Mental Math
In
Mathematics 4

NOVA SCOTIA
Education
English Program Services

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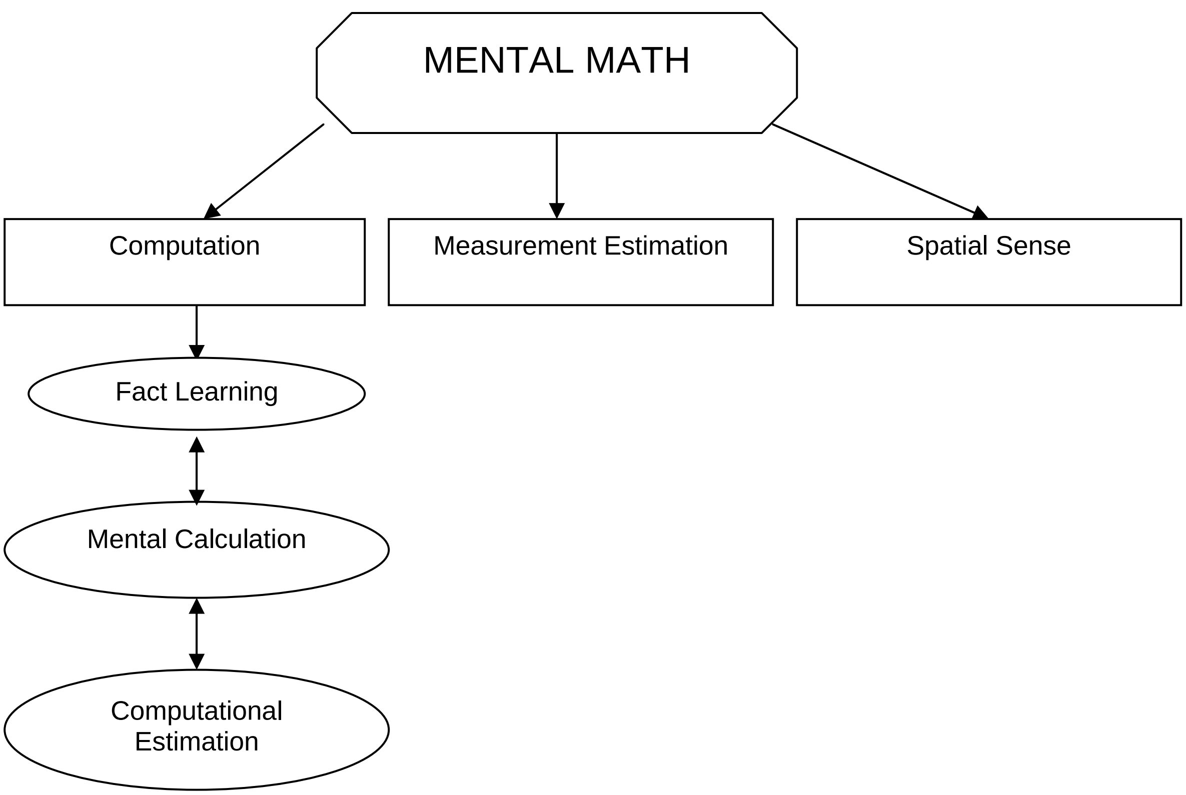
Contents

Introduction

Welcome to your grade-level mental math booklet. After the Department of Education released the *Time to Learn* document in which at least five minutes of mental math was required daily in grades 1 – 9, it was decided to support teachers by clarifying and outlining the specific mental math expectations at each grade. Therefore, grade-level booklets for computational aspects of mental math were prepared and released in draft form in the 2006–2007 school year. Building on these drafts, the current booklets describe the mental math expectations in computation, measurement, and geometry at each grade. These resources are supplements to, not replacements of, the Atlantic Canada mathematics curriculum. You should understand that the expectations for your grade are based on the full implementation of the expectations in the previous grades. Therefore, in the initial years of implementation, you may have to address some strategies from previous grades rather than all of those specified for your grade. It is critical that a school staff meets and plans on an on-going basis to ensure the complete implementation of mental math.

Definitions

In the mathematics education literature, there is not consensus on the usage of some of the words and expressions in mental math. In order to provide uniformity in communication in these booklets, it is important that some of these terms be defined. For example, the Department of Education in Nova Scotia uses the term *mental math* to encompass the whole range of mental processing in all strands of the mathematics curriculum. *Mental math* is broken into three categories in the grade-level booklets: *mental computation*, *measurement estimation*, and *spatial sense*. *Mental computation* is further broken down into *fact learning, mental calculation*, and *computational estimation*.



Fact learning refers to the acquisition of the 100 number facts related to the single digits 0 to 9 for each of the four operations. When students know these facts, they can quickly retrieve them from memory (usually in 3 seconds or less). Ideally, through practice over time, students will achieve automaticity; that is, they will have instant recall without using strategies. *Mental calculation* refers to getting exact answers by using strategies to do the calculations in one's head, while *computational estimation* refers to getting approximate answers by using strategies to do calculations in one's head.

While each category in computations has been defined separately, this does not suggest that the three categories are entirely separable. Initially, students develop and use strategies to get quick recall of the facts. These strategies and the facts themselves are the foundations for the development of other mental calculation strategies. When the facts are automatic, students are no longer employing strategies to retrieve them from memory. In turn, the facts and mental calculation strategies are the foundations for computational estimation strategies. Actually, attempts at computational estimation are often thwarted by the lack of knowledge of the related facts and mental calculation strategies.

Measurement estimation is the process of using internal and external visual (or tactile) information to get approximate measures, or to make comparisons of measures, without the use of measurement instruments.

Spatial sense is an intuition about shapes and their relationships, and an ability to manipulate shapes in one's mind. It includes being comfortable with geometric descriptions of shapes and positions.

Rationale for Mental Math

In modern society, the development of mental skills needs to be a major goal of any mathematics program for two major reasons. First, in their day-to-day activities, most people's computational, measurement, and spatial needs can be met by having well developed mental strategies. Secondly, because technology has replaced paper-and-pencil as the major tool for complex tasks, people need to have well developed mental strategies to be alert to the reasonableness of the results generated by this technology.

PART 1

Computation

The Implementation of Mental Computation

General Approach

In general, a computational strategy should be introduced in isolation from other strategies, a variety of different reinforcement activities should be provided until it is mastered, the strategy should be assessed in a variety of ways, and then it should be combined with other previously learned strategies.

A. Introducing a Strategy

The approach to highlighting a computational strategy is to give the students an example of a computation for which the strategy would be useful to see if any of the students already can apply the strategy. If so, the student(s) can explain the strategy to the class with your help. If not, you could share the strategy yourself. The explanation of a strategy should include anything that will help students see the pattern and logic of the strategy, be that concrete materials, visuals, and/or contexts. The introduction should also include explicit modeling of the mental processes used to carry out the strategy, and explicit discussion of the situations for which the strategy is most appropriate and efficient. Discussion should also include situation for which the strategy would not be the most appropriate and efficient one. Most important is that the logic of the strategy should be well understood before it is reinforced; otherwise, it's long-term retention will be very limited.

B. Reinforcement

Each strategy for building mental computational skills should be practised in isolation until students can give correct solutions in a reasonable time frame. Students must understand the logic of the strategy, recognize when it is appropriate, and explain the strategy. The amount of time spent on each strategy should be determined by the students' abilities and previous experiences.

The reinforcement activities for a strategy should be varied in type and should focus as much on the discussion of how students obtained their answers as on the answers themselves. The reinforcement activities should be structured to insure maximum participation. At first, time frames should be generous and then narrowed as students internalize the strategy. Student participation should be monitored and their progress assessed in a variety of ways to help determine how long should be spent on a strategy.

After you are confident that most of the students have internalized the strategy, you need to help them integrate it with other strategies they have developed. You can do this by providing activities that includes a mix of number expressions, for which this strategy and others would apply. You should have the students complete the activities and discuss the strategy/strategies that could be used; or you should have students match the number expressions included in the activity to a list of strategies, and discuss the attributes of the number expressions that prompted them to make the matches.

Language

Students should hear and see you use a variety of language associated with each operation, so they do not develop a single word-operation association. Through rich language usage students are able to quickly determine which operation and strategy they should employ. For example, when a student hears you say, "Six plus five", "Six and five", "The total of six and five", "The sum of six and five", or "Five more than six", they should be able to quickly determine that they must add 6 and 5, and that an appropriate strategy to do this is the *double-plus-one* strategy.

Context

You should present students with a variety of contexts for each operation in some of the reinforcement activities, so they are able to transfer the use of operations and strategies to situations found in their daily lives. By using contexts such as measurement, money, and food, the numbers become more real to the students. Contexts also provide you with opportunities to have students recall and apply other common knowledge that should be well known. For example, when a student hears you say, "How many days in two weeks?" they should be able to recall that there are seven days in a week and that double seven is 14 days.

Number Patterns

You can also use the recognition and extension of number patterns can to reinforce strategy development. For example, when a student is asked to extend the pattern "30, 60, 120, …,", one possible extension is to double the previous term to get 240, 480, 960. Another possible extension, found by adding multiples of 30, would be 210, 330, 480. Both possibilities require students to mentally calculate numbers using a variety of strategies.

Examples of Reinforcement Activities

Reinforcement activities will be included with each strategy in order to provide you with a variety of examples. These are not intended to be exhaustive; rather, they are meant to clarify how language, contexts, common knowledge, and number patterns can provide novelty and variety as students engage in strategy development.

C. Assessment

Your assessments of computational strategies should take a variety of forms. In addition to the traditional quizzes that involve students recording answers to questions that you give one-at-a-time in a certain time frame, you should also record any observations you make during the reinforcements, ask the students for oral responses and explanations, and have them explain strategies in writing. Individual interviews can provide you with many insights into a student's thinking, especially in situations where pencil-and-paper responses are weak.

Assessments, regardless of their form, should shed light on students' abilities to compute efficiently and accurately, to select appropriate strategies, and to explain their thinking.

Response Time

Response time is an effective way for you to see if students can use the computational strategies efficiently and to determine if students have automaticity of their facts.

For the facts, your goal is to get a response in 3-seconds or less. You would certainly give students more time than this in the initial strategy reinforcement activities, and reduce the time as the students become more proficient applying the strategy until the 3-second goal is reached. In subsequent grades, when the facts are extended to 10s, 100s and 1000s, you should also ultimately expect a 3-second response.

In the early grades, the 3-second response goal is a guideline for you and does not need to be shared with your students if it will cause undue anxiety.

With mental calculation strategies and computational estimation, you should allow 5 to 10 seconds, depending upon the complexity of the mental activity required. Again, in the initial application of these strategies, you would allow as much time as needed to insure success, and gradually decrease the wait time until students attain solutions in a reasonable time frame.

Integration of Strategies

After students have achieved competency using one strategy, you should provide opportunities for them to integrate it with other strategies they have learned. The ultimate goal is for students to have a network of mental strategies that they can flexibly and efficiently apply whenever a computational situation arises. This integration can be aided in a variety of ways, some of which are described below.

You should give them a variety of questions, some of which could be done just as efficiently by two or more different strategies and some of which are most efficiently done by one specific strategy. It is important to have a follow-up discussion of the strategies and the reasons for the selection of specific strategies.

You should take every opportunity that arises in regular math class time to reinforce the strategies learned in mental math time.

