a) Some practice items for numbers in the 10s and 100s:

- 45 + 29
- The total of 19 kg and 65 kg
- 277 + 499
- $298 more than $465
- $\square$ – 598 = 342

b) Some practice items for numbers in the 1000s and 10 000s:

- 6 476 increased by 999
- The sum of 1 998 km and 6 425 km
- 32 000 + 49 000
- The sum of 18 000 and 46 000
- The total of 78 200 km and 9 900 km
- $56 000 increased by $18 000

B. Subtraction — Mental Calculation

Subtraction Facts Applied to Multiples of Powers of 10 (Extension)

This strategy applies to calculations involving the subtraction of two numbers with the same place values and with only one non-zero digit. Students applied this strategy in previous grades to numbers in the tens, hundreds, and thousands. In grade 5, the application of this strategy should be extended to numbers in tens of thousands and in tenths. The strategy involves subtracting the single non-zero digits as if they were the single-digit subtraction facts and then attaching the appropriate place-value name and symbols. For tens of thousands, however, some students may prefer to subtract the tens parts of the tens of thousands (see example 4 below). This strategy should be reviewed and modeled with base-10 blocks so students understand that 7 blocks subtract 3 blocks will be 4 blocks whether those blocks are small cubes, rods, flats, or large cubes.

Since this strategy rests on students' knowledge of subtraction facts, the facts should be reviewed and consolidated. If some students need remediation on the subtraction facts, information can be found in the document for mental math in grade 3. The principal strategy advocated for these facts is the *Think-Addition* strategy, although the *Back-Through-10* strategy and the *Up-Through-10* strategy are also helpful when the minuends are greater than 10.

After students have achieved competency using this strategy, you should provide opportunities for them to integrate it with other strategies.

Examples

For 80 – 30, think: 8 tens subtract 3 tens is 5 tens, or 50; or think: 8 subtract 3 is 5, but this is 5 tens, so the answer is 50.

For 1500 – 600, think: 15 hundreds subtract 6 hundreds is 9 hundreds, or 900; or think: 15 subtract 6 is 9, but this is 9 hundreds, so the answer is 900.

For 6 000 – 2 000, think: 6 thousands subtract 2 thousands is 4 thousands, or 4 000; or think: 6 subtract is 4, but this is 4 thousands, so the answer is 4 000.

For 90 000 – 40 000, think: 9 subtract 4 is 5, but this is tens of thousands, so the answer is 50 000; or think: 90 thousand subtract 40 thousand is 50 thousand or 50 000.

For 0.8 – 0.5, think: 8-tenths subtract 5-tenths is 3-tenths, or 0.3; or think: 8 subtract 5 is 3, but this is tenths, so the answer is 0.3.

For 1.4 – 0.7, think: 14-tenths – 7-tenths is 7-tenths, or 0.7; or think: 14 subtract 7 is 7, but this is tenths, so the answer is 0.7.

Examples of Some Practice Items

a) Some practice items for numbers in the 10s, 100s, and 1000s:

- 120 – 70
- $20 less than $90
- 700 kg decreased by 300 kg
- The difference between 1100 km and 400 km
- 6000 minus 1000
- $13 000 less $6000
- □ + 4 000 = 12 000

b) Some practice items for numbers in the 10 000s:

- 40 000 – 10 000
- 80 000 minus 20 000
- The difference between $90 000 and $50 000
- 120 000 km decreased by 30 000
- 60 000 + $\square$ 000 = 130 000

c) Some practice items for numbers in the tenths:

- 0.7 kg – 0.2 kg
- The difference between 1.5 km and 0.6 km
- 0.5 m less than 0.8 m
- 1.6 kg decreased by 0.9 kg
- 0.8 + $\square$ = 1.2

Quick Subtraction (Extension)

This strategy is actually the *Front-End* strategy applied to subtraction questions that involve no regrouping. If questions only require two subtractions to get an answer, students should be able to do them mentally. However, questions involving three, or more, subtractions should be presented visually with students quickly recording their answers on paper. While this is a pencil-and-paper strategy for these questions because answers will always be recorded on paper before answers are read, it is included here as a mental math strategy because most students will do all the subtractions in their heads starting at the front end.

This strategy requires students to holistically examine each question to confirm there will be no regrouping. This habit of holistically examining each question as a first step in determining the most efficient strategy needs to pervade all mental math lessons. (One suggestion for an activity during the discussion of this strategy is to present students with a list of 20 questions, some of which do require regrouping, and direct students to apply quick subtraction to the appropriate questions and leave out the other ones.) It is important to present examples of these subtraction questions in both horizontal and vertical formats. Students should have applied this strategy to 3-digit and 4-digit numbers up to the end of grade 4, so in grade 5 they should apply it to 5-digit numbers as well. The numbers should include decimal examples as well as whole number examples.

Most likely, students will subtract the digits in corresponding place values of the minuend and subtrahend without consciously thinking about the names of the place values. Therefore, in the discussion of the questions, you should encourage students to read the numbers correctly and to use place-value names. This will reinforce place-value concepts at the same time as subtraction is reinforced.

If there are only two combinations in the process, students should be able to hold the answer in their memories and orally state it. For more than two combinations students should record each place value and read the answer.

After students have achieved competency using this strategy, you should provide opportunities for them to integrate it with other strategies.

Examples

For 560 – 120, think: 500 – 100 is 400 and 60 – 20 is 40, so the answer is 440. (Record the answer if required.)

For 568 – 135, think and record each difference: Subtract 100 from 500, 30 from 60, and 5 from 8 to get 433; or think and record each resultant digit: 5 – 1 = 4, 6 – 3 = 3, 8 – 5 = 3, so the answer is 433 (four hundred thirty-three).

For 4 070 – 3 030, think: 4 000 – 3000 is 1 000 and 70 – 30 is 40, so the answer is 1 040. (Record answer if required.)

For 4568 – 1135, think and record each difference: Subtract 1000 from 4000, 100 from 500, 30 from 60, and 5 from 8 to get 3433; or think and record each resultant digit: 4 – 1 = 3, 5 – 1 = 4, 6 – 3 = 3, 8 – 5 = 3, so the answer is 3433 (three thousand, thirty-three).

For 87 000 – 32 000, think: 80 000 – 30 000 is 50 000 and 7 000 – 2 000 is 5 000 so the answer is 55 000. (Record if required.)

For 25 786 – 12 125, think and record each subtraction: Subtract 10 000 from 20 000, 2000 from 5000, 100 from 700, 20 from 80, and 5 from 6 to get 13 661; or think and record each difference: 2 – 1 = **1**, 5 – 2 = **3**, 7 – 1 = **6**, 8 – 2 = **6**, and 6 – 5 = **1**, so the answer is 13 661(thirteen thousand, six hundred sixty-six).

For 345.8 – 112.4, think and record each subtraction: Subtract 100 from 300, 10 from 40, 2 from 5, and 4-tenths from 8 tenths to get 233.4; or think and record each digit: 3 – 1 = **2**, 4 – 1 = **3**, 5 – 2 = **3**, and 8 – 4 = **4**, so the answer is 233.4 (two hundred thirty-three and four-tenths).

Examples of Some Practice Items

a) Some practice items that can be done mentally with recording an option:

- 56 – 21
- 604 – 203
- 590 – 230
- 6 700 – 1 100
- 4 080 – 1 020
- 14 000 – 2 000
- 38 000 – 1 500

b) Some practice items for numbers in the 100s, and 1000s:

- $\begin{array}{r} 537 \\ \underline{-101} \end{array}$
- 304 fewer people than 8605 people
- $3245 less than $7366
- The difference between 1225 km and 3575 km
- Subtract 575 from 3889

c) Some practice items for numbers in the 10 000s:
- 45 678 – 21 543
- $\begin{array}{r} 83\ 419 \\ -\ \underline{21\ 417} \end{array}$
- The difference between $96 475 and $5 125
- 75 575 km decreased by 31 235

d) Some practice items for numbers involving tenth/hundredths:
- 213.7 kg – 101.2 kg
- The difference between 456.9 km and 45.6 km
- 45.12 m less than 57.75 m
- $\begin{array}{r} 575.86 \\ -\underline{125.36} \end{array}$

Back-Through a Multiple of a Power of 10 (Extension)

This strategy involves subtracting a part of the subtrahend to get to the nearest multiple of a power of ten, and then subtracting the rest of the subtrahend. This strategy is most effective when the subtrahend is relatively small compared to the minuend.

In previous grades, students would have been introduced to this strategy as *Back-Through 10* and *Back-Through 10s/100s*. In grade 5, the strategy is extended to going back through thousands and tens of thousands, so the strategy name reflects the more generalized application.

After students have achieved competency using this strategy, you should provide opportunities for them to integrate it with other strategies.

Examples

For 35 – 8, think: 35 subtract 5 (one part of the 8) is 30, and 30 subtract 3 (the other part of the 8) is 27.

For 530 – 70, think: 530 subtract 30 (one part of the 70) is 500, and 500 subtract 40 (the other part of the 70) is 460.

For example: For 8 600 – 700, think: 8 600 subtract 600 (one part of the 700) is 8 000 and 8 000 subtract 100 (the rest of the 700) is 7 900.

For 74 000 – 9 000, think: 74 000 subtract 4 000 (one part of the 9 000) is 70 000, and 70 000 subtract 5 000 (the rest of the 9 000) is 65 000.

Examples of Some Practice Items

a) Some practice items for numbers in the 10s, 100s, and 1000s:
- $\begin{array}{r} 57 \\ -\underline{8} \end{array}$
- 9 fewer people than 92 people
- $40 less than $210
- The difference between 630 km and 80 km
- Subtract 600 from 2 300
- 7 500 less 700
- $\square$ + 500 = 3 200

b) Some practice items for numbers in the 10 000s:

- 45 000 – 8 000
- 83 400 minus 600
- The difference between $42 000 and $7 000
- 33 000 km decreased by 5 000 km

Up-Through a Multiple of a Power of 10 (Extension)

This strategy involves finding the difference between the two numbers in two steps starting from the smaller: first, find the difference between the subtrahend and the next multiple of a power of ten, then find the differenc*e* between that multiple of a power of ten and the minuend, and finally add these two differences to get the total difference. This strategy is particularly effective when the two numbers involved are quite close together, although in making change in money situations, this is the principal strategy that traditionally has been used, regardless of the difference. For example, to get the change from a $20-bill for an item that costs $6.95, you select a nickel to get to $7, a $1 coin and a $2 coin to get to $10, and a $10-bill to get to $20.

In previous grades, students would have been introduced to this strategy as *Up-Through 10* and *Up-Through 10s/100s*. In grade 5, because the strategy is extended to going up through thousands, tens of thousands, ones, and tenths, the strategy name reflects the more generalized application.

After students have achieved competency using this strategy, you should provide opportunities for them to integrate it with other strategies.

Examples

For example: For 84 – 77, think: It is 3 from 77 to 80 and 4 from 80 to 84, so the total difference is 3 plus 4, or 7.

For 613 – 594, think: It is 6 from 594 to 600 and 13 from 600 to 613, so the total difference is 6 plus 13, or 19.

For 2 310 – 1 800, think: It is 200 from 1 800 to 2 000 and 310 from 2 000 to 2 310, so the total difference is 200 plus 310, or 510.

For 57 000 – 49 000, think: It is 1 000 from 49 000 to 50 000 and 7 000 from 50 000 to 57 000, so the total difference is 1 000 + 7 000, or 8 000.

Examples of Some Practice Items

a) Some practice items for numbers in the 10s, 100s, and 1000s:

- $\begin{array}{r} 57 \\ \underline{-48} \end{array}$
- 92 – 86
- $140 less than $210
- The difference between 630 km and 580 km
- 2 400 minus 1 700
- 8 500 decreased by 7 800

b) Some practice items for numbers in the 10 000s:

- 45 000 – 38 000
- 83 000 less 79 000
- The difference between $42 000 and $35 000
- 35 000 km subtract 26 000 km
- 63 000 + $\square$ = 71 000

Break Up and Bridge (Extension)

This strategy involves starting with the minuend in its entirety and subtracting the values in the place values of the subtrahend, one-at-a-time, starting with the largest. If students were modelling subtraction on a number line, they would probably naturally use this strategy. This strategy was introduced in grade 4 and applied to 2-digit and 3-digit whole numbers. In grade 5, the strategy should be extended to 4-digit and 5-digit whole numbers.

After students have achieved competency using this strategy, you should provide opportunities for them to integrate it with other strategies.

Examples

For 92 – 26, think: Start with 92 and subtract 20 (the tens place of 26) to get 72, and then subtract 6 (the ones place in 26) from 72 to get 66.

For 745 – 207, think: Start with 745 and subtract 200 (the hundreds place in 207) to get 545, and then subtract 7 (the ones place in 207) from 545 to get 538.

For 860 – 370, think: Start with 860 and subtract 300 (the hundreds place in 370) to get 560, and then subtract 70 (the tens place in 370) from 560 to get 490. (*Likely a* Back-Through-100s *strategy would be used in the last step.)*

For 8 300 – 2 400, think: Start with 8 300 and subtract 2 000 to get 6 300, and then subtract 400 from 6 300 to get 5 900. (*Likely a* Back-Through-1000s *strategy would be used in the last step.)*

For 5 750 – 680, think: Start with 5 750 and subtract 600 to get 5 150, and then subtract 80 from 5 150 to get 5 070. (*Likely a* Back-Through-100 *strategy would be used in the last step.)*

For 47 000 – 28 000, think: Start with 47 000 and subtract 20 000 to get 27 000, and then subtract 8 000 from 27 000 to get 19 000. (*Likely a* Back-Through-10 000s *strategy would be used in the last step.)*

For 24 500 – 2 700, think: Start with 24 500 and subtract 2 000 to get 22 500, and then subtract 700 from 22 500 to get 21 800. (*Likely a* Back-Through-100s *strategy would be used in the last step.)*

Examples of Some Practice Items

a) Some practice items for numbers in the 10s and 100s:

- $\begin{array}{r} 74 \\ \underline{-36} \end{array}$
- 53 – 25
- $306 less than $870
- The difference between 640 km and 170 km
- 750 minus 260

- 803 decreased by 306

b) Some practice items for numbers in the 1000s:

- 5 400 – 1 500
- 7 100 less 2 600
- The difference between $8 020 and $3 050
- $\begin{array}{r} 6\ 425 \\ -\underline{307} \end{array}$
- $\square$ + 7 500 = 12 200

c) Some practice items for numbers in the 10 000s:

- 63 000 – 25 000
- The difference between 66 500 km and 18 000 km
- $75 500 – $4 900
- 10 600 less than 32 100

Compensation (Extension)

This strategy for subtraction involves changing the subtrahend to the next multiple of a power of ten, carrying out the subtraction, and then adjusting the answer to compensate for the difference between the original subtrahend and the multiple of a power of ten that was used. Students should understand that the subtrahend is changed to make it more compatible, and that they have to hold in their memories the amount of that change. In the last step, it is helpful if they remind themselves that they subtracted too much, so they will have to add that amount back on. This strategy is most effective when the digit in the lowest non-zero place value is an 8 or a 9.

In grade 4, students applied this strategy to 2-digit and 3-digit whole numbers. In grade 5, this strategy should be extended to numbers in the 4-digit and 5-digit whole numbers.

After students have achieved competency using this strategy, you should provide opportunities for them to integrate it with other strategies.

Examples

For 36 – 8, think: 36 – 10 = 26, but I subtracted 2 too many; so, I add 2 to 26 and get 28.

For 85 – 29, think: 85 – 30 = 55, but I subtracted 1 too many; so, I add 1 to 55 to get 56.

For 145 – 99, think: 145 – 100 = 45, but I subtracted 1 too many; so, I add 1 to 45 to get 46.

For 750 – 190, think: 750 – 200 = 550, but I subtracted 10 too many; so, I add 10 to 550 to get 560.

For 5 700 – 997, think: 5 700 – 1000 is 4 700, but I subtracted 3 too many; so, I add 3 to 4 700 to get 4 703.

For 3 600 – 990, think: 3 600 – 1000 is 2 600, but I subtracted 10 too many; so, I add 10 to 2 600 to get 2 610.

For 24 000 – 995, think: 24 000 – 1000 is 23 000, but I subtracted 5 too many; so, I add 5 to 23 000 to get 23 005.

For 56 000 – 980, think: 56 000 – 1000 is 55 000, but I subtracted 20 too many; so, I add 20 to 55 000 to get 55 020.

For 47 000 – 19 000, think: 47 000 – 20 000 is 27 000, but I subtracted 1000 too many; so, I add 1000 to 27 000 to get 28 000.

Examples of Some Practice Items

a) Some practice items for numbers in the 10s and 100s:

- $\begin{array}{r} 57 \\ -\underline{29} \end{array}$
- 92 less 38
- $399 less than $875
- The difference between 630 km and 298 km
- 450 minus 190
- 830 decreased by 380

b) Some practice items for numbers in the 1000s:

- 5 700 – 997
- 4 500 less 1 990
- The difference between $7 500 and $2 900
- 6 500 km subtract 1 980 km
- 3 999 + $\square$ = 5 200

c) Some practice items for numbers in the 10 000s:

- 23 000 – 1 997
- The difference between 33 000 km and 2 980 km
- $64 000 – $9 900
- Subtract 29 000 from 92 000

Balancing For a Constant Difference (Extension)

For subtraction questions that require regrouping, this strategy can be used most effectively. By adding the same amount to both numbers in order to get the subtrahend to a multiple of a power of 10, any regrouping is eliminated, so the subtraction is much easier to do. This strategy needs to be carefully introduced because students need to be convinced it actually works! They need to understand that by adding the same amount to both numbers, the two new numbers have the same difference as the original two numbers. Examining possible numbers on a metre stick that are a fixed distance apart can help students with the logic of this strategy. (For example, place a highlighter that is more than 10 cm long against a metre stick so that its bottom end is at the 18-cm mark, note where its top end is located, and write the subtraction sentence that gives the length of the highlighter. Now move the marker up the metre stick until the bottom end of the highlighter is at the 20-cm mark, and write the subtraction sentence that now gives the length of the marker. Ask, Is the length of the highlighter the same in both number sentences? Which subtraction would be easier to do?)

This strategy was introduced in grade 4 and applied to 2-digit and 3-digit whole numbers. In grade 5, the strategy should be extended to 4-digit and 5-digit whole numbers.

Because both numbers change in carrying out this strategy, many students may need to record the changed minuend to keep track, especially for numbers greater than 2-digit. This strategy should be compared to the *Compensation* strategy so students see how it is alike and how it is different.

The questions selected for reinforcement of this strategy would include those that appear to require more than two combinations and one regrouping because the strategy eliminates the need to regroup and can reduce the number of combinations. (See the second example below as an illustration.)

After students have achieved competency using this strategy, you should provide opportunities for them to integrate it with other strategies.

Note: This strategy can lead to a very effective pencil-and-paper strategy for questions in which the minuends are multiples of powers of ten. These questions traditionally required subtracting with regrouping from one, or more, zeros; however, if 1 is subtracted from both numbers, the questions will require no regrouping. For example, for 4 000 – 3 467, if 1 is subtracted from both the minuend and the subtrahend, the question becomes 3 999 – 3 466 which is then much easier to subtract by *Quick Subtraction*. This strategy would not be part of mental math time; rather, it could be a discussion in regular math class time.

Examples

For 87 – 19, think: If 1 is added to both numbers, the question becomes 88 – 20 which is easier to subtract to get 68.

For 345 – 198, think: If 2 is added to both numbers, the question becomes 347 – 200 which is easier to subtract to get 147.

For 5 600 – 1 990, think: If 10 is added to both numbers, the question becomes 5 610 – 2000 which is easier to subtract to get 3 610.

For 6000 – 1700, think: If 300 is added to both numbers, the question becomes 6300 – 2000 which is easier to subtract to get 4300.

For 7 800 – 3 998, think: If 2 is added to both numbers, the questions becomes 7 802 – 4 000 which is easier to subtract to get 3 802.

For 45 000 – 19 000, think: If 1 000 is added to both numbers, the question becomes 46 000 – 20 000 which is easier to subtract to get 26 000.

For 67 000 – 29 999, think: If 1 is added to both numbers, the question becomes 67 001 – 30 000 which is easier to subtract to get 37 001.

For 52 000 – 9 800, think: If 200 is added to both numbers, the question becomes 52 200 – 10 000 which is easier to subtract to get 42 200.

Examples of Some Practice Items

a) Some practice items for numbers in the 10s and 100s:

- $\begin{array}{r} 77 \\ -\underline{39} \end{array}$
- 53 – 28
- \$399 less than \$875
- The difference between 600 km and 190 km
- 750 minus 290
- 830 decreased by 380

b) Some practice items for numbers in the 1000s:

- 5 400 – 997
- 7 000 less 3 100
- The difference between $8 500 and $3 900
- $\begin{array}{r} 6\ 500 \\ -\underline{1\ 980} \end{array}$

c) Some practice items for numbers in the 10 000s:

- 43 000 – 2 997
- The difference between 66 000 km and 4 980 km
- $75 000 – $9 900
- Subtract 38 000 from 92 000
- $\square$ + 19 900 = 40 500

C. Multiplication — Fact Learning

Multiplication Fact Learning Strategies (Review)

While the acquisition of the multiplication facts is an outcome for student in grade 4, there may be grade 5 students who have not mastered them. For all students, quick recall of these facts is an essential prerequisite for acquisition of the division facts. Therefore, the facts and related strategies should be reviewed before other multiplication or division strategies are undertaken. For you information and use, the following information is copied from the mental math document for grade 4.

In grade 4, students are to know the multiplication facts with at least a 3-second response by the end of the year. This is done through learning a series of strategies, each of which addresses a cluster of facts. Each strategy is introduced, reinforced, and assessed before being integrated with previously learned strategies. It is important that students understand the logic and reasoning of each strategy, so the introductions of the strategies are very important. As students master each cluster of facts for a strategy, it is recommended that they record these learned facts on a multiplication chart. By doing this, they visually see their progress and are aware of which facts they should be practicing. What follows is a suggested sequence for these strategies.

A. The Twos Facts (Doubles)

This strategy involves connecting the addition doubles to the related "two-times" multiplication facts. It is important to make sure students are aware of the equivalence of commutative pairs (2 × ? and ? × 2); for example, 2 × 7 is the double of 7 and that 7 × 2, while it means 7 groups of 2, has the same answer as 2 × 7. When students see 2 × 7 or 7 × 2, they should think: 7 and 7 are 14. Flash cards displaying the facts involving 2 and the times 2 function on the calculator are effective reinforcement tools to use when learning the multiplication doubles.

It is suggested that 2 × 0 and 0 × 2 be left until later when all the zeros facts will be done.

Examples

For 2 × 9, think: This is 9 plus 9, so the answer is 18.

For 6 × 2, think: This 6 plus 6, so the answer is 12.

B. The Nifty Nine Facts

The introduction of the facts involving 9s should concentrate on having students discover two patterns in the answers; namely, the tens' digit of the answer is one under the number of 9s involved, and the sum of the ones' digit and tens' digit of the answer is 9. For example, for 6 × 9 = 54, the tens' digit in the product is one less than the factor 6 (the number of 9s) and the sum of the two digits in the product is 5 + 4 or 9. Because multiplication is commutative, the same thinking would be applied to 9 × 6. Therefore, when asked for 3 × 9, think: the answer is in the 20s (the decade of the answer) and 2 and 7 add to 9; so, the answer is 27. You could help students master this strategy by scaffolding the thinking involved; that is, practice presenting the multiplication expressions and just asking for the decade of the answer; practice presenting the students with a digit from 1 to 8 and asking them the other digit that they would add to your digit to get 9; and conclude by presenting the multiplication expressions and asking for the answers and discussing the steps in the strategy.

Another strategy that some students may discover and/or use is a compensation strategy, where the computation is done using 10 instead of 9 and then adjusting the answer to compensate for using 10, rather than 9. For example, for 6 × 9, think: 6 groups of 10 is 60 but that is 6 too many (1 extra in each group), so 60 subtract 6 is 54. Because this strategy involves multiplication followed by subtraction, many students find it more difficult than the two-pattern strategy.