You should include written questions in regular math time. This could be as journal entry, a quiz/test question, part of a portfolio, or other assessment for which students will get individual feedback. You might ask students to explain how they could mentally compute a given question in one or more ways, to comment on a student response that has an error in thinking, or to generate sample questions that would be efficiently done by a specified strategy.

A. Addition — Mental Calculation

Facts Applied to Multiples of 10, 100, and 1000 (Extension)

At the beginning of grade 4, it is important that students review the addition facts to 18 and the fact learning strategies. Knowing the addition facts with a 3-second, or less, response time is a grade 2 expectation. These facts are then applied to 10s and 100s in grade 3. In grade 4, they should be extended to 1000s. It would be beneficial to connect these sums to the addition of two groups of base-10 blocks. For example, for 5 small cubes and 6 small cubes or 5 rods and 6 rods or 5 flats and 6 flats or 5 large cubes and 6 large cubes, the results will all be 11 blocks, be they 11 ones, 11 tens, 11 hundreds, or 11 thousands. The sums of 10s are a little more difficult than the sums of 100s and 1000s because when the answer is more than ten 10s, students have to translate the number. For example, for 70 + 80, 7 tens and 8 tens are 15 tens, or *one hundred fifty*, while 700 +800 is 7 hundreds and 8 hundreds which is 15 hundreds, or *fifteen hundred*.

Examples

The following are the grade 2 addition fact strategies with examples as well as examples of the same facts applied to 10s, 100, and 1000s:

a) Doubles Facts (4 + 4, 40 + 40, 400 + 400, and 4000 + 4000)

b) Plus One (Next Number) Facts (5 + 1, 50 + 10, 500 + 100, 5000 + 1000)

c) 1-Apart (Near Double) Facts (3 + 4, 30 + 40, 300 + 400, 3000 + 4000)

d) Plus Two (Next Even/Odd) Facts (7 + 2, 70 + 20, 700 + 200, 7000 + 2000)

e) Plus Zero (No Change) Facts (8 + 0, 80 + 0, 800 + 0, 8000 + 0)

f) Make 10 Facts (9 + 6, 90 + 60, 900 + 600, 9000 + 6000
8 + 4, 80 + 40, 800 + 400, 8000 + 4000)

g) The Last 12 Facts with some possible strategies (may be others):

   i) 2-Apart (Double Plus 2) Facts (5 + 3, 50 + 30, 500 + 300, 5000 + 3000)

   ii) Plus Three Facts (6 + 3, 60 + 30, 600 + 300, 6000 + 3000)

   iii) Make 10 (with a 7) Facts (7+ 4, 70 + 40, 700 + 400, 7000 + 4000)

Examples of Some Practice Items

a) Some practice items for numbers in the 10s **and** 100s:

- 90 + 80
- 70 increased by 20
- $300 more than $600
- 400 girls and 400 boys. How many children?

b) Some practice items for numbers in the 1000s:

- 1 000, 4 000, 7 000, _______
- 2 000 + 3 000
- 5 000 more than 9 000
- I had $7 000 and earned $4 000. How much do I have now?

Front-End Addition (Extension)

This strategy is applied to questions that involve two combinations of non-zero digits, one combination of which may require regrouping. The strategy involves first adding the digits in the highest place-value position, then adding the non-zero digits in another place-value position, and doing any needed regrouping. This was applied to all two 2-digit numbers in grade 3 and should be extended in grade 4 to those two 3-digit and 4-digit whole number calculations that will require only two combinations.

Examples

For 37 + 26, think: 30 and 20 is 50, 7 and 6 is 13, and 50 plus 13 is 63.

For 450 + 380, think: 400 and 300 is 700, 50 and 80 is 130, and 700 plus 130 is 830.

For 3300 + 2800, think: 3000 and 2000 is 5000, 300 and 800 is 1100, and + 5000 and 1100 is 6100.

For 2 070 + 1 080, think: 2 000 and 1 000 is 3 000, 70 and 80 is 150, and 3 000 and 150 is 3 150.

Examples of Some Practice Items

a) Examples of Some Practice Items for Numbers in the 10s:

- 53 + 29 =
- 1 dozen more than 19 eggs
- The total of 15 and 66
- Increase 74 cm by 19 cm

b) Examples of Some Practice Items for Numbers in the 100s:

- 190 + 430 =
- I read 340 pages in one novel and 280 pages in another novel. How many pages did I read in total?
- Teesha likes to travel. She drove 290 km the first day and 120 km the following day. What is the sum of the km Teesha travelled?
- Dan walked halfway around the rectangular ball field. He walked 470m down one side and 360m down another side. How far did Dan walk?
-   $607
<u>+ $309</u>

c) Examples of Some Practice Items for Numbers in the 1000s:

- 4 200 plus 5 900
- The sum of $6400 and $2800
- Increase 4 700 by 2 400
- Sylvia went on an airplane trip. She flew 1900 km the first day and 2300 km the second day. How far did Sylvia fly in total?

Quick Addition (Extension)

This strategy is actually the *Front-End* strategy applied to questions that involve more than two combinations and with no regrouping needed. The questions are always presented visually and students quickly record their answers on paper. While this is a pencil-and-paper strategy because answers will always be recorded on paper before answers are read, it is included here as a mental math strategy because most students will do all the combinations in their heads starting at the front end.

This strategy requires students to holistically examine each question to confirm there will be no regrouping. This habit of holistically examining each question as a first step in determining the most efficient strategy needs to pervade all mental math lessons. (One suggestion for an activity during the discussion of this strategy is to present students with a list of 20 questions, some of which do require regrouping, and direct students to apply quick addition to the appropriate questions and leave out the other ones.) It is important to present examples of these addition questions in both horizontal and vertical formats. Students should have applied this strategy to 3-digit and 4-digit numbers up to the end of grade 4, so in grade 5 they should apply it to 5-digit numbers as well. The numbers should include decimal examples as well as whole number examples.

Most likely, students will add the digits in corresponding place values of the two addends without consciously thinking about the names of the place values. Therefore, in the discussion of the questions, you should encourage students to read the numbers correctly and to use place-value names. This will reinforce place value concepts at the same time as addition.

Examples

For 543 + 256, think and record each resultant digit: 5 and 2 is 7, 4 and 5 is 9, and 3 and 6 is 9, so the answer is 799 (seven hundred ninety-nine); or think: 500 and 200 is 700, 40 and 50 is **9**0, 3 and 6 is **9** to get 799.

For 2 341 increased by 3 415, think and record each resultant digit: 2 and 3 is 5, 3 and 4 is 7, 4 and 1 is 5, and 1 and 5 is 6, so the answer is 5 756 (five thousand, seven hundred fifty-six); or think: 2 000 and 3 000 is **5** 000, 300 and 400 is 700, 40 and 10 is **5**0, 1 and 5 is **6** to get 5 756.

Examples of Some Practice Items

a) Examples of Some Practice Items for Review of Numbers in the 100s:

- $715 + $123 =
- 314
<u>+ 263</u>
- 770 increased by 129
- One bulletin board has a length of 870 cm. Another bulletin board has a length of 109 cm. What is the combined length of the two bulletin boards?

b) Examples of Some Practice Items for Numbers in the 1000s:

- The total of 6 621 km and 2 100 km
- 1 452 increased by 8 200
- $4 678 + $3 211
- 6334
<u>+ 2200</u>
- 3700 more than 5200
- The sum of 6245 and 1712 is

Finding Compatibles (Extension)

This strategy for addition involves looking for pairs of numbers that combine easily to make a sum that will be easy to work with. In grade 4, this should involve searching for pairs of numbers that add to 1000, the next power of ten beyond 10 and 100 that were the focus in grade 3. Some examples of common compatible numbers are 1 and 9, 40 and 60, 300 and 700, and 75 and 25. (In some resources, these compatible numbers are referred to as *friendly* numbers or *nice* numbers.) You should be sure that students are convinced that the numbers in an addition expression can be combined in any order (the associative property of addition).

Examples

For 3 + 8 + 7 + 6 + 2, think: 3 and 7 is 10, 8 and 2 is 10, so 10 and 10 and 6 is 26.

For 25 + 47 + 75, think: 25 and 75 is 100, so 100 plus 47 is 147.

For 400 + 720 + 600, think: 400 and 600 is 1000, and 1000 plus 720 is 1720.

Examples of Some Practice Items

a) Examples of Some Practice Items for Numbers in the 1s and 10s (Grade 3):

- 6 + 9 + 4 + 5 + 1 =
- Students measured the capacity of 5 different containers in mL. Find the total capacity. The containers were 7 mL, 1 mL, 3 mL, 5 mL, and 9 mL.
- 60 + 30 + 40 =
- How much money would Elijah need to buy one of each candy bar which cost 75, 95, 25 cents?

b) Examples of Some Practice Items for Numbers in the 100s (Grade 4):

- 300 plus 437 plus 700
- What is the total mass of three bunches of bananas that weigh 310 g, 600 g, and 400 g?
- What is the sum of $750 + $250 + $330?
- Susan walked 700 metres on Monday, half a kilometre on Tuesday, and 300 metres on Wednesday. How many metres did she walk altogether?
- $\begin{array}{r} 200 \\ 225 \\ +\ 800 \\ \hline \end{array}$

Break Up and Bridge (Extension)

This strategy involves starting with the first number in its entirety and adding the place values of the second number, one-at-a-time, starting with the largest place value. In grade 4, this involves extending the practice items to numbers involving hundreds. Remember that the practice items should only include sums that involve two combinations and one regrouping.

In the introduction of this strategy, you should model both numbers with base-10 blocks and model their addition by combining the blocks, starting with the largest blocks of the second number, in the same way you would combine the symbols for *Break Up and Bridge*. Similarly, modelling on a number line would have you start with the first number in its entirety.

Example

For 45 + 36, think: 45 and 30 (from the 36) is 75, and 75 plus 6 (the rest of the 36) is 81. In short, 45 + 36 = (45 + 30) + 6

For 537 + 208, think: 537 and 200 is 737, and 737 plus 8 is 745.

Examples of Some Practice Items

a) Examples of Some Practice Items for Numbers in the 10s (Grade 3):

- 37 plus 45 equals
- The total number of days in February plus 35
- 66
+ 27

b) Examples of Some Practice Items for Numbers in the 100s (Grade 4):

- 325 + 280 =
- $439 plus $209 =
- 576 increased by 206 =
- William had 2 new containers of yogurt. One held 350 mL of yogurt and the other held 125 mL of yogurt. How many mL of yogurt did William have?
- Kim went to the store and bought two packages of rice. One was labelled as having 487 g and the other as having 220 g. What is the total mass of rice that Kim bought?

Compensation (Extension)

This strategy involves changing one number in the addition question to a nearby multiple of ten or hundred, carrying out the addition using that multiple of ten or hundred, and adjusting the answer to compensate for the original change. Students should understand that the number is changed to make it more compatible, and that they have to hold in their memories the amount of the change. In the last step, it is helpful if they remind themselves that they added too much so they will have to take away that amount. Some students may have used this strategy when learning their facts involving 9s in grade 2; for example, for 9 + 7, they may have found 10 + 7 and then subtracted 1.

Examples

For 52 + 39, think: 40 is easier to work with than 39. Then 52 plus 40 is 92, but I added 1 too many; so, to compensate I subtract one from my answer, 92, to get 91.

For “the sum of 345 and 198”, think: 200 is easier to work with than 198. Then 345 + 200 is 545, but I added 2 too many; so, I subtract 2 from 545 to get 543.