While 2 × 9 and 9 × 2 could be done by this strategy, these two nines facts were already handled by the twos facts. This *nifty-nine* strategy is probably most effective for factors 3 to 9 combined with the factor 9, leaving the 0s and 1s for later strategies.

Examples

For 5 × 9, think: The answer is in the 40s, and 4 and 5 add to 9, so 45 is the answer.

For 9 × 9, think: The answer is in the 80s, and 8 and 1 add to 9, so 81 is the answer.

C. The Fives Facts

Many students probably have been using a skip-counting-by-5 strategy when 5 has been a factor; however, this strategy is difficult to apply in 3 seconds, or less, for all combinations, and often results in students' using fingers to keep track. Therefore, students need to adopt a more efficient strategy.

If the students know how to read the various positions of the minute hand on an analog clock, it is easy to make the connection to the multiplication facts involving 5s. For example, if the minute hand is on the 6 and students know that means 30 minutes after the hour, then the connection to 6 × 5 = 30 is easily made. This is why you may see the Five Facts referred to as the "clock facts." This would be the best strategy for students who can proficiently tell time on an analog clock.

Another possible strategy involves the patterns in the products. While most students have observed that the Five Facts have a 0 or a 5 as a ones' digit, some have also noticed other patterns. One pattern is that the ones' digit is a 0 if the number of 5s involved is even or the ones' digit is 5 if the number of 5s involved is odd. Another pattern is that the tens' digit of the answer is half the numbers of 5s involved, or half the number of 5s rounded down. For example, the product of 8 and 5 ends in 0 because there are 8 fives and the tens' digit is 4 because 4 is half of 8; therefore, 8 × 5 is 40. The product of 7 and 5 ends in 5 because 7 is odd and the tens' digit is 3 because 3 is half of 7 rounded down; therefore, 7 × 5 is 35.

While these strategies apply to 2 × 5, 5 × 2, 5 × 9, and 9 × 5, these facts were also part of the *twos facts*, and *nines facts*. The *fives facts* involving zeros are probably best left for the zeros facts since the minute-hand approach has little meaning for 0.

Examples

For 5 × 8, think: When the minute hand is on 8, it is 40 minutes after the hour, so the answer is 40.

For 3 × 5, think: When the minute hand is on 3, it is 15 minutes after the hour, so the answer is 15.

D. The Ones Facts

While the ones facts are the "no change" facts, it is important that students understand why there is no change. Many students get these facts confused with the addition facts involving 1. To understand the ones facts, knowing what is happening when we multiply by one is important. For example 6 × 1 means *six groups of 1* or 1 + 1 + 1 + 1 + 1 + 1 and 1 × 6 means *one group of 6*. It is important to avoid teaching arbitrary rules such as "any number multiplied by one is that number". Students will come to this rule on their own given opportunities to develop understanding. Be sure to present questions visually and orally; for example, "4 groups of 1" and 4 × 1; and "1 group of 4" and 1 × 4.

While this strategy applies to 2 × 1, 1 × 2, 1 × 5, and 5 × 1, these facts have also been handled previously with the other strategies.

Examples

For 8 × 1, think: Eight 1s make 8.

For 1 × 7, think: One 7 is 7.

E. The Tricky Zeros Facts

As with the ones facts, students need to understand why these facts all result in zero because they are easily confused with the addition facts involving zero; thus, the zeros facts are often "tricky." To understand the zeros facts, students need to be reminded what is happening by making the connection to the meaning of the number sentence. For example: 6 × 0 means "six 0's or "six sets of nothing." This could be shown by drawing six boxes with nothing in each box. 0 × 6 means "zero sets of 6." This is much more difficult to conceptualize; however, if students are asked to draw two sets of 6, then one set of 6, and finally zero sets of 6, where they don't draw anything, they will realize why zero is the product. Similar to the previous strategy for teaching the ones facts, it is important not to teach a rule such as "any number multiplied by zero is zero". Students will come to this rule on their own, given opportunities to develop understanding.

Examples

For 7 × 0, think: Having seven zeros means having a total of zero.

For 0 × 8, think: Having no eights means having zero.

F. The Threes Facts

The way to teach the threes facts is to develop a "double plus one more set" strategy. You could have students examine arrays with three rows. If they cover the third row, they easily see that they have a "double" in view; so, adding "one more set" to the double should make sense to them. For example, for 3 × 7, think: 2 sets of 7(double) plus one set of 7 or (7 × 2) + 7 = 14 + 7 = 21. This strategy uses the doubles facts that should be well known before this strategy is introduced; however, there will need to be a discussion and practice of quick addition strategies to add on the third set.

While this strategy can be applied to all facts involving 3, the emphasis should be on 3 × 3, 3 × 4, 4 × 3, 3 × 6, 6 × 3, 3 × 7, 7 × 3, 3 × 8, and 8 × 3, all of which have not been addressed by earlier strategies.

Examples

For 3 × 6, think: Two sixes make 12, plus one more six is 18.

For 4 × 3, think: Two fours make 8, plus one more 4 is 12.

G. The Fours Facts

The way to teach the fours facts is to develop a "double-double" strategy. You could have students examine arrays with four rows. If they cover the bottom two rows, they easily see they have a "double" in view and another "double" covered; so, doubling twice should make sense. For example: for 4 × 7, think: 2 × 7(double) is 14 and 2 × 14 is 28. Discussion and practice of quick mental strategies for the doubles of 12, 14, 16 and 18 will be required for students to master their fours facts. (One efficient strategy is front-end whereby you double the ten, double the ones, and add these two results together. For example, for 2 × 16, think: 2 times 10 is 20, 2 times 6 is 12, so 20 and 12 is 32.)

While this strategy can be applied for all facts involving 4, the emphasis should be on 4 × 4, 4 × 6, 6 × 4, 4 × 7, 7 × 4, 4 × 8, and 8 × 4, all of which have not been addressed by earlier strategies.

Examples

For 4 × 6, think: Double six is 12, and double 12 is 24.

For 8 × 4, think: Double eight is 16, and double 16 is 32.

H. The Last Nine Facts

After students have worked on the above seven strategies for learning the multiplication facts, there are only ***nine*** facts left to be learned. These include: 6 × 6; 6 × 7; 6 × 8; 7 × 7; 7 × 8; 8 × 8; 7 × 6; 8 × 7; and 8 × 6. At this point, the students themselves can probably suggest strategies that will help with quick recall of these facts. You should put each fact before them and ask for their suggestions.

Among the strategies suggested might be one that involves decomposition and the use of helping facts.

Examples

For 6 × 6, think: 5 sets of 6 is 30 plus 1 more set of 6 is 36.

For 6 × 7 or 7 × 6, think: 5 sets of 6 is 30 plus 2 more sets of 6 is 12, so 30 plus 12 is 42.

For 6 × 8 or 8 × 6, think: 5 sets of 8 is 40 plus 1 more set of 8 is 48. Another strategy is to think: 3 sets of 8 is 24 and double 24 is 48.

For 7 × 7, think: 5 sets of 7 is 35, 2 sets of 7 is 14, so 35 and 14 is 49. (This is more difficult to do mentally than most of the others; however, many students seem to commit this one to memory quite quickly, perhaps because of the uniqueness of 49 as a product.)

For 7 × 8, think: 5 sets of 8 is 40, 2 sets of 8 is 16, so 40 plus 16 is 56. (Some students may notice that 56 uses the two digits 5 and 6 that are the two counting numbers before 7 and 8.)

For 8 × 8, think: 4 sets of 8 is 32, and 32 doubled is 64. (Some students may know this as the number of squares on a chess or checker board.)

D. Multiplication — Mental Calculation

Multiplication Facts Applied to Multiples of Powers of 10 (New)

After the multiplication facts and the related strategies are reviewed, or at the same time, these facts should be extended to the 10s, 100s, and 1000s, with one non-zero digit in these numbers, multiplied by 1-digit numbers. A simple strategy for these extensions is to combine the single non-zero digits as if they were single-digit multiplication facts and then attach the appropriate number of zeros to the result. Students, however, should be encouraged to approach these questions as modeled in the "think lines" in the examples provided, so the place value of the answers are known before any multiplication is undertaken. It would be beneficial to connect these products to groups of base-10 blocks. For example, 6 groups of 3 small cubes, 6 groups of 3 rods, 6 groups of 3 flats, and 6 groups of 3 large cubes all result in 18 blocks, whether they are 1s, 10s, 100s, or 1000s.

After students have achieved competency using this strategy, you should provide opportunities for them to integrate it with other strategies.

Examples

For 3 × 70, think: The answer will be tens and the number of those tens is 3 × 7 or 21, so the answer is 21 tens or 210.

For 6 × 900, think: The answer will be hundreds and the number of those hundreds is 6 × 9 or 54, so the answer is 54 hundreds, or 5400.

For 4 × 6 000, think: The answer will be thousands and the number of those thousands is 4 × 6 or 24, so the answer is 24 thousands, or 24 000.

Examples of Some Practice Items

- 6 × 80
- The number of minutes in 4 hours
- 300 × 9
- The product of 5 and 700
- 3 times $9 000
- Seven 8000s
- 7 000 × $\square$ = 21 000

The multiplication facts should also be applied to the product of two multiples of 10 (less than 100). The strategy for finding the product of two multiples of 10 is to realize that the product will be a number of hundreds and that number of hundreds is found by multiplying the two single digits in the tens places. To convince students that these questions always result in hundreds, model some products, such as 20 × 30 and 30 × 40, as arrays with base-10 flats. For example, 20 × 30 would be modelled as 2 rows of 3 flats: the dimensions of the rectangle formed by these 6 flats would be 20 by 30; therefore, 20 × 30 is 600.

This strategy should be introduced in grade 5 before students begin the introduction of pencil-and-paper strategies for products of two 2-digit numbers, so they can estimate these products before carrying out a pencil-and-paper strategy.

Examples

For 30 × 80, think: The product of two tens is hundreds, so 3 times 8 is 24 hundreds, or 2 400.

For 50 × 70, think: The product of two tens is hundreds, so 5 times 7 is 35 hundreds, or 3 500.

For 60 × 50, think: The product of two tens is hundreds, so 6 × 5 is 30 hundreds, or 3000—three thousand.

Examples of Some Practice Items

- 20 × 70
- 60 times 30
- The product of 40 and 90
- 30 cartons @ $50 per carton

Front-End Multiplication (New)

This strategy involves mentally multiplying each place value, starting with the largest, and mentally combining the results to get the answer. In grade 5, this strategy is applied to all 2-digit numbers and to 3-digit numbers that have a zero in the ones or in the tens place.

After students have achieved competency using this strategy, you should provide opportunities for them to integrate it with other strategies.

Examples

For 4 × 32, think: 4 times 30 is 120 and 4 times 2 is 8, so the answer is 128.

For 3 × 75, think: 3 times 70 is 210 and 3 times 5 is 15, so the answer is 210 plus 15 or 225 (two hundred twenty-five).

For 8 × 510, think: 8 times 500 is 4000 and 8 times 10 is 80, so the answer is 4080.

For 3 × 506, think: 3 times 500 is 1500 and 3 times 6 is 18, so the answer is 1518.

For 6 × 230, think: 6 times 200 is 1200 and 6 times 30 is 180, so the answer is 1200 plus 180 or 1380 (thirteen hundred eighty).

Examples of Some Practice Items

- 4 × 82
- The number of hours in 3 days
- 310 × 9
- The product of 5 and 707
- 6 times $930
- 650 × 7

Quick Multiplication (New)

This pencil-and-paper strategy is applied to questions in which there are more than two sub-products but with no regrouping needed. In grade 5, the questions will involve 3-digit and 4-digit numbers multiplied by single-digit numbers. The questions are usually presented visually rather than orally, and students always record their answers on paper. It is included here as a mental math strategy because students should do all the combinations in their heads starting at the front end, recording only the results of each sub-product. Most likely, students will simply multiply the digits of the number, starting at the front end, without being conscious of the place value names; therefore, when you call upon them to state their answers, insist that they read the number using correct place-value terminology. In any discussion of the strategy, be sure to use the place value names as shown in the "think line" in the examples provided.

After students have achieved competency using this strategy, you should provide opportunities for them to integrate it with other strategies.

Examples

For 423 × 2, think: 2 times 400 is 800, 2 times 20 is 40, 2 times 3 is 6, and 800 + 40 + 6 = 846; or think and record resultant digits: 2 times 4 is 8, 2 times 2 is 4, and 2 times 3 is 6, so the answer is 846 (eight hundred forty-six).

For 3 × 2 132, think: 3 times 2 000 is 6 000, 3 times 100 is 300, 3 times 30 is 90, 3 times 2 is 6, and 6 000 + 300 + 90 + 6 is 6 396; or think and record resultant digits: 3 times 2 is 6, 3 times 1 is 3, 3 times 3 is 9, 3 times 2 is 6, so the answer is 6396 (six thousand, three hundred ninety-six).

Examples of Some Practice Items

- 4 × 821
- The product of 3 and 523
- $3 111 × 9
- The product of 2 and 7 432

Multiplying by Powers of 10

Multiplying by 10, 100, and 1000 (Extension)

This strategy was introduced in grade 4 using 10 and 100, and is extended to 1000 in grade 5. The strategy involves keeping track of how the place values have changed as a result of being multiplied by powers of ten. Introduce these products by considering base-10 block representations. For example, for 10 × 53, display 5 rods and 3 small cubes to represent 53, and think: 10 sets of 5 rods would be 50 rods, or 5 flats, and 10 sets of 3 small cubes would be 30 small cubes, or 3 rods; so, 5 flats and 3 rods represents 530. Through a few similar examples, it becomes clear that multiplying by 10 changes all the place values of a number by one position to the left, because the product is ten times as much.

The most obvious application of these products is the conversion among various units of measure in SI; however, there should only include the most common conversions.

After students have achieved competency using this strategy, you should provide opportunities for them to integrate it with other strategies.

Examples

- For 10 × 67, think: The 6 tens will change to 6 hundreds and the 7 ones will change to 7 tens, so the answer is 670.
- For 10 × 430, think: The 4 hundreds will change to 4 thousands and the 3 tens will change to 3 hundreds, so the answer is 4 300.

Similarly, through modeling with base-10 blocks, it can be shown that multiplying by 100 changes all the place values of a number by two positions to the left because the product is one hundred times as much.

Examples

For 100 × 86, think: The 8 tens will change to 8 thousands and the 6 ones will change to 6 hundreds, so the answer is 8 600; or think: It is easy to see that this is 86 hundreds, or 8600. (*This thinking will hold for all products of 100 with 2-digit whole numbers*.)

For 100 × 580, think: The 5 hundreds will change to 5 ten thousands and the 8 tens to 8 thousands, so the answer is 58 000.

For 100 × 409, think: The 4 hundreds will change to 4 ten thousands and the 9 ones to 9 hundreds, so the answer is 40 900.

Using a calculator to multiply various numbers by 1000, students should detect that the pattern of changing place values continues with multiplying by 1000 changing all the place values of a number by three positions to the left because the product is 1000 times as much. For products of 1000 with 2-digit and 3-digit whole numbers, the results are easy to determine because the 2-digit and 3-digit whole numbers are the number of sets of thousands.

Examples

For 1000 × 86, think: It is easy to see that this will be 86 thousands, or 86 000.

For 1000 × 460, think: It is easy to see that this will be 460 thousands, or 460 000.

Note: While some students may see the pattern that one zero gets attached to the original number when multiplying by 10, and two zeros get attached when multiplying by 100, this is not the best way to introduce these products. Later, when students are working with decimals, such as 100 × 0.12, attaching two zeros will not result in the correct answer while using the “two-place-value-change strategy” will.

Examples of Some Practice Items

- 10 × 89
- The number of decimetres in 452 m
- The product of 65 and 100
- The number of centimetres in 125 m
- 78 × 1000
- The number of metres in 350 km

Multiplying by 0.1, 0.01, and 0.001 (New)

This is equivalent to dividing by 10, 100, and 1000. These products can be found by continuing the pattern established by multiplying by 10, 100, and 1000—keeping track of the effect on the place values of the other factor. Through modelling some product, such as 0.1 × 45 and 0.1 × 130 using base-10 blocks, students will see that hundreds become tens, tens become ones, and ones become tenths because the product is one-tenth as much: multiplying by 0.1 changes all the place values of the other factor by one place to the right.

Similarly, it can be established that multiplying by 0.01 changes all the place values of the other factor by two place values to the right, because the product is one-hundredth as much; and that multiplying by 0.001 changes all the place values of the other factor by three place values to the right, because the product is one-thousandth as much.

To carry out these products mentally, you can just note the effect on the largest place value of the other factor and be confident that the other place values will change accordingly.

After students have achieved competency using this strategy, you should provide opportunities for them to integrate it with other strategies.

Examples

For 0.1 × 5, think: By multiplying by 1-tenth ones become tenths (one place value to the right), so the answer is 0.5 (5-tenths).

For 0.1 × 76, think: By multiplying by 1-tenth tens become ones, so the answer is 7.6 (seven and 6-tenths).

For 0.1 × 340, think: By multiplying by 1-tenth hundreds become tens, so the answer is 34 (thirty-four).

For 0.1 × 567, think: By multiplying by 1-tenth, hundreds become tens, so the answer is 56.7 (fifty-six and 7-tenths).

For 0.01 × 60, think: By multiplying by 1-hundredth tens become tenths (two place values to the right), so the answer is 0.6 (six-tenths).

For, 0.01 × 37, think: By multiplying by 1-hundredth tens become tenths, so the answer is 0.37 (37-hundredths).

For 0.01 × 480, think: By multiplying by 1-hundredth hundreds become ones (two place values to the right), so the answer is 4.8 (four and 8-tenths).

For 0.001 × 125, think: By multiplying by 1-thousandth hundreds become tenths (three place values to the right), so the answer is 0.125 (125-thousandths).

For 0.001 × 90, think: By multiplying by 1-thousandth tens become hundredths (three place values to the right), so the answer is 0.09 (9-hundredths).

Examples of Some Practice Items

- 0.1 × 20
- 76 times 0.1
- 1-tenth of $458
- The product of 0.01 and 90
- 32 km × 0.01
- 001 of $180
- 0.001 × 7
- 32 × 0.001
- 1-thousandth of 750
- times 425 km

Compensation (New)

This strategy for multiplication involves changing one of the factors to a nearby multiple of a power of 10, carrying out the multiplication, and adjusting the answer to compensate for the change that was made. This strategy is most effective when one of the factors has an 8 or 9 in its lowest place value, so that the adjustment at the end of the process is easily done mentally.

After students have achieved competency using this strategy, you should provide opportunities for them to integrate it with other strategies.

Examples

For 6 × 39, think: 6 times 40 is 240, but this is six more than it should be because 1 more was put into each of the six groups; therefore, the answer is 240 – 6 = 234.

For 9 × 26, think: 10 times 26 is 260, but this is one group of 26 too much; therefore, the answer is 260 – 26 = 234.

For 7 × 198, think: 7 times 200 is 1400, but this is 14 more than it should be because there were 2 extra in each of the 7 groups; therefore, the answer is 1400 – 14 = 1 386.

For 9 × 150, think: 10 times 150 is 1500, but this is one group of 150 too many; therefore the answer is 1500 – 150 = 1350.

Examples of Some Practice Items

- 5 × 69
- 29 times $7
- The area of a rectangle that is 9 cm by 45 cm
- 6 × 399
- The product of 490 and 4
- 9 trips @ 260 km per trip

Finding Compatible Factors (New)

This strategy for multiplication involves looking for pairs of factors whose product is a power of ten and re-associating these factors to make the overall calculation easier. Students need to be convinced that the order in which the multiplications take place does not matter because of the associative property of multiplication.

After students have achieved competency using this strategy, you should provide opportunities for them to integrate it with other strategies.

Examples

For 2 × 68 × 5, think: 2 times 5 is 10, and 10 times 68 is 680.

For 25 × 63 × 4, think: 4 times 25 is 100, and 100 times 63 is 6 300.

For 2 × 78 × 50, think: 2 times 50 is 100, and 100 times 78 is 7 800.

For 5 × 450 × 2, think: 2 times 5 is 10, and 10 times 450 is 4 500.

For 4 × 56 × 250, think: 4 times 250 is 1000, and 1000 × 56 is 56 000.

Examples of Some Practice Items

- 5 × 87 × 2
- 4 × 65 × 25
- 2 × 320 × 5
- 25 × 92 × 4
- 50 × 48 × 2
- 250 × 56 × 4
- 2 × 23 × 500

E. Division Fact Learning

It could be argued that there is no such thing as division facts. When faced with a division question, most people scan their memories for the related multiplication fact. For example, if asked to find 36 ÷ 4, they ask themselves: 4 times what number is 36? In fact, this is the strategy that would have been introduced when the division concept was introduced and reinforced in the regular classroom. This strategy, however, rests on a thorough knowledge and a quick recall of the multiplication facts. Therefore, you should be sure that your students have mastered their multiplication facts before they attempt to get a 3-second, or less, response to the division facts.

To help students achieve a 3-second, or less, response time for the division facts, it might be helpful to break down the facts into clusters rather than try to reinforce all the facts at once. These clusters can relate to the corresponding multiplication fact clusters (See section C). For example, the 17 non-zero *Twos Facts* in multiplication have 17 corresponding division facts: 18 ÷ 2, 18 ÷ 9, 16 ÷ 2, 16 ÷ 8, 14 ÷ 2, 14 ÷ 7, 12 ÷ 2, 12 ÷ 6, 10 ÷ 2, 10 ÷ 5, 8 ÷ 2, 8 ÷ 4, 6 ÷ 2, 6 ÷ 3, 4 ÷ 2, 2 ÷ 2, 2 ÷ 1. Discuss these related facts; reinforce them until the 3-second, or less, response time is achieved; introduce and reinforce the cluster of 15 new related *Nifty Nines* division facts (18 ÷ 2 and 18 ÷ 9 were already in the first cluster) until the 3-second response time for these facts is achieved; and integrate and reinforce these 32 facts. Continue isolating and reinforcing other clusters and integrating them until students have achieved a 3-second, or less, response time for the full set of division facts.

Because 19 of the 100 multiplication facts involve zero and division by zero is undefined, there are only 81 corresponding division facts.

Possible cluster of division facts:

- From *The Twos Facts in Multiplication (17)*: 18 ÷ 2, 18 ÷ 9, 16 ÷ 2, 16 ÷ 8, 14 ÷ 2, 14 ÷ 7, 12 ÷ 2, 12 ÷ 6, 10 ÷ 2, 10 ÷ 5, 8 ÷ 2, 8 ÷ 4, 6 ÷ 2, 6 ÷ 3, 4 ÷ 2, 2 ÷ 2, 2 ÷ 1
- *From The Nifty Nines Facts in Multiplication (15)*: 81 ÷ 9, 72 ÷ 9, 72 ÷ 8, 63 ÷ 9, 63 ÷ 7, 54 ÷ 9, 54 ÷ 6, 45 ÷ 9, 45 ÷ 5, 36 ÷ 9, 36 ÷ 4, 27 ÷ 9, 27 ÷ 3, 9 ÷ 9, 9 ÷ 1
- *From The Fives Facts in Multiplication (13)*: 40 ÷ 5, 40 ÷ 8, 35 ÷ 5, 35 ÷ 7, 30 ÷ 5, 30 ÷ 6, 25 ÷ 5, 20 ÷ 5, 20 ÷ 4, 15 ÷ 5, 15 ÷ 3, 5 ÷ 5, 5 ÷ 1
- *From The Ones Facts in Multiplication (11)*: 8 ÷ 1, 8 ÷ 8, 7 ÷ 1, 7 ÷ 7, 6 ÷ 1, 6 ÷ 6, 4 ÷ 1, 4 ÷ 4, 3 ÷ 1, 3 ÷ 3, 1 ÷ 1
- *From The Threes Facts in Multiplication (9)*: 24 ÷ 3, 24 ÷ 8, 21 ÷ 3, 21 ÷ 7, 18 ÷ 3, 18 ÷ 6, 12 ÷ 3, 12 ÷ 4, 9 ÷ 3
- *From The Fours Facts in Multiplication (7)*: 32 ÷ 4, 32 ÷ 8, 28 ÷ 4, 28 ÷ 7, 24 ÷ 4, 24 ÷ 6, 16 ÷ 4
- *The Last Facts (9)*: 64 ÷ 8, 56 ÷ 8, 56 ÷ 7, 49 ÷ 7, 48 ÷ 8, 48 ÷ 6, 42 ÷ 7, 42 ÷ 6, 36 ÷ 6

F. Division — Mental Calculation

Dividing by 10, 100, and 1000 (New)

Dividing by 10, 100, and 1000 is equivalent to multiplying by 0.1, 0.01, and 0.001. Through modeling of some questions with base-10 blocks and dialogue, students should be convinced that "one-tenth of a quantity" has the same result as "dividing the quantity by 10." Therefore, the pattern of effects on the place values of the dividends will be the same: logically dividing a quantity by 10, 100, and 1000 will result in smaller quantities because the quantity is being shared. Dividing by 10 has the effect of changing the place values of the dividend one place to the right; dividing by 100, two places to the right; and dividing by 1000, three places to the right.