Examples of Some Practice Items

a) Examples of Some Practice Items for Numbers in the 10s:

- $43 + $9 =
- 8 more than 56 =
- The sum of 65 + 29 =
- 44 cents plus 28 cents =

b) Examples of Some Practice Items for Numbers in the 100s:

- 255 + 49 =
- The total of the number of days in one year and 18 days
- 526 increased by 799
- I bought 355 mL of grape juice and 298 mL of orange juice. How much juice did I buy?

Make Multiples of 10, 100, or 1000 (Extension)

This strategy involves changing both addends in an addition question by distributing part of one addend to the other addend in order to make that addend a multiple of ten, hundred, or thousand. Students should understand that this strategy centers on getting a more compatible addend. A common error occurs when students forget that both addends have changed; therefore, some students may need to record one addend as an interim step. This strategy should be compared to the compensation strategy to see how it is alike and how it is different.

For single digit sums, if one addend is an 8 or 9, then a 2 or a 1 is taken from the other addend to turn the 8 or the 9 into a 10, and then 10 is added to what was left from the other number. The *Make-10* strategy would have been used in grade 2 to learn some of the addition facts. As well, the *Make-10s* strategy can be extended to numbers with ones digits of 7.

Example

For 9 + 6, think: If 1 is taken from the 6 and given to the 9, the question becomes 10 + 5, which is easy to add to get 15.

In grade 3, students would have applied this same strategy to sums involving single-digit numbers added to 2-digit numbers as a *Make-10s* strategy. An interesting pattern for students to notice is that the ones digit in the answer is always the part of the single-digit addend that was left after its other part was taken away to help make the multiple of ten.

Example

For 58 + 6, think: If 2 is taken from the 6 and given to the 58, the question becomes 60 + 4, which is easy to add to get 64.

In grade 4, the strategy should be extended to *Make 100s* and *Make 1000s*. Modelling some examples of the numbers with base-10 blocks, combining the blocks physically in the same way you would mentally, will help students understand the logic of the strategy.

Examples

For 390 + 59, think: If 10 is taken from 59 and given to 390, the question becomes 400 + 49, which is easy to add to get 449.

For 3400 + 3900, think: If 100 is taken from the 3400 and given to 3900, the question becomes 3300 + 4000, which is easy to add to get 7300.

Examples of Some Practice Items

a) Examples of Some Practice Items for Numbers in the 10s:
- 5 + 49 =
- 6 greater than 27
- 1 dozen muffins plus 19 muffins

b) Examples of Some Practice Items for Numbers in the 100s:
- 696 + 78 =
- The sum of 57 and 870 is
- There are 490 students at East River School. A new class of 29 students was added to the school. Now how many students are at East River School?

c) Examples of Some Practice Items for Numbers in the 1000s:

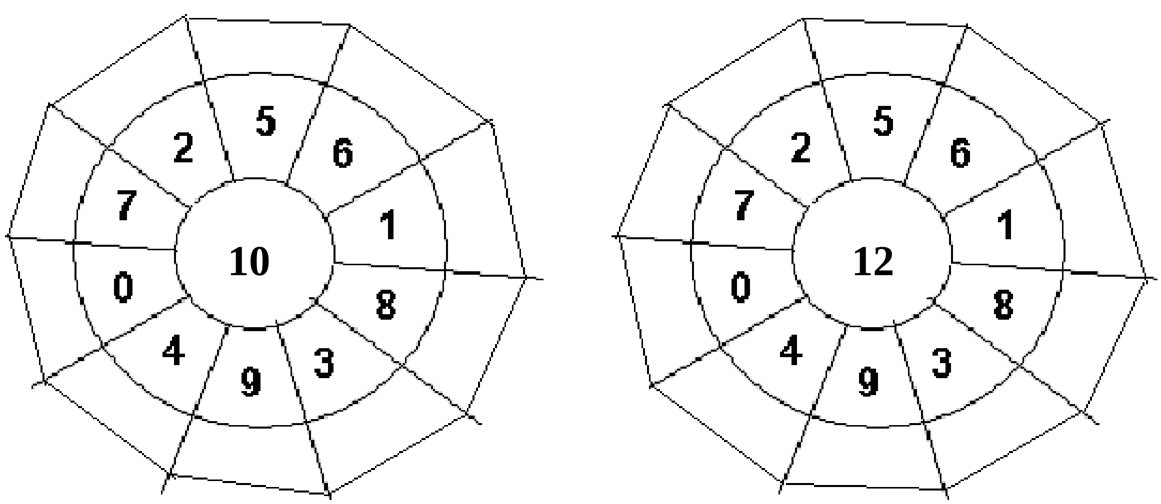
- 2 800 + 400 =
- The total of 5 900 and 1 600 =
- 1800 km farther than 6 300 km
- $1 900 more than $3 500

B. Subtraction — Fact Learning

It is an expectation that the majority of students will have achieved a 3-second, or less, response time for all the subtractions facts by the end of grade 3; however, these facts will still need to be mastered by some students in grade 4 and should be reviewed by all students. Therefore, the following has been taken from the grade 3 mental math booklet to assist you with the strategies for, and development of, subtraction facts:

As a first step in abandoning counting back on their fingers or by using materials, students in grades 1 and 2 used visualization of ten frames to help them get answers to subtraction facts with minuends of 10 or less. The principal mental strategy for the subtraction facts, however, should be *Think Addition*. This strategy should have been introduced in the development of subtraction in regular math class, but with no expectation of quick responses. In mental math time, however, it is important that students have achieved a reasonable response time for the addition facts before trying to use *Think Addition* to get a quick response time for the subtraction facts.

As a get-ready activity, select a sum and ask students a series of questions related to that sum. For example, select 12 as a sum and ask students: 6 and what make 12? 9 and what make 12? 7 and what make 12? 4 and what make 12? You could also ask questions such as: If 7 and 5 make 12, what two subtraction facts do you know? If 9 and 3 make 12, what two subtraction facts do you know? You could also prepare webs by placing different sums in the middle and getting students to complete them; for example, if you place 10 and 12 in the middle of webs (see below), ask students to complete the webs by thinking of the other numbers that would add to the given numbers to get the sum in the middle.



Think Addition (New)

This strategy involves students using their knowledge of addition facts to find the answers to subtraction facts since addition and subtraction are inverse operations. Students would have experience with fact families to draw upon. For example, they would know that 4 + 1 = 5, 1 + 4 = 5, 5 – 4 = 1, and 5 – 1 = 4 are a family of facts. If students are given one of these number sentences, they should readily state the other three related number sentences.

The strategy involves asking oneself what number would be added to the subtrahend to get the minuend.

Examples

For 9 – 4, think: What number would be added to 4 to get 9? Recall that 4 + 5 = 9, so 9 – 4 = 5

For 14 – 6, think: What number would be added to 6 to get 14? Recall that 6 + 8 = 14, so 14 – 6 = 8.

Examples of Some Practice Items

- 10 – 4 =
- It is 4 o'clock. How many more hours until 12 o'clock?
- 8 less 6 =
- 12 minus 5
- 9-L container with 3 L missing. What's left?
- 6 pencils. 4 of them are dull. How many are sharp?
- What is the difference between $14 and $9?

If your students are very competent with their addition facts, you could start applying the *Think-Addition* strategy to all the subtraction facts. However, you may want to apply this strategy to clusters of subtraction facts that are related to the addition fact clusters. Two examples of such clusters are provided; however, you can refer to the addition facts and create the clusters of corresponding subtraction facts.

Examples of Clusters of Subtraction Facts

a) Subtraction Facts Related to Double Facts in Addition

This strategy uses the addition double facts to help find the answers to related subtraction facts.

Examples

For 12 – 6, think: 6 plus what number makes 12? 6 and 6 are 12, so 12 – 6 = 6.

For 18 – 9, think: 9 plus what number makes 18? 9 and 9 are 18, so 18 – 9 = 9.

Examples of Some Practice Items

- 8 – 4
- $6 – $3 =
- 14 days less a week of days
- The difference between a score of 16 and 8
- 10 points minus 5 points

b) Subtraction Facts Related to 1-Apart (Near Doubles) Facts in Addition

This strategy also uses the addition double facts and near-double facts to help find the answers to related subtraction combinations. When the part being subtracted is close to half of the total, we can think of an addition double fact and make the necessary adjustments.

Example

For 9 – 4, think: 4 plus what number makes 9? Since 4 and 4 is 8, 4 and 5 must be 9, so 9 – 4 = 5.

For 13 – 7, think: 7 plus what number makes 13? Since 7 and 7 is 14, 7 and 6 must be 13, so 13 – 7 = 6

Examples of Some Practice Items

- 9 kg of potatoes and 4 kg of them eaten. How many kg of potatoes left now?
- 13 – 6
- Five less than nine
- $15 – ? = $7
- 17 children. 8 boys. How many girls?

Quick Counting Back (New)

While the *Counting-Back* strategy, as a direct modelling of a "take away" meaning for subtraction, was a prevalent strategy used throughout the development of the concept of subtraction, it is very inefficient and cumbersome, especially if more than 1, 2, or 3 have to be subtracted. Therefore, it should not be encouraged as a mental strategy except, perhaps, for the subtraction of 1, 2, or 3. Even then, the counting back needs to be done quickly and in one's head, without any reliance on fingers or other external props. Students may also associate "subtract 1" with a call for the number before, and "subtract 2" with a call for the even/odd number before: if so, they abandon any direct counting back.

A very common error that students make when they count back is to start counting from the given number rather than the number before it. For example, if they are to subtract 2 from 6, they say, 6, 5, and give the answer as 5. Modelling on the number line can help students understand the logic of starting to count from the number before the one given.

Examples

For 6 – 2, think: 5, 4, so the answer is 4.

For 7 – 1, think: 6, so the answer is 6.

For 8 – 3, think: 7, 6, 5, so the answer is 5.

Examples of Some Practice Items

- 9 – 1
- $10 – $2
- One less than 6
- The difference between 3 km and 8 km
- 7 kg reduced by 1 kg

Back-Through 10 (New)

This strategy is a good one for students to have in their repertoire as an alternative to *Think Addition* when the minuend is greater than 10. It involves subtracting in two steps: one part of the subtrahend is subtracted to get to ten and the other part of the subtrahend is then subtracted from 10. This strategy is most effective when only 1 or 2 has to be subtracted from 10 in the second step.

Examples

For 15 – 7, think: To take 7 from 15, first take 5 from 15 to get to 10, and then take 2 (the rest of the 7) from 10 to get 8. In symbols: 15 – 7 = (15 – 5) – 2 = 10 – 2 = 8

For 14 – 5, think: To take 5 from 14, first take 4 from 14 to get to 10, and then take 1 (the rest of 5) from 10 to get 9. In symbols: 14 – 5 = (14 – 4) – 1 = 10 – 1 = 9

Examples of Some Practice Items

- 15 – 6
- 4 less than 12
- The difference between $5 and $13
- 15 questions with 7 correct. How many incorrect?
- 16 minus 8
- Subtract 3 from 11

Up-Through 10 (New)

This strategy, like the *Back-Though-10* strategy, is another alternative to *Think Addition* when the minuend is greater than 10. This strategy, however, involves finding the difference between the two numbers in two steps: first, the difference between the subtrahend and 10 is found, then the difference between 10 and the minuend is found, and finally these two differences are added to give the total difference. This strategy is most effective when the subtrahend is 8 or 9.