To carry out these divisions mentally, just note the effect on the largest place value of the dividend and be confident that the other place values will change accordingly.

The most relevant application for these divisions is the changing units in SI. For example, dividing the number of decimetres by 10 will give the equivalent number of metres, dividing the number of centimetres by 100 will give the equivalent number of metres, and dividing the number of metres by 1000 will give the equivalent number of kilometres.

Examples

For 60 ÷ 10, think: 6 tens will become 6 ones, so the answer is 6.

For 32 ÷ 10, think: 3 tens will become 3 ones, so the answer is 3.2.

For 500 ÷ 10, think: 5 hundreds will become 5 tens, so the answer is 50.

For 850 ÷ 10, think: 8 hundreds will become 8 tens, so the answer is 85.

For 6 700 ÷ 10, think: 6 thousands will become 6 hundreds, so the answer is 670.

For 92 ÷ 100, think: 9 tens will become 9 tenths, so the answer is 0.92.

For 260 ÷ 100, think: 2 hundreds will become 2 ones, so the answer is 2.6.

For 7 000 ÷ 100, think: 7 thousands will become 7 tens, so the answer is 70.

For 4 700 ÷ 100, think: 4 thousands will become 4 tens, so the answer is 47.

For 46 ÷ 1000, think: 4 tens will become 4 hundredths, so the answer is 0.046.

For 650 ÷ 1000, think: 6 hundreds will becomes 6 tenths, so the answer is 0.65.

For 2 500 ÷ 1000, think: 2 thousands will become 2 ones, so the answer is 2.5.

For 75 000 ÷ 1000, think: 7 ten thousands will become 7 tens, so the answer is 75.

Examples of Some Practice Items

a) Some practice items for divisors of 10:

- 58 m ÷ 10
- The number of metres in 302 dm
- 7 300 km ÷ 10
- 4 050 divided by 10
- $56 000 shared by 10 people
- The number of groups of 10 in 80 700

b) Some practice items for divisors of 100:

- 61 ÷ 100
- 420 divided by 100
- $9 300 ÷ 100
- Mass of 1 package when 100 packages is 3 080 kg
- The number of metres in 85 000 cm
- The number of dollars in 30 800 cents

c) Some practice items for divisors of 1000:

- 58 ÷ 1000
- $302 divided by 1000
- The number of kilometres in 7 400 metres
- 4 050 ÷ 1000
- The number of 1000-kg loads in 56 000 kg
- The number of kilograms in 80 700 g

G. Computational Estimation

Students should habitually estimate answers before attempting pencil-and-paper, or calculator, computations, in order to be alert to the reasonableness of answers. Usually, these need only be "ball-park" estimates, especially when using a calculator where typical input errors result in place value mistakes that can be detected from those "ball-park" estimates. There are also many instances in life when an estimate is all that is needed: such estimates should be as close as you can get to the actual answer.

The language of estimation should be used throughout estimation lessons. Some of the common words and phrases are: *about, just about, between, a little more than, a little less than, close, close to,* and *near*.

It is also important for students to hear and see a variety of contexts for each estimation strategy, so they are able to transfer the use of estimation and strategies to situations found in their daily lives.

Appropriate reinforcement questions would involve the combining of two numbers under a single operation; however, keep in mind that students are expected to be able to get exact answers mentally in addition and subtraction when there are only two combinations involved with one regrouping, and to use quick addition for questions involving any number of combination with no regrouping. Therefore, the estimation questions should use numbers that require more than two combinations and regrouping. Reinforcement questions that involve more than two numbers and operations would also be appropriate.

Front-End Estimation (Extension)

This strategy is the simplest of all the estimation strategies for addition, subtraction, and multiplication. It involves combining only the digits in the highest place value of each number to get a "ball-park" estimate. As such, these combinations will only require the use of the basic facts. While this strategy could be applied to division questions if the divisor is a factor of the highest place value of dividend, division estimation is better done by a rounding strategy.

These front-end estimates are adequate in many circumstances, particularly before using a calculator. For addition and multiplication, the actual answer will always be more than the front-end estimates since the digits in the other place values are disregarded. In subtraction, without considering the other digits, you don't know if the actual answer is more, or less, than the front-end estimate.

Examples

To estimate 4 276 + 3 237, think: 4 000 plus 3 000 is 7 000, so the estimate is more than 7 000.

To estimate 37 260 + 28 142, think: 30 000 plus 20 000 is 50 000, so the estimate is more than 50 000.

To estimate 8.95 + 3.21, think, 8 plus 3 is 11, so the estimate is more than 11.

To estimate 0.439 + 0.278, think, 4-tenths plus 2-tenths is 6-tenths, so the estimate is more than 0.06.

To estimate 5 475 – 3 128, think: 5000 subtract 3000 is 2000, so the estimate is about 2 000.

To estimate 58 123 – 22 144, think: 50 000 subtract 20 000 is 30 000, so the estimate is about 30 000.

To estimate 8.88 – 4.27, think, 8 subtract 4 is 4, so the estimate is about 4.

To estimate 0.927 – 0.618, think, 9-tenths subtract 6-tenths is 3-tenths, so the estimate is about 0.3.

For example: To estimate 938 × 4, think: 900 times 4 is 3 600, so the estimate is more than 3 600.

To estimate 5 × 1 437, think: 5 times 1 000 is 5 000, so the estimate is more than 5 000.

To estimate 8 × 8 548, think: 8 times 8 000 is 64 000, so the estimate is more than 64 000.

To estimate 36 × 82, think: 30 times 80 is 2400, so the estimate is more than 2 400.

Examples of Some Practice Items

a) Some practice items for addition:

- These amounts are going to be added on a calculator: $43, $56, $37, $91, and $25. What should you expect the sum to be more than?
- Estimate $439 + $562
- Approximately what is the sum of 1296 km and 6388 km?
- About what is the result if 34 568 kg is increased by 15 789 kg?
- Estimate the sum of 5.69 m and 2.13 m
- Estimate: 0.256 kg plus 0.425 kg

b) Some practice items for subtraction:

- Estimate 425 kg – 139 kg
- Approximately how much less is $229 than $715?
- About how much longer is 5 675 km than 3 757 km?
- Estimate the difference between 73 568 and 35 225.
- Estimate

c) Some practice items for multiplication:

- Estimate: 6 × \$629
- What is the approximate product of 2 725 and 8?
- Estimate: 43 250 × 5
- What is the approximate area of a 43 cm by 65 cm rectangle?
- About how much does Sally earn for 7 hours @ \$7.45/hour?

Rounding (Extension)

This most common estimation strategy involves rounding one, or both, numbers to their highest place values, or to compatible numbers, so the calculation is more easily done mentally. Rounding numbers to the highest place value enables students to keep track of the rounded numbers and do the calculation in their heads using basic facts; however, rounding to two highest place values would require most students to record the rounded number(s) before performing the calculation mentally. Rounding to one or to two highest place values depends upon how close your estimate needs to be to the actual answer.

If the digit before the highest place value to which you are rounding is (a) less than 5, you would normally round down; (b) greater than 5, you would normally round up; (c) exactly a 5, you would round up or down depending upon the overall effect it would have on the answer. For example, in addition if both addends have a 5 to be rounded, it is best to round one up and one down to minimize the effect of rounding. This is also true if both numbers are close to a 5; for example, 648 + 747 would be rounded to 700 + 700 or 600 + 800. In subtraction, on the other hand, if both the minuend and subtrahend have 5s to be rounded, both numbers should be rounded up because you are looking for the difference between the two numbers; therefore, you don't want to increase this difference by rounding one up and one down. (This will require careful introduction for students to be convinced. So often students only associate subtraction with *take-away* and need to be reminded that subtraction also finds the *difference* between two numbers. Help them make the connection to the Balancing-for-a-Constant- Difference strategy in mental calculation for subtraction.)

Examples

To estimate 348 + 229, think: 348 rounds to 300 and 229 rounds to 200, so 300 plus 200 is an estimate of 500; or think: Round 348 to 350 and record, round 229 to 230 and record, and the estimate is 350 plus 230, or 580.

To estimate 4276 + 3937, think: 4276 rounds to 4000 and 3937 rounds to 4000, so 4000 plus 4000 is an estimate of 8000; or think: Round 4276 to 4300 and record, round 3937 to 3900 and record, and the estimate is 4300 + 3900, or 8200.

To estimate 594 – 216, think: 594 rounds to 600 and 216 rounds to 200, so 600 subtract 200 is an estimate of 400; or think: Round 594 to 590 and record, round 216 to 220 and record, and the estimate is 590 – 220, or 370.

To estimate 6237 – 2945, think: 6237 rounds to 6000 and 2945 rounds to 3000, so 6000 subtract 3000 is an estimate of 3000; or think: Round 6237 to 6200 and record, round 2945 to 2900 and record, and the estimate is 6200 – 2900, or 3300.

To estimate 7 × 642, think: 642 rounds to 600, so 7 times 600 is an estimate of 4200; or think: Round 642 to 640 and record, so the estimate is 7 × 640, or 4480.

To estimate 8 × 6 930, think: 6 930 rounds to 7 000, so 8 times 7 000 is an estimate of 56 000; or think: Round 6 930 to 6 900 and record, so the estimate is 8 × 6 900, or 55 200.

When rounding multiplication questions with two 2-digit factors, round as usual, but if the ones digits are both 5 (or more), consider rounding the smaller factor up and the larger factor down. Some examples of these questions:

> To estimate 25 × 65, think: Round 25 (the smaller factor) to 30 and 65 to 60, so 30 × 60 is an estimate of 1800. (*Note: This is a closer estimate than 20 × 60, 20 × 70, or 30 × 70.)*
>
> To estimate 76 × 36, think: 76 (the larger number) rounds down to 70 and 36 (the smaller number) rounds up to 40, so 70 × 40 is an estimate of 2800. (*Note: This is a closer estimate than 80 × 40 or 80 × 30.)*
>
> To estimate 43 × 78, think: Round 43 to 40 and 78 to 80, so 40 × 80 is an estimate of 3200. (*Note: This uses usual rounding strategies*.)

The strategy for rounding in division questions with single-digit divisors is to round the dividends to compatibles with the divisors.

> To estimate 471 ÷ 6, think: Round 471 to 480, a compatible with 6 in division, so 480 ÷ 6 gives an estimate of 80.
>
> To estimate 822 ÷ 9, think: Round 822 to 810, a compatible with 9 in division, so 810 ÷ 9 gives an estimate of 90.

Examples of Some Practice Items

a) Some practice items for addition:
   - Estimate: 28 + 57 + 19 + 75
   - Estimate the result when 223 is increased by 583
   - Estimate the total of two deposits: $8 879 and $4 238.
   - Approximately what is the sum of $36 789 and $44 329?

b) Some practice items for subtraction:
   - Estimate the difference between 562 and 185
   - Estimate: 1266 – 387
   - What number is close to the difference between 4 567 and 3 188?
   - About what is 88 767 minus 36 409?
   - Approximately how much less is $27 755 than $36 225?

c) Some practice items for multiplication:
   - Estimate: 4 × 579
   - What is the approximate product of 823 and 6?
   - Estimate the cost of 5 TVs @ $1 749 each
   - About what is the result of 3 × 18 725?
   - Estimate: 32 × 68
   - What is the approximate area of a 35 cm by 55 cm rectangle?
   - Approximately what is the cost of 26 printers at $66 each?

d) Some practice items for division:
   - Estimate: 116 ÷ 6
   - Approximately what is $650 divided by 8?
   - About what is the daily average if 1 435 km was travelled in 7 days?

Adjusted Front-End Estimation (Extension)

This strategy is often used as an alternative to rounding to get closer estimates for addition questions. It involves getting a Front-End estimate and then adjusting that estimate to get a better, or closer, estimate by either (a) by clustering all the values in the other place values to eyeball whether there would be enough together to account for an adjustment, or (b) by considering the second highest place values. The adjustment by method (b) often results in a closer estimate than method (a), and would likely only be bettered by the strategy of rounding to the two highest place values.

Method (b) can also be applied to multiplication questions to bypass any consideration of rounding. The 2-digits to be multiplied will always be visible in the given question, so there will no need to record digits as might occur in the case of rounding to 2-digits.

Examples

To estimate 434 + 547, (a) think: 400 plus 500 is 900, but eye-balling 34 and 47 suggests about another 100, so the adjusted estimate is 900 + 100, or 1000; or (b) think: 400 plus 500 is 900, but 30 plus 40 is 70, so the estimate is 900 + 70, or 970.

To estimate 512 + 219, (a) think: 500 plus 200 is 700, and eyeballing 12 and 19 suggests no adjustment of another 100, so the estimate is 700; or (b) think: 500 plus 200 is 700 and 10 plus 10 is 20, so the adjusted estimate is 700 + 20, or 720.

To estimate 4 678 + 2 712, (a) think: 4 000 plus 2 000 is 6 000, but eyeballing 678 and 712 suggests another 1000, so the adjusted estimate is 6 000 + 1 000, or 7 000; or (b) think: 4 000 plus 2 000 is 6 0000 and 600 plus 700 is 1 300, so the adjusted estimate is 6 000 plus 1 300, or 7 300.

To estimate 3 275 + 5 187, (a) think: 3 000 and 5 000 is 8 000, and eyeballing 275 and 187 suggests no adjustment of another 1 000, so the estimate is 8 000; or (b) think: 3 000 plus 5 000 is 8 000 and 200 plus 100 is 300, so the estimate is 8 000 + 300, or 8 300.

To estimate 41 679 + 25 342, (a) think: 40 000 plus 20 000 is 60 000, but eyeballing 1 679 and 5 342 suggests another 10 000, so the adjusted estimate is 60 000 + 10 000, or 70 000; or (b) think: 40 000 plus 20 000 is 60 000 and 1 000 plus 5 000 is 6 000, so the estimate is 60 000 +
6 000, or 66 000.

To estimate 3.225 + 1.236, (a) think: 3 plus 1 is 4 and eyeballing 0.225 and 0.236 does not suggest another 1, so the estimate is 4; or (b) think: 3 plus 1 is 4 and 0.2 plus 0.2 is 0.4, so the estimate is 4 + 0.4, or 4.4.

To estimate 4 × 429, think: 4 times 400 is 1600 and 4 times 20 is 80, so the estimate is 1600 + 80, or 1680.

To estimate 2 357 × 6, think: 2 000 times 6 is 12 000 and 300 times 6 is 1 800, so the estimate is 12 000 + 1 800, or 13 800.

To estimate 5 × 2.189, think: 5 times 2 is 10 and 5 times 0.1 is 0.5, so the estimate is 10 + 0.5, or 10.5.

Examples of Some Practice Items

a) Some practice items for addition:

- Estimate: \$625 + \$429
- What is the approximate sum of 2 457 km and 5 289 km?
- About what is 32 675 kg more than 45 135 kg?
- Estimate 3.675 m more than 9.185 m

b) Some practice items for multiplication:

- Estimate 3 × \$575
- What is the approximate product of 8 and 2 456?
- About what is 5 times 6 237 kg?
- Estimate: 4.445 × 7

Clustering of Near Compatibles (Extension)

When estimating the addition of a list of numbers, it is sometimes useful to look for two or three numbers that can be grouped to almost make 10s, 100s or 1000s (compatible numbers).

Examples

For 44 + 33 + 62 + 71, think: 44 plus 62 is almost 100 and 33 plus 71 is almost 100, so the estimate is 100 + 100, or 200.

For 692 + 604 + 298, think: 692 plus 298 is almost 1000, so the estimate is 1 000 + 600, or 1600.

Examples of Some Practice Items

- Estimate: \$625 + \$429 + \$712
- Approximately what is the sum of 256, 212, 749, and 807?
- Estimate: 3 125 kg + 4 389 kg + 6 998 kg
- Estimate the sum of 0.125 m, 0.478 m, 0.923 m, 0.522 m, and 0.345 m

PART 2

Measurement Estimation

The Implementation of Measurement Estimation

General Approach

For the most part, a measurement estimation strategy would be reinforced and assessed during mental math time in the grades following its initial introduction. The goal in mental math is to increase a student's *competency* with the strategy. It is expected that measurement estimation strategies would be introduced as part of the general development of measurement concepts at the appropriate grade levels. Each strategy would be explored, modeled, and reinforced through a variety of class activities that promote understanding, the use of estimation language, and the refinement of the strategy. As well, the assessment of the strategy during the instructional and practice stages would be ongoing and varied.

Competency means that a student reaches a reasonable estimate using an appropriate strategy in a reasonable time frame. While what is considered a reasonable estimate will vary with the size of the unit and the size of the object, a general rule would be to aim for an estimate that is within 10% of the actual measure. A reasonable time frame will vary with the strategy being used; for example, using the *benchmark strategy* to get an estimate in metres might take 5 to 10 seconds, while using the *chunking strategy* might take 10 to 30 seconds, depending upon the complexity of the task.

A. Introducing a Strategy in Regular Classroom Time

Measurement estimation strategies should be introduced when appropriate as part of a rich and carefully sequenced development of measurement concepts. For example, in grade 3, the distance from the floor to most door handles is employed as a *benchmark* for a metre so students can use a *benchmark strategy* to estimate lengths of objects in metres. This has followed many other experiences with linear measurement in earlier grades: in grade primary, students compared and ordered lengths of objects concretely and visually; in grade 1, students estimated lengths of objects using non-standard units such as paper clips; in grade 2, students developed the standard unit of a metre and were merely introduced to the idea of a *benchmark* for a metre.

The introduction of a measurement estimation strategy should include a variety of activities that use concrete materials, visuals, and meaningful contexts, in order to have students thoroughly understand the strategy. For example, a meaningful context for the *chunking strategy* might be to estimate the area available for bookshelves in the classroom. Explicit modeling of the mental processes used to carry out a strategy should also be part of this introduction. For example, when demonstrating the *subdivision strategy* to get an estimate for the area of a wall, you would orally describe the process of dividing the wall into fractional parts, the process of determining the area of one part in square metres, and the multiplication process to get the estimate of the area of the entire wall in square metres. During this introductory phase, you should also lead discussions of situations for which a particular strategy is most appropriate and efficient, and of situations for which it would not be appropriate, or efficient.

B. Reinforcement in Mental Math Time

Each strategy for building measurement estimation skills should be practised in isolation until students can give reasonable estimates in reasonable time frames. Students must understand the logic of the strategy, recognize when it is appropriate, and be able to explain the strategy. The amount of time spent on each strategy should be determined by the students' abilities, progress, and previous experiences.

The reinforcement activities for a strategy should be varied in type and should focus as much on the discussion of how students obtained their answers, as on the answers themselves. The reinforcement activities should be structured to ensure maximum participation. At first, time frames should be generous and then narrowed as students internalize the strategy and become more efficient. During estimation activities, you should use a variety of answer formats to help students overcome any hesitation they might have in estimating and to help them refine their estimates. For example, questions might ask students to select a range for the estimate from a given set of ranges; to decide if the estimate is *more than, less than, or about* a given measure; or to select the estimate from a list of given possibilities. Eventually, through experiences, they should be able to determine a reasonable single-number estimate on their own. Student participation should be monitored and their progress assessed in a variety of ways. This will help determine the length of time that should be spent on a strategy.

During the reinforcement activities, the actual measures should not be determined every time an estimate is made. You do not want your students to think that an estimate is always followed by measurement with an instrument: there are many instances where an estimate is all that is required. When students are first introduced to an activity, it is helpful to follow their first few estimates with a determination of the actual measurement in order to help them refine their estimation abilities. Afterwards, however, you should just confirm the reasonable estimates, having determined them in advance

Most of the reinforcement activities in measurement will require the availability of many objects and materials because students will be using some objects and materials as benchmarks and will be estimating the attributes of others. To do this, they must see and/or touch those objects and materials.

After you are confident that most students have achieved a reasonable competency with the strategy, you need to help them integrate it with other strategies that have been developed. You should do this by providing activities that include a mix of attributes to be estimated in different units using a variety of contexts. You should have the students complete the activities and discuss the strategy/strategies that were, or could have been, used. You should also have students match measurement estimation tasks to a list of strategies, and have them discuss the reasoning for their matches.

C. Assessment

Your assessments of measurement estimation strategies should take a variety of forms. Assessment opportunities include making and noting observations during the reinforcements, as well as students' oral and written responses and explanations. Individual interviews can provide you with many insights into a student's thinking about measurement tasks. As well, traditional quizzes that involve students recording answers to estimation questions that you give one-at-a-time in a certain time frame can be used.

Assessments, regardless of their form, should shed light on students' abilities to estimate measurements efficiently and accurately, to select appropriate strategies, and to explain their thinking.

Measurement Estimation Strategies

The Benchmark Strategy

This strategy involves mentally matching the attribute of the object being estimated to a known measurement. For example, to estimate the width of a large white board, students might mentally compare its width to the distance from the doorknob to the floor. This distance that is known to be about 1 metre is a *benchmark*. When students mentally match the width of the white board to this benchmark, they may estimate that the width would be about two of these benchmarks; therefore, their estimate would be 2 metres. In mathematics education literature you will often see reference made to *personal referents.* These are benchmarks that individuals establish using their own bodies; for example, the width of a little finger might be a personal referent for 1 cm, a hand span a referent for 20 cm, and a hand width a referent for 1 dm. These benchmarks have the advantage of being portable and always present whenever and wherever an estimate is needed.

The Chunking Strategy (Starting in Grade 5)

This strategy involves mentally dividing the attribute of an object into a number of chunks, estimating each of these chunks, and adding the estimates of all the chunks to get the total estimate. For example, to estimate the length of a wall, a student might estimate the distance from one corner to the door, estimate the width of the door, estimate the distance from the door to the bookcase, and estimate the length of the bookcase which extends to the other corner. All these individual estimates would be added to get the estimate of the length of the wall.

The Unitizing Strategy (Starting in Grade 5)

This strategy involves establishing an object (that is smaller than the object being estimated) as a unit, estimating the size of the attribute of this unit, and multiplying by the number of these units it would take to make the object being measured. For example, students might get an estimate for the length of a wall by noticing it is about five bulletin boards long. The bulletin board becomes the unit. Students estimate the length of the bulletin board and multiply this estimate by five to get an estimate of the length of the wall.

The Subdivision Strategy (Starting in Grade 6)

This strategy involves mentally dividing an object repeatedly in half until a more manageable part of the object can be estimated and then this estimate is multiplied by the appropriate factor to get an estimate for the whole object. For example, to estimate the area of a table top, students might mentally divide the table top into fourths, estimate the area of that one-fourth in square decimeters, and then multiply that estimate by four to get the estimate of the area of the entire table top.