Examples

For 12 – 9, think: From 9 to 10 is 1, and from 10 to 12 is 2, so the total difference is 1 plus 2, or 3. In symbols: 12 – 9 = (10 – 9) + (12 – 10) = 1 + 2 = 3

For 14 – 8, think: From 8 to 10 is 2, and from 10 to 14 is 4, so the total difference is 2 plus 4, or 6. In symbols: 14 – 8 = (10 – 8) + (14 – 10) = 2 + 4 = 6

Examples of Some Practice Items

- 14 – 9
- $11 minus $8
- The difference between 9 and 15
- How much less is 8 km than 12 km?
- 8 subtracted from 13
- 16 children, 9 of them boys. How many girls?

C. Subtraction — Mental Calculation

Facts Applied to Multiples of 10, 100, and 1000 (Extended)

This strategy applies to calculations involving the subtraction of two numbers that are both multiples of 10, 100, or 1000. A simple strategy for these questions is to combine the front-end digits as if they were subtraction facts, and then attach the appropriate place-value name and symbols. This strategy should be modeled with base-10 blocks so students understand that 7 blocks subtract 3 blocks will be 4 blocks whether those blocks are small cubes, rods, flats, or large cubes.

Since this strategy rests on students' knowledge of subtraction facts, the facts should be reviewed and consolidated. This strategy was introduced in grade 3 to the tens and hundreds that are related to the subtraction facts with minuends of 10 or less.

Example

For 80 – 30, think: 8 tens subtract 3 tens is 5 tens, or 50; or think: 8 subtract 3 is 5, but this is 5 tens, so the answer is 50.

For 1500 – 600, think: 15 hundreds subtract 6 hundreds is 9 hundreds, or 900; or think: 15 subtract 6 is 9, but this is 9 hundreds, so the answer is 900.

For 6 000 – 2 000, think: 6 thousands subtract 2 thousands is 4 thousands, or 4 000; or think: 6 subtract is 4, but this is 4 thousands, so the answer is 4 000.

Examples of Some Practice Items

a) Examples of Some Practice Items for numbers in the 10s:
- 90 – 10 =
- $60 less than $70 =
- 770 decreased by 50

b) Examples of Some Practice Items for numbers in the 100s:
- $700 – $300 =
- 400 fewer than 600 =
- Susan walked 700 metres. How much farther must she walk to reach one kilometre?
- What is the difference in mass between an 800 g bag of raisins and a 300 g bag of sunflower seeds?

c) Examples of Some Practice Items for numbers in the 1000s:
- 8 000 – 5 000 =
- The century before 1400
- The difference between 10 000 and 7 000
- What should I add to 13 000 to get 17 000?
- My trip will be a total of 6 000 km. I already travelled 4 000 km. How much farther do I have to go?

Quick Subtraction (Extension)

This strategy is actually the *Front-End* strategy applied to subtraction questions that involve no regrouping. If questions only require two subtractions to get an answer, students should be able to do them mentally. However, questions involving three, or more, subtractions should be presented visually with students quickly recording their answers on paper. While this is a pencil-and-paper strategy for these questions because answers will always be recorded on paper before answers are read, it is included here as a mental math strategy because most students will do all the subtractions in their heads starting at the front end.

This strategy requires students to holistically examine the demands of each question as a first step in choosing a strategy: this habit of thinking needs to pervade all mental math lessons. The practice items are presented visually instead of orally. It is important to present these subtraction questions both horizontally and vertically. In grade 3, students would have applied this to two 2-digit and two 3-digit numbers in mental math; this is extended to 4-digit numbers in grade 4. This is the only strategy that would involve 4-digit numbers in mental calculation in grade 4.

Most likely, students will subtract the digits in corresponding place values of the minuend and subtrahend without consciously thinking about the names of the place values. Therefore, in the discussion of the questions, you should encourage students to read the numbers correctly and to use place-value names. This will reinforce place value concepts at the same time as subtraction is reinforced.

Examples

For 87 – 23, think: I see there is no regrouping needed, so I simply subtract 20 from 80 and 3 from 7, recording as I do each subtraction, to get 64.

For 568 – 135, think: I see there is no regrouping needed, so I simply subtract 100 from 500, 30 from 60, and 5 from 8, recording as I do each subtraction, to get 43.

For 4 568 – 1 135, think: I see there is no regrouping needed, so I simply subtract 1000 from 4000, 100 from 500, 30 from 60, and 5 from 8, recording as I do each subtraction, to get 3 433.

Examples of Some Practice Items

a) Examples of Some Practice Items for numbers in the 10s:
   - 38 – 25 =
   - 85
– 31
   - How many hours less is a day than 76 hours?
   - The teacher had 27 dm of yarn for a craft. The students used 15 dm of yarn. How many dm of yarn are remaining?

b) Examples of Some Practice Items for numbers in the 100s:
   - 537
-101
   - 304 fewer people than 8605 people
   - $475 less than $699
   - It is 745 m from Jan's house to the school. She walks 23 m to meet Stephen at his house and then they continue on to school. How far is it from Stephen's house to the school?

Examples of Some Practice Items for numbers in the 1000s:

- 2 457 – 2 111
- $3 245 less than $7 366
- The difference between 1 225 km and 3 575 km
- Subtract 575 from 3 889

Back Through Multiples of 10 and 100 (Extension)

This strategy extends the *Back-Through-10* strategy students learned in grade 3 for fact learning. This strategy involves subtracting a part of the subtrahend to get to the nearest multiple of ten or hundred, and then subtracting the rest of the subtrahend. Note: This strategy is most effective when the subtrahend is much less than the minuend.

Examples

For 14 – 6, think: From 14 to 10 is 4, leaving 2 more from the 6 to subtract, so 10 subtract 2 is 8.

For 75 – 8, think: From 75 to 70 is 5, leaving 3 more from the 8 to subtract, so 70 subtract 3 is 67.

For 530 – 70, think: From 530 to 500 is 30, leaving 40 more from the 70 to subtract, so 500 subtract 40 is 460.

Examples of Some Practice Items

a) Examples of Some Practice Items for numbers in the 10s:

- 15 – 6 =
- The difference between $97 and $8
- If you borrowed a book for 34 days and returned it a week early how long did you have the book?
- A nickel less than 53 cents

b) Examples of Some Practice Items for numbers in the 100s:

- 850 – 70 =
- Claire poured 80 mL from her 970 mL bottle of water. How much water is left in the bottle?
- If a school day is 320 minutes long, how long have you been in school when it is one hour before the end of the day?

Up Through Multiples of 10 and 100 (Extension)

This strategy is an extension of the *Up-Through-10* strategy that students learned in grade 3 to help learn the subtraction facts. This strategy involves finding the difference between the two numbers in two steps from the smaller: first, find the difference between the subtrahend and the next multiple of ten or hundred, then find the difference between that multiple of ten or hundred and the minuend, and finally add these two differences to get the total difference. Note: This strategy is most effective when the two numbers involved are quite close together.

Examples

For 12 – 9, think: It is 1 from 9 to 10 and 2 from 10 to 12; therefore, the difference is 1 plus 2, or 3.

For 84 – 77, think: It is 3 from 77 to 80 and 4 from 80 to 84; therefore, the difference is 3 plus 4, or 7.

For 610 – 594, think: It is 6 from 594 to 600 and 10 from 600 to 610; therefore, the difference is 6 plus 10, or 16.

Examples of Some Practice Items

a) Examples of Some Practice Items for numbers in the 10s:

- 95 – 86 =
- What's left of $46 if $38 is spent?
- What is the difference between 23 minutes and a quarter of an hour?

b) Examples of Some Practice Items for numbers in the 100s:

- 715 – 698 =
- If I had run 498 m when I fell in a 510 m race, how far was I from the finish line?
- There are 846 seats in the theatre and 799 of them are filled. How many more people can come to sit in the theatre?

Break Up and Bridge (New)

This strategy involves starting with the first number in its entirety and subtracting the values in the place values of the second number, one-at-a-time, starting with the largest place value. 2-digit examples are easily modelled on a hundreds chart and/or a metre stick. If subtraction is modelled on a number line, it is natural to model it in the same manner as this strategy.

Examples

For 92 – 26, think: Start with 92 and subtract 20 (the tens place of 26) to get 72, and then subtract 6 (the ones place in 26) from 72 to get 66.

For 745 – 207, think: Start with 745 and subtract 200 (the hundreds place in 207) to get 545, and then subtract 7 (the ones place in 207) from 545 to get 538.

For 860 – 370, think: Start with 860 and subtract 300 (the hundreds place in 370) to get 560, and then subtract 70 (the tens place in 370) from 560 to get 490.

Examples of Some Practice Items

a) Examples of Some Practice Items for Numbers in the 10s:

- 73 – 35 =
- Jason owned 71 books. Robin owned 33 books. How many more books did Jason have than Robin?
- Juan needed 76 cm of string for his project. He cut it from a piece that was 93 cm long. How much extra string will he have?
- What is the difference between 156 and 47?

b) Examples of Some Practice Items for Numbers in the 100s:

- 736 – 308 =
- 840 subtract 250
- $960 – $380 =
- There were 640 sheets of art paper in the art room. Our class used 150 sheets of it for a project. How many sheets of this paper are left in the art room?

Compensation (New)

This strategy for subtraction involves changing the subtrahend to the nearest multiple of ten or hundred, carrying out the subtraction, and adjusting the answer to compensate for the original change. Students should understand that the number is changed to make it more compatible, and that they have to hold in their memories the amount of that change. In the last step, it is helpful if they remind themselves that they subtracted too much, so they will have to add that amount back on.

Examples

For 56 – 18, think: 20 is easier to work with than 18. Then 56 subtract 20 is 36, but I subtracted 2 too many; so, I add 2 back on to get 38 as the answer.

For 85 – 29, think: 30 is easier to work with than 29. Then 85 subtract 30 is 55, but I subtracted 1 too many; so, I add 1 back on to get 56 as the answer.

For 145 – 99, think: 100 is easier to work with than 99. Then 145 subtract 100 is 45, but I subtracted 1 too many; so, I add 1 back on to get 46 as the answer.
to get 46.

For 756 – 198, think: 200 is easier to work with than 198. Then 756 subtract 200 is 556, but I subtracted 2 too many; so, I add 2 back on to get 558 as the answer.

Examples of Some Practice Items

a) Examples of Some Practice Items for numbers in the 10s:

- 15 – 8 =
- $\begin{array}{r} 83 \\ \underline{-\ 28} \end{array}$
- $19 less than $73
- The difference between 63 km and 39 km

b) Examples of Some Practice Items for numbers in the 100s:

- 673 – 99 =
- The difference between $854 and $399
- Kelly's hockey stick weighs 637 g and her little sister's hockey stick weighs 398 g. How much less does Kelly's little sister's hockey stick weigh?
- After reading 298 pages of her book that has 513 pages, how many pages does Melissa have left to read?

Balancing for a Constant Difference (New)

In subtraction situations that require regrouping, this strategy can be used most effectively. By adding the same amount to both numbers in order to get the minuend to a ten or a hundred, regrouping is eliminated. This strategy needs to be carefully introduced because students need to be convinced it actually works! They need to understand that by adding the same amount to both numbers, the two new numbers have the same difference as the original numbers. Examining possible numbers on a metre stick that are a fixed distance apart can help students with the logic of this strategy. (For example, place a highlighter that is more than 10 cm long against a metre stick so that its bottom end is at the 18-cm mark, note where its top end is located, and write the subtraction sentence that gives the length of the highlighter. Repeat by placing the bottom end of the highlighter at the 20-cm mark. Ask, Is the length of the highlighter the same in both number sentences? Which subtraction would be easier to do?) Note: Because both numbers change in carrying out this strategy, many students may need to record the changed minuend to keep track, especially for numbers greater than 2-digit.