H. Length, Perimeter, and Area—Measurement Estimation

In grades 3 and 4, students developed understanding of the benchmark for the millimetre, centimetre, decimetre, and metre, and the relationships among them. In grade 4, students developed understanding of the concept of perimeter as the distance around the boundary of a closed geometric figure, using non-standard and the standard unit of centimetres. Students have also developed understanding of the concept of area using non-standard units and the standard units of square centimetres and square decimetres. They are now ready to use these understandings to review and develop their estimation competencies for these units of measure.

Benchmarks for Centimetre, Decimetre, and Metre (Extended)

In grades 3 and 4, students have estimated lengths and distances in whole number centimetres, decimetres, and metres. In grade 5, students should extend these estimations to include tenths of decimetres and tenths of metres. Student should gain competency with estimating lengths of objects that are between 1 dm and 10 dm and giving these estimates in tenths of metres, and with estimating lengths between 1 and 10 cm and giving these estimates in tenths of decimetres.

Students could use the length of a small cube in the base-10 blocks as a benchmark for 1 cm and the length of a rod in the base-10 blocks as a benchmark for 1 dm. As personal referents, they could use the width of their small fingers for 1 cm and the width of their hands for 1 dm. For a metre, students can use the height of the doorknob, or the length of a metre stick as a benchmark for a metre. Each student could stand against a metre stick and note the part of his/her body that is 1 m above the floor: this could serve as a personal referent. Placing ten base-10 rods against a metre stick would help reinforce that 10 dm = 1 m or that 1 dm = 0.1 m. Similarly, placing 10 small cubes against a rod in the base-10 blocks would reinforce that 10 cm = 1 dm or that 1 cm = 0.1 dm.

Examples of Some Practice Activities

- Cut off a number of varying lengths of string, rope, and ribbon. As you display each one, ask students if they would estimate its length to be about 0.4 m, 0.6 m, or 0.8 m. Have them record their estimates on their individual white boards, and to show you when directed. Ask them to share how they made their decisions. For the first two or three lengths, follow the discussion with the measurement of the actual lengths using a metre stick to help refine their estimation skills.
- Select 8 to 10 objects in the room. For each object, prepare three possible estimates from which you will ask students to select the one they think is the most reasonable. For example, ask: Which is the most reasonable estimate for this eraser: 0.5 dm, 0.7 dm, or 0.9 dm? Which is the most reasonable estimate for the length of this textbook: 2 dm, 3 dm, or 4 dm? After each question, have students discuss how they made their decisions.
- Provide students with a length and ask them to name objects in the room that are approximately that length. For example, ask them to name an object that is about 2 m long, an object that is about 7 dm high, an object that is less than 10 cm long, an object that is about 3 dm long, and an object that is approximately 0.5 m long.
- Select ten objects that have lengths between 0.1 m and 2 m. Advise students that the lengths they will estimate will be between these two measures. Display, or point to, each one and ask students to record their estimates in tenths of metres. For the initial two or three objects, determine the range of estimates that the students have made, discuss the strategies they used to get the estimates, find the actual length, and discuss the range of estimates that would qualify as good estimates. (For example, students might have estimated the length of a table in the classroom and produced estimates from 1.1 m to 1.8 m. When the table length was measured with a metre stick, it was actually 1.4 m. You could point out that estimates of 1.3 m and 1.5 m would be

considered very good, and estimates of 1.2 m and 1.6 m would be acceptable.) Through these experiences students should be helped to refine their estimation skills.

Applying Benchmarks to Perimeter (New)

Students should gain competency estimating the perimeter of rectangular shapes in non-standard and in standard units of centimetres, decimetres, and metres. This is just an application of the benchmarks they have already established for centimetres, decimeters, and metres.

They should be able to choose the most appropriate unit depending on the size of the object. While initially students might estimate each side and add them, the estimation process should include estimating one length and one width, finding the sum of these two estimates, and then doubling the sum to get the estimated perimeter. As such, part of this process will include recording the estimates made for the lengths and widths before the calculations are done mentally.

Examples of Some Practice Items Involving Non-Standard Units

- Will the perimeter of a math text in post-it notes be about 6, 14, or 20?
- Will a desktop have a perimeter in pencil lengths of between 10 – 14 or 16 – 20?
- Estimate the perimeter of a math text in eraser lengths. Ask students to respond by giving range, e.g., 20 – 25 eraser lengths.
- Estimate the perimeter of a piece of loose leaf using markers. Ask students to respond with a single number.

Examples of Some Practice Items Involving Standard Units

- Will the top of this marker box have a perimeter of about 20 cm, 50 cm, or 70cm?
- Will the cover of your math text have a perimeter between 60 and 70 cm, or between 90 and 100 cm?
- Will the perimeter of a binder top be *more than*, *less than*, or *about* 120 cm?
- Which is the best estimate of the perimeter of the classroom: 18 m? 28 m? 38 m?
- Estimate the perimeter of the top of this whiteboard eraser in centimetres.
- Which is the best estimate of the perimeter of the teacher’s desk: 20 dm? 30 dm? or 40 dm?
- Estimate the perimeter of your desk top in decimetres, with your estimate being within a range of two decimetres.
- Estimate the perimeter of one windowpane in decimetres.
- Estimate the perimeter of the door in metres.

Benchmarks for Millimetre (New)

Students should gain competency with estimating attributes of objects, such as length, width, and thickness, in millimetres. The attributes should be those for which millimetres is a realistic unit to use. Students could use the thickness of a dime as a benchmark for 1 mm. As well, students could also use the relationship 10 mm = 1 cm to estimate some attributes. Since a personal referent for 1 cm is the width of a little finger, comparisons to this 10-mm finger width could be used to generate estimates in millimetres. For example, if the width of an object was a little more than two of these finger widths, a student might estimate the width as 22 or 23 mm.

Examples of Some Practice Activities

- As you point out objects in the classroom, or that you hold up, ask students to decide if each object is longer/wider/thicker than 5 mm. After they make their decision on each one, have them explain how they made their decisions.
- Ask students to visualize 1 millimetre. Ask them to look around the classroom for something that is less than 1 mm, something that is about 1 mm, and something that is more than 1 mm, and to record them. Working as partners, have them share their lists. Any controversies can be settled by using a dime as a comparison tool.
- Display a variety of small materials, such small nails, screws, tacks, and buttons, one-at-a-time and ask students if its length is more than 15 mm, less than 15 mm, or about 15 mm. Ask students to write their estimate on their individual whiteboards. After each estimate, use a ruler to determine the actual length.
- Ask a variety of questions, such as: Will the thickness of this sheet of Bristol board be closer to 1 mm or 2 mm? Will the length this tack be closer to 10 mm or 20 mm? Have students record their answers. Discuss how decisions were made.
- Provide students with intervals for the estimates of a variety of objects in the classroom, and ask them to select the interval they think best represents the estimate. For example, ask: Is the thickness of this calculator between 3 and 5 mm, 8 and 10 mm, or 13 and 15 mm? Is the length of this battery 20 – 30 mm, 30 – 40 mm, or 40 – 50 mm?
- Present students with a variety of estimation situations and get them to commit to one estimate. Afterwards, facilitate a discussion of the estimates and the strategies used to make a decision. For example, present this situation: Sharon estimated that the thickness of a regular notebook is about 4 mm, while Donna estimates it is about 8 mm. Whose estimate do you think is best? Why?

Benchmarks for Area in Square Centimetres and Square Decimetres (New)

Student should gain competency in estimating areas of rectangular surfaces of objects using square centimetres (cm^2) and square decimetres (dm^2). They should be able to choose the most appropriate unit depending on the size of the object.

Students could use one face of a small base-10 cube as a benchmark for one square centimetre. A personal referent might be the surface of the nail on their smallest finger. They could use the large face of a base-10 flat as a benchmark for one square decimetre. This also helps establish that 1 dm^2 is equivalent to 100 cm2. Another useful benchmark to establish is that the area of a sheet of loose leaf is about 600 cm^2 or 6 dm^2.

For smaller surfaces students would probably visualize and count the number of square units needed to completely cover the surface. However, for relatively larger surfaces, students should be encouraged to visualize the surface covered by an array of square units, so they would estimate the number of square units in one row and multiply that estimate by their estimate of the number of rows. For example, to estimate the area of a table top, they might estimate that it would take about 10 flats to go along the length of the table and that there would be about 5 rows of those flats needed to cover the table top, so the estimate of its area would be 5 x 10, or 50 dm^2.

Examples of Some Practice Activities

- Display a number of objects, one-at-a-time, that would have areas appropriately measured in square centimetres. For each surface, provide students with a number of square centimetres and ask then to decide if the best estimate of its area would be *less than*, *more than*, or *about* that number. After each estimate, discuss the strategies used to make the selection. For example: Will the area of a small tissue package be *more than*, *less than*, or *about* 60 cm^2?
- Display a number of objects, one-at-a-time, that would have areas appropriately measured in square centimetres. Ask students to choose a best estimate from among three that you provide. After each estimate, discuss the strategies used to make the selection, and model the visualization process. For example: Is the area of a small crayon box closer to 35 square centimetres, 70 square centimetres or 105 square centimetres?
- Select some surfaces within the classroom that would have their areas appropriately measured in square decimetres. As you direct students' attention to each surface, ask them to select the better of two ranges of estimates that you provide. After each estimate, discuss the strategies used to make the decision. For the first few, you could have a pair of students model covering the surface with a few flats to confirm the reasonableness of the estimates. For example: Which is the better estimate of the area of the top of your student desk: between 42 and 46 square decimetres or between 24 to 28 square decimetres?
- Select some surfaces within the classroom that would have their areas appropriately measured in square decimetres. As you direct students' attention to each surface, ask them to select the best estimate from three that you provide. After each estimate, discuss the strategies used to make the decision. For example: Which is the best estimate of the area of a math text cover: 5 dm^2, 8 dm^2, or 10 dm^2?
- Select a variety of surfaces that you will point to or display, all of which would have their areas measured in either square centimetres or square decimetres. Determine their actual areas in advance and decide on a range of acceptable estimates for each one. Ask students to estimate each area by giving a single number and the appropriate unit. For example: Estimate the area of the top of the teacher's desk. What is the approximate area of this computer screen? About how many units is the area of the window in the classroom door?

Chunking for Length, Perimeter, and Area (New)

When objects are large or are composites of different shapes, it is sometimes difficult to estimate the measures of their attributes using just the benchmark strategy. In these situations, one helpful strategy to use is the chunking strategy. This strategy involves mentally dividing the attribute of an object into a number of chunks, estimating each of these chunks using benchmarks, and adding the estimates of all the chunks to get the total estimate. The objects for which students are asked to estimate attributes should lend themselves to using this chunking strategy rather than the unitizing strategy (see below).

Students should record the measure of each chunk and mentally add the chunks to get the final estimate.

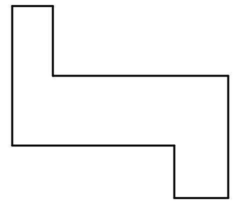
Examples

To estimate the length of a wall, a student might estimate (in metres) the distance from one corner to the door, estimate the width of the door, estimate the distance from the door to the bookcase, and estimate the length of the bookcase which extends to the other corner. All these individual estimates would be added to get the estimate of the length of the wall.

To estimate the area of the door to the classroom, a student might estimate (in square decimetres) the area of the window in the door, the area below the window, and the area above the window. After the student has recorded each individual chunk of area, he/she would add all the chunks to get the estimate of the area of the door.

Examples of Some Practice Activities

- On sheets of paper, draw some polygons (such as the example given) whose perimeters would be measured in centimetres and areas would be measured in square centimetres. These polygons should lend themselves to being mentally subdivided into two or more other polygons, the areas of which can be estimated. Ask students to estimate the perimeter and area of each polygon.



- Describe the path that someone walked in the classroom and ask students to estimate the distance walked. For example, Josey walked from the teacher's desk to the pencil sharpener, then to the door, and back to her desk. How many metres did Josey walk? Discuss strategies used to determine the distance. (Repeat describing other walks.)
- Gather a few boxes that would have their face areas measured in square decimetres (such as a shoebox and a box for a computer keyboard). Show students the boxes, one-at-a-time. Ask them to estimate how many square decimetres of paper would be needed to cover all the faces of this box. Discuss the strategies used to decide on the estimate.