Examples

For 87 – 19, think: If I add 1 to 19, I get 20 which is easier to work with; so, I must add 1 to both numbers to get 88 – 20. Now it's easy to get 68, the answer.

For 76 – 28, think: If I add 2 to 28, I get 30 which is easier to work with; so, I must add 2 to both numbers to get 78 – 30. Now it's easy to get 48, the answer.

For 345 – 197, think: If I add 3 to 197, I get 200 which is easier to work with; so, I must add 3 to both numbers to get 348 – 200. Now it's easy to get 148, the answer.

Examples of Some Practice Items

a) Examples of Some Practice Items for numbers in the 10s:

- 85 – 18 =
- 29 cm shorter than 68 cm
- The difference between $47 and $91
- At week 28 in 2007, how many weeks are left in the year?

b) Examples of Some Practice Items for numbers in the 100s:

- 649 – 299 =
- I had $829 in my bank account and I took out $495. How much money is left in my bank account?
- Every week 577 cartons of milk are delivered to the school. On Monday students drank 96 cartons of milk. How many cartons of milk were left for the rest of the week?
- $\begin{array}{r} 912 \\ \underline{-\ 797} \end{array}$

D. Addition and Subtraction — Computational Estimation

The ability to estimate computations is a major goal of any modern computational program. For most people in their everyday lives, an estimate is all that is needed to make decisions, and to be alert to the reasonableness of numerical claims and answers generated by others and with technology. The ability to estimate rests on a strong and flexible command of facts and mental calculation strategies.

Before attempting pencil-and-paper or calculator computations, students must find "ball-park" estimates, so they are alert to the reasonableness of those pencil-and-paper or calculator answers. You should also model this process before you personally do any calculations in front of the class, and you should constantly remind your students to estimate before calculating.

While teaching estimation strategies, it is important to use the language of estimation. Some of the common words and phrases are: *about, just about, between, a little more than, a little less than, close, close to,* and *near.*

Front-End Estimation (Extension)

This strategy involves adding or subtracting only the values in the highest place-value positions to get a "ball-park" estimate. Such estimates are adequate in many circumstances, including getting an estimate before computations with technology in order to be alert to the reasonableness of the answers. For addition, because only the highest place values are added, the estimated front-end sum will always be less than the actual sum. For subtraction,

Examples

To estimate 213 + 397, think: 200 + 300 = 500, so the estimate is about 5 hundred.

To estimate 392 – 153, think: 300 – 100 = 200, so the estimate is about 2 hundred.

To estimate 437 + 548, think: 400 + 500 = 900, so the estimate is about 9 hundred

To estimate 534 – 254, think: 500 – 200 = 300, so the estimate is about 3 hundred.

To estimate 4 276 + 3 237, think: 4000 + 3000 = 7 000, so the estimate is about 7 thousand.

To estimate 7896 – 2347, think: 7000 – 2000 = 5000, so the estimate is 5 thousand.

Examples of Some Practice Items

a) Examples of Some Practice Items for Estimating Sums of Numbers in the 100s:

- Estimate the sum of 234 and 438
- Estimate the total of 779 + 128
- If there were 423 apples in one bin and 443 apples in another bin, about how many apples would there be altogether?
- Examples of Some Practice Items for Estimating Differences of Numbers in the 100s:
- Estimate the difference between 327 and 142
- If there were 928 cars in the parking lot and by noon 741 of these cars had left about how many of the original cars would be left in the parking lot?
- The movie theatre made 248 bags of popcorn and sold 109 bags. About how many bags of popcorn are left to sell?

c) Examples of Some Practice Items for Estimating Sums of Numbers in the 1000s:

- Estimate the sum of 1 324 and 8 265
- Approximately what is the total of 5 719 + 4 389?
- If 4 096 people attended the fair on Saturday and 3 227 people attended the fair on Sunday, about how many people attended the fair on the weekend?

d) Examples of Some Practice Items for Estimating Differences of Numbers in the 1000s:

- Estimate the difference between 6 237 and 2 945
- Approximately how much further is 5 475 km than 3 128 km?
- There were 4 308 people at the hockey game. At intermission 1 489 people went to buy food. About how many people did not go to buy food during intermission?

Adjusted Front-End Estimation (Extension)

This strategy begins by getting a front-end estimate and then adjusting that estimate to get a better, or closer, estimate by either (a) considering the second highest place values or (b) by clustering all the values in the other place values to "eyeball" whether there would be enough together to account for an adjustment. These two adjustment strategies will not always result in the same adjustment being made.

Examples

To estimate 437 + 545 by (a), think: 400 plus 500 is 900, but this can be adjusted by thinking 30 and 40 is 70 which is closer to another 100; so, the adjusted estimate would be 900 + 100 = 1000.

OR

To estimate 437 + 545 by (b), think: 400 plus 500 is 900, but this can be adjusted by "eyeballing" that 37 and 45 would be close to another 100; so, the adjusted estimate would be 900 + 100 = 1000.

To estimate 3297 + 2285 by (a), think: 3000 plus 2000 is 5000, and 200 plus 200 is only 400, which is not close to another 1000; so, the estimate is 5000. However, clustering 297 and 285 would suggest about 600, so another 1000 would be added to give an estimate of 6000.

To estimate 382 – 116 by (b), think: 300 subtract 100 is 200, and 80 – 10 is 70 that is close to another 100; so, the adjusted estimate is 300.

OR

To estimate 382 – 116, think: 300 subtract 100 is 200, and "eyeballing" 82 – 16 suggests another 100 estimate; so, the adjusted estimate is 200 + 100 = 300.

To estimate 5674 – 2487 by (a), think: 5000 subtract 2000 is 3000, and 600 – 400 is 200 that is not close to another thousand; so, the estimate stays at 3000.

OR

To estimate 5674 – 2487 by (b), think: 5000 subtract 2000 is 3000, and "eyeballing" 674 – 487 suggests there is not another thousand estimate; so, the estimate stays at 3000.

Examples of Some Practice Items

a) Examples of Some Practice Items for Estimating Sums:

- Estimate the sum of 256 and 435
- Kevin had 519 basketball cards and Dakota gave him 146 new cards. About how many basketball cards does Kevin have now?
- The radio station was given 2 298 new CDs last month and 1 289 new CDs this month. About how many new CDs did they receive in the past two months?

b) Examples of Some Practice Items for Estimating Differences:

- Estimate the difference between 645 and 290
- If Todd and Tami were sharing a 720 m length of rope and Todd took 593 m for his boat, about how many meters were left for Tami?
- Smith's grocery store has 1223 cans of tuna and Jackson's grocery store has 2932 cans of tuna. About how many more cans of tuna are in Jackson's grocery store?

Rounding (Extension)

Rounding in Addition

This strategy involves rounding each number to the highest, or the highest two, place values and adding the rounded numbers. Rounding to the highest place value would enable most students to keep track of the rounded numbers and do the calculation in their heads; however, rounding to two highest place values would probably require most students to record the rounded numbers before performing the calculation mentally.

When the digit 5 is involved in the rounding procedure for numbers in the 10s, 100s, and 1000s, the number can be rounded up or down. However, the decision to round up or to round down should be based upon the effect the rounding will have in the overall calculation. For example, if both numbers to be added have a 5, 50, or 500, rounding one number *up* and one number *down* will minimize the effect the rounding will have on the estimation. Also, if both numbers are close to 5, 50, or 500, it may be better to round one up and one down.

Examples

For 85 + 65 + 98, think: Round 98 to 100. Since 85 and 65 both have 5s, it would be best to round one up and one down to get 80 and 70, and add to get 150 and then add the 100 to get 250.

For 378 + 230, think: 378 rounds to 400 and 230 rounds to 200; so, 400 plus 200 is 600.

For 4 520 + 4 610, think: Since both numbers are close to 5000, it would be best to round them to 4 000 and 5 000, and add to get 9 000.

Examples of Some Practice Item

a) Examples of Some Practice Items for Rounding in Addition of Numbers in the 100s:

- Estimate 426 + 587 =
- Estimate the sum of 238 and 469
- Approximately what is 667 more than 579?
- Kieva sold 523 sandwiches during the morning of the School Fair and 679 sandwiches in the afternoon. About how many sandwiches were sold at the fair?
- Estimate $219
+ $392

b) Examples of Some Practice Items for Rounding in Addition of Numbers in the 1000s:

- Estimate 5 184 + 2 958
- On Friday night 4 550 ticket were sold for the hockey game and 4 850 tickets were sold for Saturday night's hockey game. About how many tickets were sold for the two hockey games on the weekend?

- My Aunt's house is 1370 km away from my house. To get to my Nan's house you have to go 2440 km past my Aunt's house. About how far away from my house does my Nan live?
- Estimate the sum of $7 780 and $3 140
- What is the approximate total if you increase 6 118 by 3 950?

Rounding in Subtraction

For rounding in subtraction situations, the process is similar to that for addition, except for the situations in which both numbers involve 5, 50, or 500, and in situations in which both numbers are close to 5, 50, or 500. For rounding in these situations, both numbers should be rounded up because you are looking for the difference between the two numbers; therefore, you don't want to increase this difference by rounding one up and one down. This will require careful introduction for students to be convinced. So often students only associate subtraction with *take-away* and need to reminded that subtraction also finds the *difference* between two numbers. (Help them make the connection to the *Balancing-for-a-Constant-Difference* strategy in mental math.)

Examples

To estimate 591 – 209, think: 591 rounds to 600 and 209 rounds to 200; so, 600 subtract 200 is 400.

To estimate 6237 – 2945, think: 6237 rounds to 6000 and 2945 rounds to 3000; so, 6 000 subtract 3000 is 3 000.

To estimate 5549 – 3487, think: Parts of both numbers are close to 500, so round both up. Then 6000 subtract 4000 is 2000.

Examples of Some Practice Items

a) Examples of Some Practice Items for Rounding in Subtraction of Numbers in the 100s:
   - Estimate 427 – 192 =
   - Clark has collected 94 coupons. He needs 266 coupons to get a bike. About how many more coupons does Clark need?
   - There are 587 students at Fairway School. There are 834 students at Middle School. About how many more students go to Middle School than Fairway School?
   - Estimate 846 decreased by 758

b) Examples of Some Practice Items for Rounding in Subtraction of Numbers in the 1000s:
   - Estimate the difference between 4768 and 3068
   - A big television costs $1754. If I have $879 about how much more money will I need to buy the television?
   - Estimate 6341 km – 1892 km =
   - 4507 CDs were delivered to the store and 1593 CDs were sold. About how many CDs are still for sale at the store?

Clustering of Near Compatibles (New)

This strategy is useful when you need to estimate the sums and differences in a list of numbers. You examine the list to search for pairs of numbers that are near known compatibles that make 100s or 1000s. These pairs provide estimates for 100 or 1000, and are combined with other such estimates, as well as estimates of any leftovers, to get a total estimate for the list.

Examples

For 44 + 33 + 62 + 71, think: 44 and 62 is about 100, and 33 and 71 is about 100; so, the estimate would be about 100 plus 100, or 200.

For 208 + 489 + 812 + 529 + 956, think: 208 and 812 is about 1000, 489 and 529 is about 1000, and 956 is almost another 1000; so, the estimate is about 1000 plus 1000 plus 1000, or 3000.

For 612 – 289 + 397, think: 612 and 397 is about 1000, and 1000 subtract about 300 gives an estimate of 700.

Examples of Some Practice Items

- Estimate: 32 + 62 + 96 + 71 + 41
- Estimate: 239 – 43 + 54 - 62
- The following are the amounts collected by the team members for the charity: $256 + $602 + $902 + $746 + $399 + $106. Approximately how much was collected for the charity?
- These are attendances for five nights: 246, 299, 748, 235, and 703. About how many attended altogether?

F. Multiplication— Fact Learning

Multiplication Fact Learning Strategies

In grade 4, students are to know the multiplication facts with at least a 3-second response by the end of the year. This is done through learning a series of strategies, each of which addresses a cluster of facts. Each strategy is introduced, reinforced, and assessed before being integrated with previously learned strategies. It is important that students understand the logic and reasoning of each strategy, so the introductions of the strategies are very important. As students master each cluster of facts for a strategy, it is recommended that they record these learned facts on a multiplication chart. By doing this, they visually see their progress and are aware of which facts they should be practicing. What follows is a suggested sequence for these strategies.

A. The Twos Facts (Doubles)

This strategy involves connecting the addition doubles to the related "two-times" multiplication facts. It is important to make sure students are aware of the equivalence of commutative pairs (2 × ? and ? × 2); for example, 2 × 7 is the double of 7 and that 7 × 2, while it means 7 groups of 2, has the same answer as 2 × 7. When students see 2 × 7 or 7 × 2, they should think: 7 and 7 are 14. Flash cards displaying the facts involving 2 and the times 2 function on the calculator are effective reinforcement tools to use when learning the multiplication doubles.

It is suggested that 2 × 0 and 0 × 2 be left until later when all the zeros facts will be done.

Examples

For 2 × 9, think: This is 9 plus 9, so the answer is 18.
For 6 × 2, think: This 6 plus 6, so the answer is 12.

B. The Nifty Nine Facts

The introduction of the facts involving 9s should concentrate on having students discover two patterns in the answers; namely, the tens' digit of the answer is one under the number of 9s involved, and the sum of the ones' digit and tens' digit of the answer is 9. For example, for 6 × 9 = 54, the tens' digit in the product is one less than the factor 6 (the number of 9s) and the sum of the two digits in the product is 5 + 4 or 9. Because multiplication is commutative, the same thinking would be applied to 9 × 6. Therefore, when asked for 3 × 9, think: the answer is in the 20s (the decade of the answer) and 2 and 7 add to 9; so, the answer is 27. You could help students master this strategy by scaffolding the thinking involved; that is, practice presenting the multiplication expressions and just asking for the decade of the answer; practice presenting the students with a digit from 1 to 8 and asking them the other digit that they would add to your digit to get 9; and conclude by presenting the multiplication expressions and asking for the answers and discussing the steps in the strategy.

Another strategy that some students may discover and/or use is a compensation strategy, where the computation is done using 10 instead of 9 and then adjusting the answer to compensate for using 10, rather than 9. For example, for 6 × 9, think: 6 groups of 10 is 60 but that is 6 too many (1 extra in each group), so 60 subtract 6 is 54. Because this strategy involves multiplication followed by subtraction, many students find it more difficult than the two-pattern strategy.

While 2 × 9 and 9 × 2 could be done by this strategy, these two nines facts were already handled by the twos facts. This *nifty-nine* strategy is probably most effective for factors 3 to 9 combined with the factor 9, leaving the 0s and 1s for later strategies.

Examples

For 5 × 9, think: The answer is in the 40s, and 4 and 5 add to 9, so 45 is the answer.

For 9 × 9, think: The answer is in the 80s, and 8 and 1 add to 9, so 81 is the answer.

C. The Fives Facts

Many students probably have been using a skip-counting-by-5 strategy when 5 has been a factor; however, this strategy is difficult to apply in 3 seconds, or less, for all combinations, and often results in students' using fingers to keep track. Therefore, students need to adopt a more efficient strategy.

If the students know how to read the various positions of the minute hand on an analog clock, it is easy to make the connection to the multiplication facts involving 5s. For example, if the minute hand is on the 6 and students know that means 30 minutes after the hour, then the connection to 6 × 5 = 30 is easily made. This is why you may see the Five Facts referred to as the "clock facts." This would be the best strategy for students who can proficiently tell time on an analog clock.

Another possible strategy involves the patterns in the products. While most students have observed that the Five Facts have a 0 or a 5 as a ones' digit, some have also noticed other patterns. One pattern is that the ones' digit is a 0 if the number of 5s involved is even or the ones' digit is 5 if the number of 5s involved is odd. Another pattern is that the tens' digit of the answer is half the numbers of 5s involved, or half the number of 5s rounded down. For example, the product of 8 and 5 ends in 0 because there are 8 fives and the tens' digit is 4 because 4 is half of 8; therefore, 8 × 5 is 40. The product of 7 and 5 ends in 5 because 7 is odd and the tens' digit is 3 because 3 is half of 7 rounded down; therefore, 7 × 5 is 35.

While these strategies apply to 2 × 5, 5 × 2, 5 × 9, and 9 × 5, these facts were also part of the *twos facts*, and *nines facts*. The *fives facts* involving zeros are probably best left for the zeros facts since the minute-hand approach has little meaning for 0.

Examples

For 5 × 8, think: When the minute hand is on 8, it is 40 minutes after the hour, so the answer is 40.

For 3 × 5, think: When the minute hand is on 3, it is 15 minutes after the hour, so the answer is 15.

D. The Ones Facts

While the ones facts are the "no change" facts, it is important that students understand why there is no change. Many students get these facts confused with the addition facts involving 1. To understand the ones facts, knowing what is happening when we multiply by one is important. For example 6 × 1 means *six groups of 1* or 1 + 1 + 1 + 1 + 1 + 1 and 1 × 6 means *one group of 6*. It is important to avoid teaching arbitrary rules such as "any number multiplied by one is that number". Students will come to this rule on their own given opportunities to develop understanding. Be sure to present questions visually and orally; for example, "4 groups of 1" and 4 × 1; and "1 group of 4" and 1 × 4.

While this strategy applies to 2 × 1, 1 × 2, 1 × 5, and 5 × 1, these facts have also been handled previously with the other strategies.

Examples

For 8 × 1, think: Eight 1s make 8.

For 1 × 7, think: One 7 is 7.

E. The Tricky Zeros Facts

As with the ones facts, students need to understand why these facts all result in zero because they are easily confused with the addition facts involving zero; thus, the zeros facts are often "tricky." To understand the zeros facts, students need to be reminded what is happening by making the connection to the meaning of the number sentence. For example: 6 × 0 means "six 0's or "six sets of nothing." This could be shown by drawing six boxes with nothing in each box. 0 × 6 means "zero sets of 6." This is much more difficult to conceptualize; however, if students are asked to draw two sets of 6, then one set of 6, and finally zero sets of 6, where they don't draw anything, they will realize why zero is the product. Similar to the previous strategy for teaching the ones facts, it is important not to teach a rule such as "any number multiplied by zero is zero". Students will come to this rule on their own, given opportunities to develop understanding.

Examples

For 7 × 0, think: Having seven zeros means having a total of zero.

For 0 × 8, think: Having no eights means having zero.

F. The Threes Facts

The way to teach the threes facts is to develop a "double plus one more set" strategy. You could have students examine arrays with three rows. If they cover the third row, they easily see that they have a "double" in view; so, adding "one more set" to the double should make sense to them. For example, for 3 × 7, think: 2 sets of 7(double) plus one set of 7 or (7 × 2) + 7 = 14 + 7 = 21. This strategy uses the doubles facts that should be well known before this strategy is introduced; however, there will need to be a discussion and practice of quick addition strategies to add on the third set.

While this strategy can be applied to all facts involving 3, the emphasis should be on 3 × 3, 3 × 4, 4 × 3, 3 × 6, 6 × 3, 3 × 7, 7 × 3, 3 × 8, and 8 × 3, all of which have not been addressed by earlier strategies.

Examples

For 3 × 6, think: Two sixes make 12, plus one more six is 18.

For 4 × 3, think: Two fours make 8, plus one more 4 is 12.

G. The Fours Facts

The way to teach the fours facts is to develop a "double-double" strategy. You could have students examine arrays with four rows. If they cover the bottom two rows, they easily see they have a "double" in view and another "double" covered; so, doubling twice should make sense. For example: for 4 × 7, think: 2 × 7(double) is 14 and 2 × 14 is 28. Discussion and practice of quick mental strategies for the doubles of 12, 14, 16 and 18 will be required for students to master their fours facts. (One efficient strategy is front-end whereby you double the ten, double the ones, and add these two results together. For example, for 2 × 16, think: 2 times 10 is 20, 2 times 6 is 12, so 20 and 12 is 32.)

While this strategy can be applied for all facts involving 4, the emphasis should be on 4 × 4, 4 × 6, 6 × 4, 4 × 7, 7 × 4, 4 × 8, and 8 × 4, all of which have not been addressed by earlier strategies.

Examples

For 4 × 6, think: Double six is 12, and double 12 is 24.

For 8 × 4, think: Double eight is 16, and double 16 is 32.

H. The Last Nine Facts

After students have worked on the above seven strategies for learning the multiplication facts, there are only ***nine*** facts left to be learned. These include: 6 × 6; 6 × 7; 6 × 8; 7 × 7; 7 × 8; 8 × 8; 7 × 6; 8 × 7; and 8 × 6. At this point, the students themselves can probably suggest strategies that will help with quick recall of these facts. You should put each fact before them and ask for their suggestions.

Among the strategies suggested might be one that involves decomposition and the use of helping facts.

Examples

For 6 × 6, think: 5 sets of 6 is 30 plus 1 more set of 6 is 36.

For 6 × 7 or 7 × 6, think: 5 sets of 6 is 30 plus 2 more sets of 6 is 12, so 30 plus 12 is 42.

For 6 × 8 or 8 × 6, think: 5 sets of 8 is 40 plus 1 more set of 8 is 48. Another strategy is to think: 3 sets of 8 is 24 and double 24 is 48.

For 7 × 7, think: 5 sets of 7 is 35, 2 sets of 7 is 14, so 35 and 14 is 49. (This is more difficult to do mentally than most of the others; however, many students seem to commit this one to memory quite quickly, perhaps because of the uniqueness of 49 as a product.)

For 7 × 8, think: 5 sets of 8 is 40, 2 sets of 8 is 16, so 40 plus 16 is 56. (Some students may notice that 56 uses the two digits 5 and 6 that are the two counting numbers before 7 and 8.)

For 8 × 8, think: 4 sets of 8 is 32, and 32 doubled is 64. (Some students may know this as the number of squares on a chess or checker board.)

G. Multiplication — Mental Calculation

Multiplication by 10 and 100

This strategy involves keeping track of how the place values have changed. Introduce these products by considering base-10 block representations. For example, for 10 × 53, display 5 rods and 3 small cubes to represent 53, and think: 10 sets of 5 rods would be 50 rods, or 5 flats, and 10 sets of 3 small cubes would be 30 small cubes, or 3 rods; so, 5 flats and 3 rods represents 530. Through a few similar examples, it becomes clear that multiplying by 10 changes all the place values of a number by one position to the left because the product is ten times as much. Similarly, by modelling 100 × 53, it becomes clear that multiplying by 100 changes all the place values of a number by two positions to the left because the product is one hundred times as much.