Unitizing for Length, Perimeter, and Area (New)

When objects are large but somewhat uniformly shaped, the unitizing strategy can be efficiently used to estimate the measures of its attributes. This strategy involves establishing a part of the object as a unit, estimating the size of the attribute of this unit, and multiplying by the number of these units it would take to make the object being measured.

Examples

Students were asked to get an estimate for the length of a wall. They noticed the wall was about five bulletin boards long. This bulletin board became the unit. Students estimated the length of the bulletin board at 2 m, and then multiply 2 m by five to get 10 m as an estimate of the length of the wall.

To estimate the area of the shelves in the classroom bookcase, students noticed that the bookcase contained 10 shelves, all the same size. They estimated the area of one shelf to be 24 dm^2. They multiplied 24 by 10 to get an estimate of 240 dm^2 for the total bookshelf space.

Examples of Some Practice Activities

- Note some lengths in your classroom or school that could be estimated by establishing a unit. (For example, the length of the classroom might be estimated in metres by using the length of a ceiling tile or a light fixture as a unit, or by using the width of one window in a bank of windows as a unit. The length of a corridor might be estimated in metres by using the length or width of one classroom as a unit. The width of a bulletin board that has a numbers of sheets of paper posted on it might be estimated in decimetres by using as a unit the length of one of those sheets of paper. The length or width of a shelf that contains a number of textbooks might be estimated in centimetres by using a dimension of one textbook as a unit.) Present each of the lengths you have noted, one-at-a-time, and ask students to estimate it in the appropriate unit. After each one, discuss the estimates and the strategies used to get the estimates.
- Note some areas in your classroom or school that could be estimated by establishing a unit. (For example, the area of the classroom might be estimated in metres by using the length of a ceiling tile or a light fixture as a unit, or by using the width of one window in a bank of windows as a

unit. The length of a corridor might be estimated in metres by using the length or width of one classroom as a unit. The width of a bulletin board might be estimated in decimetres by using as a unit the length of one sheet of paper that is on the bulletin board. The length or width of a shelf might be estimated in centimetres by using a known hand span as a unit.) Present each of the lengths you have noted, one-at-a-time, and ask students to estimate it in the appropriate unit. After each one, discuss the estimates and the strategies used to get the estimates.

I. Volume and Capacity—Measurement Estimate

In grade 4, students developed understanding of estimating and measuring volume of rectangular prisms using non standard and standard units (cm^3). In grades 3 and 4, students developed an understanding of estimating and measuring capacity using millilitres (ml) and litres (L). They are now ready to use this understanding to develop the following estimation competencies.

Benchmarks for Volume (New)

For estimation of volume in grade 5, student competency would be the ability to recognize a reasonable estimate of the volume of relatively small objects in cubic centimetres or cubic decimetres. When given an estimate, students should judge its reasonableness by visualizing the number of small or large base-10 cubes this estimate represents, and compare the object to this visualized volume. Student would thus be using the volume of a small cube from the base-10 unit blocks as a benchmark for one cubic centimetre (1 cm^3), and a large cube from the base ten is one cubic decimetre (1 dm^3).

The ability to generate their own estimates will be developed in later grades after they have worked with volume formulas. However, the ability to recognize a reasonable estimate of volume is an important prerequisite to the development of those volume formulas.

Examples of Practice Activities

- Select a few blocks from a set of Geoblocks. As you display each block, ask students to select the most reasonable estimate of its volume from a list of three estimates that you provide. For example, display a rectangular prism (2 cm x 4 cm x 8 cm) but do not state its dimensions. Ask students to select the best estimate from 40 cm^3, 60 cm^3, and 80 cm^3. Discuss how students made their decisions.
- Gather some small boxes with lids on them. As you display each one, ask students to decide if the most reasonable estimate is more, or less, than a number of cubic decimetres that you tell them. For example, display a shoebox and ask students whether the best estimate of its volume is less than, or more than, 10 dm^3.

Benchmarks for Capacity (Extended)

For capacity estimation, student competency should centre on the reasonableness of given estimates and on making reasonable decisions involving capacities (how much a container is capable of holding) of a variety of commonly used containers, such as pop cans and bottles, and juice and milk containers. Student could use a small cube from the base-10 unit blocks as a benchmark for 1 mL, a rod as a benchmark for 10 mL, a flat as a benchmark for 100 mL, and a large cube as a benchmark for 1 L.

Of all the metric units, millilitres and litres are ones that are most commonly used in general society; therefore, students will likely bring out-of-school experiences to these estimation activities.

Example

Estimate the capacity of the fish tank using litres as the unit of measure.

J. Angles—Measurement Estimation

In grade 4, students developed understanding of using a benchmark for a right angle to determine if angles would be described as acute, right, or obtuse angle. In grade 5, this will be extended to estimating sizes of angles in degrees.

Benchmark for Angles

Student should gain competency with estimating the size of angles in relation to 0°, 90°, or 180°, using the visual image of a "square corner" as a benchmark for 90°. This would include responses such as close to 0°, 90°, or 180°, more than/a little more than 0° or 90°, a less than/little less 180°, and between 0° and 90° or between 90° and 180°.

Examples

Provide each student with two Geo Strips and a brad. Explain that you will ask them to use the Geo Strips to display examples of angles, the measures of which you will provide. For example, ask them to display an angle that is (a) a little more than 90°, (b) almost 180°, (c) almost 0°, (d) half way between 0° and 90°, and (e) approximately 90°. (This activity could be done using pipe cleaners, two pencils, or children's two arms/two fingers (one on each hand).)

Prepare an overhead acetate that contains drawings of angles that are numbered from 1 to 10. Vary the positions in space of these angles and the size of the arms of the angles. Display each angle and ask students to describe the estimate of the size of each angle using 0°, 90°, and/or 180° in their answers.

Explain that you will display a variety of angles using Geo Strips. For each angle they are to estimate its measure using 0°, 90°, and 180°, and one of the terms: *a little less than* or *a little more than*. (Be sure to vary the position in which you display the Geo Strips.)

PART 3

Spatial Sense

The Development of Spatial Sense

What is Spatial Sense?

Spatial sense is an intuition about shapes and their relationships, and an ability to manipulate shapes in one's mind. It includes being comfortable with geometric descriptions of shapes and positions.

There are seven abilities in spatial sense that need to be addressed in the classroom:

Eye-motor co-ordination. This is the ability to co-ordinate vision with body movement. In the mathematics classroom, this involves children learning to write and draw using pencils, crayons, and markers; to cut with scissors; and to handle materials.

- **Visual memory**. This is the ability to recall objects no longer in view.
- **Position-in-space perception**. This is the ability to determine the relationship between one object and another, and an object's relationship to the observer. This includes the development of associated language (over, under, beside, on top of, right, left, etc.) and the transformations (translations, reflections, and rotations) that change an object's position.
- **Visual discrimination**. This is the ability to identify the similarities and differences between, or among, objects.
- **Figure-ground perception.** This is the ability to focus on a specific object within a picture or within a group of objects, while treating the rest of the picture or the objects as background.
- **Perceptual constancy**. This is the ability to recognize a shape when it is seen from a different viewpoint, or from a different distance. This is the perception at play when students recognize similar shapes (enlargements/reductions), and when they perceive as squares and rectangles, the rhombi and parallelograms in isometric drawings.
- **Perception of spatial relationships.** This is the ability to see the relationship between/among two or more objects. This perception is central when students assemble materials to create an object or when they solve puzzles, such as tangram, pattern block, and jigsaw puzzles.

These abilities develop naturally through many experiences in life; therefore, most students come to school with varying degrees of ability in each of these seven aspects. However, students who have been identified as having a learning disability are likely to have deficits in one or more of these abilities. Classroom activities should be planned to allow most students to further develop these perceptions. Most spatial sense activities involve many, if not most, of the spatial abilities, although often one of them is emphasized.

The Classroom Context

Spatial abilities are essential to the development of geometric concepts, and the development of geometric concepts provides the opportunity for further development of spatial abilities. This mutually supportive development can be achieved through consistent and ongoing strategic planning of rich experiences with shapes and spatial relationships. Classroom experiences should also include activities specifically designed for the development of spatial abilities. These activities should focus on shapes new to the grade, as well as shapes from previous grades. As the shapes become more complex, students' spatial senses should be further developed through activities aimed at the discovery of relationships between and among properties of these shapes. Ultimately, students should be able to visualize shapes and their various transformations, as well as sub-divisions and composites of these shapes.

Spatial Sense in Mental Math Time

Following the exploration and development of spatial abilities during geometry instruction, mental math time can provide the opportunity to revisit and reinforce spatial abilities periodically throughout the school year.

Assessment of Spatial Sense Abilities

Assessment of spatial sense development should take a variety of forms. The focus in this aspect of mental math is on individual growth and development in spatial sense, rather than on an arbitrary level of competency to be achieved. You should record any observations of growth students make during the reinforcements, as well as noting students' oral and written responses and explanations. For spatial sense, traditional quizzes that involve students recording answers to questions that you give one-at-a-time in a certain time frame should play a very minor role.

K: Spatial Sense in 2-D Geometry

In grade 5, students predict and construct figures made by combining two triangles and explore how figures can be dissected and transformed into other figures. They recognize, name, describe, and construct right, obtuse, and acute triangles. Angle classification is connected to the side classification—equilateral, isosceles, scalene—that was studied in grade 4. They recognize, name, describe, and represent perpendicular lines/segments, bisectors of angles and segments, and perpendicular-bisectors of segments. They make generalizations about the properties of translations and reflections of shapes. They explore rotations of one-quarter, one-half, and three-quarter turns, using a variety of centres. They also make generalizations about the rotational symmetry property of squares and rectangles and apply these generalizations.

Examples of Spatial Sense Activities

- Provide students with individual geoboards. Inform them that you are going to provide descriptions of a number of triangles. For each triangle that is described, they are to create a triangle on their geoboards, and display it when called upon. Examples of some descriptions:
    1. It is an obtuse triangle with a pair of congruent sides.
    2. It is a right triangle with a line of reflective symmetry.
    3. It is an acute triangle with no lines of reflective symmetry.
    4. It is an isosceles triangle with one leg twice the length of the other leg.
- Direct students to draw a pair of congruent right triangles, not attached, on dot paper. The direct them to draw all the polygons they could make by combining these triangles, matching up their congruent sides. Ask them to name the polygons they draw. This activity can be repeated with two congruent acute or two congruent obtuse triangles.
- Show a simple shape, such as the outline of an arrow pointing upward, on the class whiteboard. Ask students to draw on their individual whiteboards what this shape would like after it is rotated clockwise a quarter turn (90 degrees). Have students show their rotated shapes when directed. Then have students show a half turn (180 degrees) rotation of the original figure. This activity can be repeated using other simple figures.
- Ask students to listen to the following descriptions of the relationships between two line segments. After each description have them draw the line segments on dot paper and share their drawings when directed. Examples of some descriptions
    1. Draw two line segments that are parallel.
    2. Draw a line segment AB. Draw another line segment AC.
    3. Draw a line segment PQ with a midpoint T. Draw a segment RS that intersects PQ at T.
    4. Draw a segment DF. Draw a segment DH that is perpendicular to DF.

L: Spatial Sense in 3-D Geometry

In grade 5, students draw a variety of nets for prisms and pyramids. They make and interpret isometric drawings of shapes made from cubes. They identify, describe and represent the various cross-sections of cubes and rectangular prisms.

Examples of Spatial Sense Activities

- Display nets of various prisms and pyramids, one-at-a-time, on the overhead projector. Have students name the 3-D shape associated with this net. After each shape is named, have students share how they made their decisions.
- Display a cube and describe a variety of cuts you could make with a knife. For each cut, ask students to name the shape of the cross section that would result. Repeat using a rectangular prism
- Create front right, front left, back right, and back left isometric drawings of three different shapes created with 8 cubes. Randomly number and display these 12 drawings, and ask students to examine these drawings and select the three sets of four drawings that all represent the same shape.
- On an overhead projector, display an isometric drawing of a 3-D shape made with cubes. Ask students, (a) How many cubes can you see in this drawing? (b) How many cubes are on the bottom layer? (c) How many cubes would you need to build this shape?