Note: While some students may see the pattern that one zero gets attached to the original number when multiplying by 10, and two zeros get attached when multiplying by 100, this is not the best way to introduce these products. Later, when students are working with decimals, such as 100 × 0.12, using the "two-place-value-change" approach will be more meaningful than the "attaching-two-zeros" approach, and it will more likely produce a correct answer!

Examples

For 10 × 67, think: the 6 tens will change to 6 hundreds and the 7 ones will change to 7 tens; therefore, the answer is 670.

For 100 × 86, think: the 8 tens will change to 8 thousands and the 6 ones will change to 6 hundreds; therefore, the answer is 8 600.

Examples of Some Practice Items

- 10 × 53 =
- 10 groups of 34 =
- 87 times 10 =
- 47 dm = ____ cm
- 78 m = ____ dm
- There were 66 large tables with 10 chairs at each table in the restaurant. How many people could eat at the restaurant if it was full?
- 100 × 7 =
- 100 multiplied by 74 =
- The school bought 83 packages of paper and each package held 100 sheets of paper. How many sheets of paper did the school purchase?
- 8 m = ___cm
- How many years in 20 centuries?
- How many pennies equal $6?

PART 2

Measurement Estimation

The Implementation of Measurement Estimation

General Approach

For the most part, a measurement estimation strategy would be reinforced and assessed during mental math time in the grades following its initial introduction. The goal in mental math is to increase a student's *competency* with the strategy. It is expected that measurement estimation strategies would be introduced as part of the general development of measurement concepts at the appropriate grade levels. Each strategy would be explored, modeled, and reinforced through a variety of class activities that promote understanding, the use of estimation language, and the refinement of the strategy. As well, the assessment of the strategy during the instructional and practice stages would be ongoing and varied.

Competency means that a student reaches a reasonable estimate using an appropriate strategy in a reasonable time frame. While what is considered a reasonable estimate will vary with the size of the unit and the size of the object, a general rule would be to aim for an estimate that is within 10% of the actual measure. A reasonable time frame will vary with the strategy being used; for example, using the *benchmark strategy* to get an estimate in metres might take 5 to 10 seconds, while using the *chunking strategy* might take 10 to 30 seconds, depending upon the complexity of the task.

A. Introducing a Strategy in Regular Classroom Time

Measurement estimation strategies should be introduced when appropriate as part of a rich and carefully sequenced development of measurement concepts. For example, in grade 3, the distance from the floor to most door handles is employed as a *benchmark* for a metre so students can use a *benchmark strategy* to estimate lengths of objects in metres. This has followed many other experiences with linear measurement in earlier grades: in grade primary, students compared and ordered lengths of objects concretely and visually; in grade 1, students estimated lengths of objects using non-standard units such as paper clips; in grade 2, students developed the standard unit of a metre and were merely introduced to the idea of a *benchmark* for a metre.

The introduction of a measurement estimation strategy should include a variety of activities that use concrete materials, visuals, and meaningful contexts, in order to have students thoroughly understand the strategy. For example, a meaningful context for the *chunking strategy* might be to estimate the area available for bookshelves in the classroom. Explicit modeling of the mental processes used to carry out a strategy should also be part of this introduction. For example, when demonstrating the *subdivision strategy* to get an estimate for the area of a wall, you would orally describe the process of dividing the wall into fractional parts, the process of determining the area of one part in square metres, and the multiplication process to get the estimate of the area of the entire wall in square metres. During this introductory phase, you should also lead discussions of situations for which a particular strategy is most appropriate and efficient, and of situations for which it would not be appropriate nor efficient.

B. Reinforcement in Mental Math Time

Each strategy for building measurement estimation skills should be practised in isolation until students can give reasonable estimates in reasonable time frames. Students must understand the logic of the strategy, recognize when it is appropriate, and be able to explain the strategy. The amount of time spent on each strategy should be determined by the students' abilities, progress, and previous experiences.

The reinforcement activities for a strategy should be varied in type and should focus as much on the discussion of how students obtained their answers, as on the answers themselves. The reinforcement activities should be structured to ensure maximum participation. At first, time frames should be generous and then narrowed as students internalize the strategy and become more efficient. Student participation should be monitored and their progress assessed in a variety of ways. This will help determine the length of time that should be spent on a strategy.

During the reinforcement activities, the actual measures should not be determined every time an estimate is made. You do not want your students to think that an estimate is always followed by measurement with an instrument: there are many instances where an estimate is all that is required. When students are first introduced to an activity, it is helpful to follow their first few estimates with a determination of the actual measurement in order to help them refine their estimation abilities. Afterwards, however, you should just confirm the reasonable estimates, having determined them in advance

Most of the reinforcement activities in measurement will require the availability of many objects and materials because students will be using some objects and materials as benchmarks and will be estimating the attributes of others. To do this, they must see and/or touch those objects and materials.

After you are confident that most students have achieved a reasonable competency with the strategy, you need to help them integrate it with other strategies that have been developed. You should do this by providing activities that include a mix of attributes to be estimated in different units using a variety of contexts. You should have the students complete the activities and discuss the strategy/strategies that were, or could have been, used. You should also have students match measurement estimation tasks to a list of strategies, and have them discuss the reasoning for their matches.

C. Assessment

Your assessments of measurement estimation strategies should take a variety of forms. Assessment opportunities include making and noting observations during the reinforcements, as well as students' oral and written responses and explanations. Individual interviews can provide you with many insights into a student's thinking about measurement tasks. As well, traditional quizzes that involve students recording answers to estimation questions that you give one-at-a-time in a certain time frame can be used.

Assessments, regardless of their form, should shed light on students' abilities to estimate measurements efficiently and accurately, to select appropriate strategies, and to explain their thinking.

Measurement Estimation Strategies

The Benchmark Strategy

This strategy involves mentally matching the attribute of the object being estimated to a known measurement. For example, to estimate the width of a large white board, students might mentally compare its width to the distance from the doorknob to the floor. This distance that is known to be about 1 metre is a *benchmark*. When students mentally match the width of the white board to this benchmark, they may estimate that the width would be about two of these benchmarks; therefore, their estimate would be 2 metres. In mathematics education literature you will often see reference made to *personal referents.* These are benchmarks that individuals establish using their own bodies; for example, the width of a little finger might be a personal referent for 1 cm, a hand span a referent for 20 cm, and a hand width a referent for 1 dm. These benchmarks have the advantage of being portable and always present whenever and wherever an estimate is needed.

The Chunking Strategy (Starting in Grade 5)

This strategy involves mentally dividing the attribute of an object into a number of chunks, estimating each of these chunks, and adding the estimates of all the chunks to get the total estimate. For example, to estimate the length of a wall, a student might estimate the distance from one corner to the door, estimate the width of the door, estimate the distance from the door to the bookcase, and estimate the length of the bookcase which extends to the other corner. All these individual estimates would be added to get the estimate of the length of the wall.

The Unitizing Strategy (Starting in Grade 5)

This strategy involves establishing an object (that is smaller than the object being estimated) as a unit, estimating the size of the attribute of this unit, and multiplying by the number of these units it would take to make the object being measured. For example, students might get an estimate for the length of a wall by noticing it is about five bulletin boards long. The bulletin board becomes the unit. Students estimate the length of the bulletin board and multiply this estimate by five to get an estimate of the length of the wall.

The Subdivision Strategy (Starting in Grade 6)

This strategy involves mentally dividing an object repeatedly in half until a more manageable part of the object can be estimated and then this estimate is multiplied by the appropriate factor to get an estimate for the whole object. For example, to estimate the area of a table top, students might mentally divide the table top into fourths, estimate the area of that one-fourth in square decimeters, and then multiply that estimate by four to get the estimate of the area of the entire table top.

H. Measurement Estimation — Length / Distance

In grade 3, students developed an understanding of the benchmarks for the decimetre and they had many classroom experiences using these benchmarks to develop a sense of length. They are now ready to use these benchmarks to develop the estimation competencies.

Benchmarks — Decimetre

Student should gain competency with estimating lengths of objects that are between 1 dm and 20 dm. Students could use the length of the rod in the base-10 blocks as a benchmark for 1 dm, or a personal referent such as their hand width.

Examples of Some Practice Activities

- Show students a small crayon box (its width is bout 1 decimetre). As you pinpoint other articles in the classroom, ask them to decide if each one is longer/shorter (or wider or higher) than the width of the crayon box. After they decide on each one, have them explain how they made their decisions.
- Ask students to visualize 1 decimetre. Ask them to look around the classroom for something that is less than 1 dm, something that is about 1 dm, and something that is more than 1 dm. Record their three things. Working as partners, have them share their lists. Any controversies can be settled by using a base-10 rod as a comparison tool.
- Display a variety of materials, such as a piece of loose leaf paper, and ask them if its length is more than 3 dm, or less than 3 dm. Ask students to write their estimate on their individual whiteboards. After each estimate, use a ruler to determine the actual length.
- Ask a variety of questions, such as: Will the length of your math text be closer to 2 dm or 3 dm? Will the length of your desktop be closer to 3 dm or 6 dm? Have students record their answers. Discuss how decisions were made.
- Provide students with intervals for the estimates of a variety of objects in the classroom, and ask them to select the interval they think best represents the estimate. For example, ask: Is the height of the classroom door between 2 and 3 dm, 8 and 12 dm, or 18 and 22 dm? Is the height of the filing cabinet 10 – 30 dm, 30 – 50 dm, or 50 – 70 dm?
- Present students with a variety of estimation situations and get them to commit to one estimate. Afterwards, facilitate a discussion of the estimates and the strategies used to make a decision. For example, present this situation: Francesca estimated that the height of the teacher's desk was about 4 dm. Enrico estimated its height at about 7 dm and Maria estimated that it was about 9 dm. Whose estimate do you think is best? Why?
- From where they are sitting, have students estimate the width of a classroom window in decimetres. Then have them compare their answers with partners and discuss how they determined their estimates. Finally, ask a pair of students to measure the actual width, so all students can determine how close their estimates were.

I: Measurement Estimation – Capacity

In grade 3, students developed an understanding of the benchmark for the millilitre. They are now ready to use this benchmark to develop estimation competency.

Benchmarks — Millilitre

Student should gain competency estimating the capacity of various small containers. Student could use a small cube from the base-10 unit blocks as a benchmark for 1 ml, a rod as a benchmark for 10 mL, and a flat as a benchmark for 100 mL. A personal referent for 1 mL could be their small fingers tips that include the fingernails.

Examples of Some Practice Activities

- Present students with a variety of containers and ask them to choose if the capacity of each is more than, less than, or about a specified amount. For example, ask: Is the capacity of this cereal bowl more than, less than, or about 250 mL? Is the capacity of this match box more than, less than, or about 20 mL? After each estimate, discuss the strategies for making decisions.
- Present students with a variety of containers and ask them to choose the estimate of the capacity of each within specified ranges. For example, ask: Will the capacity of this pill container be between 15 and 30 mL or 45 and 60 mL?
- Save a variety of sizes of tin cans, and take off their labels and tops. Line them up so all students can see them. Select one can and ask students to estimate its capacity. Check the estimates by filling the can with water and pouring the water into a measuring cup with mL markings. Now ask students to estimate the capacities of the other tin cans using this known capacity as a benchmark.
- Present students with a large soup container and ask them, working as partners, to estimate its capacity. When all partnerships have estimates, record them on the whiteboard to see the range of estimates. Ask some partners to explain how they arrived at their estimates. After the discussion, invite students to change their estimates if they wish. Did the range of estimates change? Check the actual capacity by measuring the number of mLs of water it contains.
- Estimate how many more mL is in a medium-size mustard jar than in a small yogurt container?

J: Measurement Estimation – Mass

In grade 3, students developed an understanding of the benchmark for one gram. They are now ready to use this benchmark to develop estimation competency.

Mass is one of the more difficult concepts in measurement because it utilizes the sense of touch rather than sight. In fact, students often go astray in making comparisons of mass because they are influenced by the visual appearance of relative sizes. When students are making comparisons of the masses of two object, they should pick them up one-after-the- other using the same hand.

Benchmarks — Gram

Student should gain confidence and accuracy with estimating the mass of very light objects. Student might use the mass of a paper clip as a referent for 1 g. The interlocking Centicubes (not the small cubes in the base-10 Blocks) that come in different colours have been constructed to be 1 gram, so these cubes make good benchmarks for 1 gram, 10 grams, and other small masses. The actual masses of a few objects that are always readily available to the students can also serve as benchmarks. For example, students could find and use the mass of a highlighter, ruler, and marker as benchmarks.

Examples of Some Practice Activities

- Ask the students about the masses of some well-known objects. For example, ask: Will the mass of a nickel be a little more, a little less, or just about 5 g? Will 10, 15, or 20 nickels have a mass of about 75g? Will the mass of a nickel be more than or less than 4 Centicubes? Will a honeybee have a mass of about 1g, 10 g, or 30 g?
- Ask students to write their estimate on their individual whiteboards and show you when they are directed to do so. Lead a discussion of how students decided.
- Present students with a variety of well-known objects and ask them to select an estimate range from a given list of ranges. For example, ask: Will the mass of a banana be about 100 to 150g or 500 to 550g or 700 to 750g?
- Display several objects of varying masses. Have the students, working as partners, lift each object and, using the benchmark of their choice, estimate the mass of each object in grams. When all partnerships have their estimates, select some partnerships to determine the actual masses using the classroom balance. Discuss the students' estimates, which objects were easier to estimate, any surprises they had, and which benchmark was most helpful.
- Place a 100-gram mass on one side of a ban balance. Have a variety of objects that are between 70 g and 130 g available for each group to handle. Focussing on one at a time, ask: Do you think this ___ has a mass greater or less than 100 g? Get each group's estimate before you place the object on the other pan to check.
- Establish the mass of an object, such as a jellybean. Ask: About how many jellybeans would have the same mass as 5 pennies? As a hexagon in the Pattern Blocks?

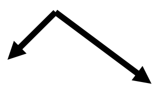
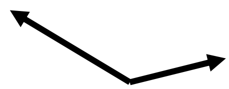
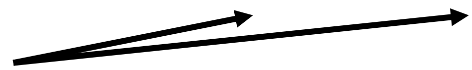
K: Measurement Estimation — Angles

In grade 2, students developed understanding of right angles as "square corners" in shapes. In grade 3, they estimated and described other angles as *less than a right angle* or *more than a right angle*. In grade 4, students should be able to quickly classify angles as right, acute, or obtuse regardless of their positions in space, the size of their arms, or whether they are vertices of polygons or independent figures. They should also associate the angle in a quarter-turn as a right angle.

Benchmarks — Angles

Students can use a corner of a file card or of other objects as a benchmark for a right angle, and compare angles to this benchmark.

Examples

Right	Obtuse	Acute
		

Examples of Some Practice Activities

- Draw a variety of different angles in different positions on an overhead acetate. Display each one for the students and ask them to estimate if angle shown is acute, obtuse, or right. Explain that they should raise their left hands if it is acute, their right hands if it is obtuse and both hands if it is right. Ask them to wait until you invite them to show their hands. Quickly check their responses to determine whether you need to stop for some discussion before you move on to the next angle. (This activity can also be done by drawing and labelling a variety of polygons that contain these types of angles. Ask students to classify each vertex that you point to or name.)
- Provide each student with two Geo Strips and a fastener. Ask them to hold a Geo Strip in each hand so the vertex of the angle in above their hands. Explain that you will describe an angle for them to make. When you invite them, they should lift up their hands to show you their angles. Some examples: Show me an acute angle that is almost a right angle. Show me an acute angle that is about one-half a right angle. Show me a right angle. Show me an obtuse angle that is almost a right. (Change the activity by asking the students to use their left hands to hold one Geo Strip horizontally with the fastener on their left side, and move only the other Geo Strip to make the angles you describe.)
- Show students an analog clock. Explain that you will give them a time and they have to state if the angle formed by the minute and hour hands at that time will be acute, right or obtuse. Ask them to record their decision on their individual whiteboards, and to show you when they are directed to do so.
- Ask students to visualize each capital letter that you will give them and to indicate if there are acute, right, and/or obtuse angle in the letters.

PART 3

Spatial Sense

The Development of Spatial Sense

What is spatial sense?

Spatial sense is an intuition about shapes and their relationships, and an ability to manipulate shapes in one's mind. It includes being comfortable with geometric descriptions of shapes and positions.

There are seven abilities in spatial sense that need to be addressed in the classroom:

- **Eye-motor co-ordination**. This is the ability to co-ordinate vision with body movement. In the mathematics classroom, this involves children learning to write and draw using pencils, crayons, and markers; to cut with scissors; and to handle materials.
- **Visual memory**. This is the ability to recall objects no longer in view.
- **Position-in-space perception**. This is the ability to determine the relationship between one object and another, and an object's relationship to the observer. This includes the development of associated language (over, under, beside, on top of, right, left, etc.) and the transformations (translations, reflections, and rotations) that change an object's position.
- **Visual discrimination**. This is the ability to identify the similarities and differences between, or among, objects.
- **Figure-ground perception.** This is the ability to focus on a specific object within a picture or within a group of objects, while treating the rest of the picture or the objects as background.
- **Perceptual constancy**. This is the ability to recognize a shape when it is seen from a different viewpoint, or from a different distance. This is the perception at play when students recognize similar shapes (enlargements/reductions), and when they perceive as squares and rectangles, the rhombi and parallelograms in isometric drawings.
- **Perception of spatial relationships.** This is the ability to see the relationship between/among two or more objects. This perception is central when students assemble materials to create an object or when they solve puzzles, such as tangram, pattern block, and jigsaw puzzles.

These abilities develop naturally through many experiences in life; therefore, most students come to school with varying degrees of ability in each of these seven aspects. However, students who have been identified as having a learning disability are likely to have deficits in one or more of these abilities. Classroom activities should be planned to allow most students to further develop these perceptions. Most spatial sense activities involve many, if not most, of the spatial abilities, although often one of them is highlighted.

Classroom Context

Spatial abilities are essential to the development of geometric concepts, and the development of geometric concepts provides the opportunity for further development of spatial abilities. This mutually supportive development can be achieved through consistent and ongoing strategic planning of rich experiences with shapes and spatial relationships. Classroom experiences should also include activities specifically designed for the development of spatial abilities. These activities should focus on shapes new to the grade, as well as shapes from previous grades. As the shapes become more complex, students' spatial senses should be further developed through activities aimed at the discovery of relationships between and among properties of these shapes. Ultimately, students should be able to visualize shapes and their various transformations, as well as sub-divisions and composites of these shapes.

Spatial Sense in Mental Math Time

Following the exploration and development of spatial abilities during geometry instruction, mental math time can provide the opportunity to revisit and reinforce spatial abilities periodically throughout the school year.

Assessment

Assessment of spatial sense development should take a variety of forms. The focus in this aspect of mental math is on individual growth and development in spatial sense, rather than on an arbitrary level of competency to be achieved. You should record any observations of growth students make during the reinforcements, as well as noting students' oral and written responses and explanations. For spatial sense, traditional quizzes that involve students recording answers to questions that you give one-at-a-time in a certain time frame should play a very minor role.

L: Spatial Sense in 2-D Geometry

In grade 4, students recognize, name, describe, and construct acute and obtuse angles (in addition to right angles); equilateral, isosceles, and scalene triangles. They sort and describe quadrilaterals according to their side, angle, and reflective symmetry properties. They explore composite figures that can be made from a given set of figures. They predict and confirm the results of various figures under slides, reflections, and quarter/half turns.

Examples of Spatial Sense Activities

- What might I look like? For each of the following example descriptions, ask students to make the shape on their geoboards, or whiteboards. Explain that after a reasonable amount of time you will direct them to show their shapes.
  - I am a quadrilateral with one line of symmetry.
  - I am a quadrilateral with two right angles and only one pair of parallel sides.
  - I am an isosceles triangle with one obtuse angle.
  - I am a concave hexagon with one right angle and two obtuse angles.
- Create additional questions to ask your students, but be sure to draw the shapes yourself in advance as a check.
- Place tangrams (labelled A to G) along the top of the class whiteboard. Create a puzzle by outlining the shape made by two tangrams on a piece of acetate. Show the students your puzzle on an overhead and ask them to identify by letter the shapes that would be used to solve the puzzle, recording their responses on their individual whiteboards. After a reasonable time, have them show you their whiteboards. Repeat this activity by creating other two-piece puzzles. This activity can be extended by creating three-piece puzzles.
- With the overhead shut off, use three, or four, overhead tangrams to create a composite shape. Explain to students that you will turn on the projector for 5 to 10 seconds during which time they should carefully examine your shape because they will be asked to replicate it after the projector is turned off. Ask students to use their tangrams to replicate the shape. After a reasonable time, ask students to compare their shapes with their partner's. Then turn the projector back on and ask them to compare their shapes with the overhead shape. Have students discuss how they remembered the shape. Repeat this activity using other composite shapes.
- On a piece of acetate, draw a scalene triangle, a translation image of this triangle, a reflection image, and a half-turn rotation image. Label the images A, B, and C in any order. Place this acetate on an overhead projector and ask students to identify by letter the triangle that shows a translation (slide) of the scalene triangle, recording their responses on their individual whiteboards. Repeat this activity asking students to identify other transformations and/or by creating additional acetates with different shapes under the three transformation.

M: Spatial Sense in 3-D Geometry

In grade 4, students draw nets for rectangular prisms, square prisms and cubes and explore prepared nets for cylinders and cones. They begin to identify and make generalizations about the number of vertices, edges, and faces of prisms, cylinders, cones, and pyramids.

Examples of Spatial Sense Activities

- On the overhead, show five configurations of six squares, three of which are nets of a cube and two of which are not nets of a cube. These should be labeled A, B, C, D, and E. Ask students to record which of the configurations are nets of a cube. Share and discuss their responses. Repeat this activity for other possible nets of cubes and for rectangular prisms.
- Prepare a variety of clue puzzles. Present the puzzles one-at-a-time, and discuss the solutions. For example, you might ask: What shapes are we? We both have six vertices. We both have some triangular faces.
- Show students a concept circle with four sections. Complete three sections with a picture of the "footprints" that would be made by the faces of three different examples of a particular type of shape. Ask the students to complete the fourth section by drawing the 'footprints" of the faces of another example of the same type of shape, or by drawing the "footprints" of a non-example of the shape. For example, draw a square and four equilateral triangles in one section, an equilateral triangle and three isosceles triangles in a second section, and a hexagon and six isosceles triangles in a third section. The students should realize that your footprints are all example of pyramids, and therefore either draw the footprints of a fourth example of a pyramid or of a shape that is not a pyramid, depending on what you asked them to do.
- Show students three isometric drawings, labelled A, B, and C, of which two are different views of the same structure and one is not. Students identify which two are drawings of the same structure.