Mental Math

In

Mathematics 3

NOVA SCOTIA
Education
English Program Services

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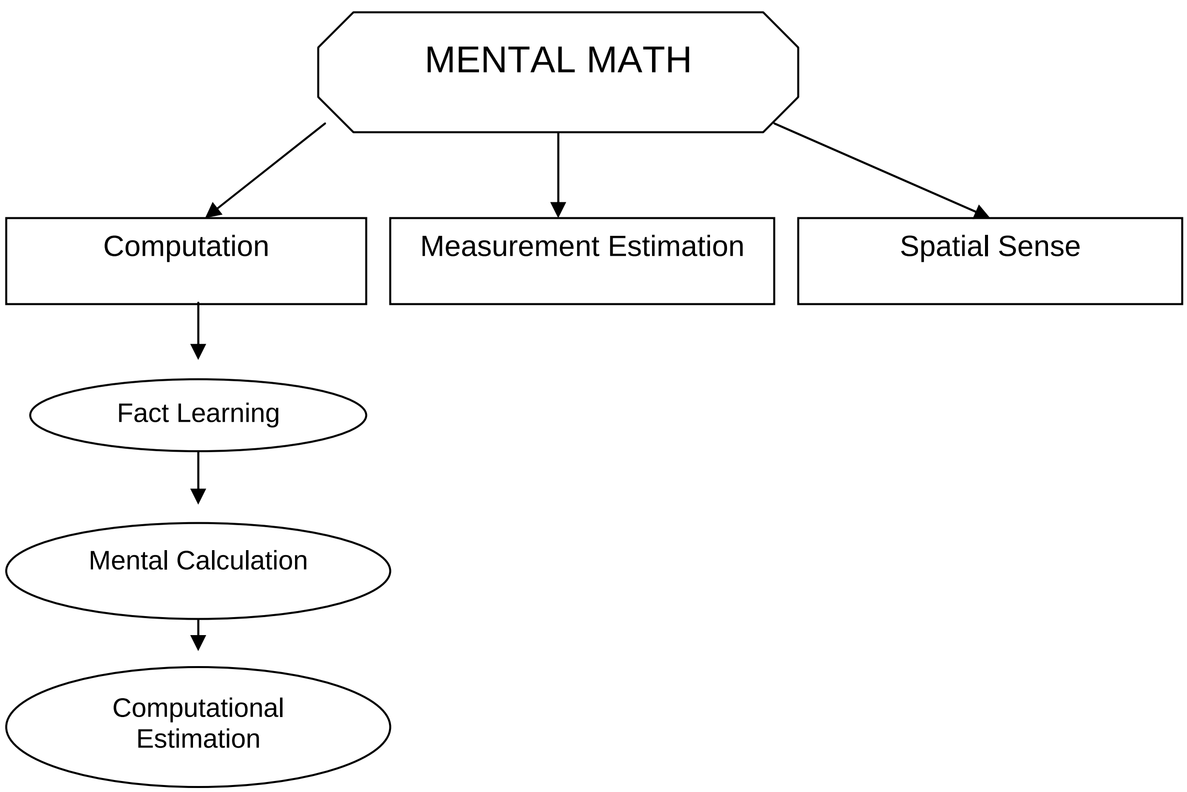
Contents

Introduction

Welcome to your grade-level mental math booklet. After the Department of Education released the *Time to Learn* document in which at least five minutes of mental math was required daily in grades 1 – 9, it was decided to support teachers by clarifying and outlining the specific mental math expectations at each grade. Therefore, grade-level booklets for computational aspects of mental math were prepared and released in draft form in the 2006–2007 school year. Building on these drafts, the current booklets describe the mental math expectations in computation, measurement, and geometry at each grade. These resources are supplements to, not replacements of, the Atlantic Canada mathematics curriculum. You should understand that the expectations for your grade are based on the full implementation of the expectations in the previous grades. Therefore, in the initial years of implementation, you may have to address some strategies from previous grades rather than all of those specified for your grade. It is critical that a school staff meets and plans on an on-going basis to ensure the complete implementation of mental math.

Definitions

In the mathematics education literature, there is not consensus on the usage of some of the words and expressions in mental math. In order to provide uniformity in communication in these booklets, it is important that some of these terms be defined. For example, the Department of Education in Nova Scotia uses the term *mental math* to encompass the whole range of mental processing in all strands of the mathematics curriculum. *Mental math* is broken into three categories in the grade-level booklets: *mental computation*, *measurement estimation*, and *spatial sense*. *Mental computation* is further broken down into *fact learning, mental calculation*, and *computational estimation*.



Fact learning refers to the acquisition of the 100 number facts related to the single digits 0 to 9 for each of the four operations. When students know these facts, they can quickly retrieve them from memory (usually in 3 seconds or less). Ideally, through practice over time, students will achieve automaticity; that is, they will have instant recall without using strategies. *Mental calculation* refers to getting exact answers by using strategies to do the calculations in one's head, while *computational estimation* refers to getting approximate answers by using strategies to do calculations in one's head.

While each category in computations has been defined separately, this does not suggest that the three categories are entirely separable. Initially, students develop and use strategies to get quick recall of the facts. These strategies and the facts themselves are the foundations for the development of other mental calculation strategies. When the facts are automatic, students are no longer employing strategies to retrieve them from memory. In turn, the facts and mental calculation strategies are the foundations for computational estimation strategies. Actually, attempts at computational estimation are often thwarted by the lack of knowledge of the related facts and mental calculation strategies.

Measurement estimation is the process of using internal and external visual (or tactile) information to get approximate measures, or to make comparisons of measures, without the use of measurement instruments.

Spatial sense is an intuition about shapes and their relationships, and an ability to manipulate shapes in one's mind. It includes being comfortable with geometric descriptions of shapes and positions.

Rationale for Mental Math

In modern society, the development of mental skills needs to be a major goal of any mathematics program for two major reasons. First, in their day-to-day activities, most people's computational, measurement, and spatial needs can be met by having well developed mental strategies. Secondly, because technology has replaced paper-and-pencil as the major tool for complex tasks, people need to have well developed mental strategies to be alert to the reasonableness of the results generated by this technology.

PART 1

Mental Computation

The Implementation of Mental Computation

General Approach

In general, a computational strategy should be introduced in isolation from other strategies, a variety of different reinforcement activities should be provided until it is mastered, the strategy should be assessed in a variety of ways, and then it should be combined with other previously learned strategies.

Introducing a Strategy

The approach to highlighting a computational strategy is to give the students an example of a computation for which the strategy would be useful to see if any of the students already can apply the strategy. If so, the student(s) can explain the strategy to the class with your help. If not, you could share the strategy yourself. The explanation of a strategy should include anything that will help students see the pattern and logic of the strategy, be that concrete materials, visuals, and/or contexts. The introduction should also include explicit modeling of the mental processes used to carry out the strategy, and explicit discussion of the situations for which the strategy is most appropriate and efficient. Discussion should also include situation for which the strategy would not be the most appropriate and efficient one. Most important is that the logic of the strategy should be well understood before it is reinforced; otherwise, it's long-term retention will be very limited.

Reinforcement

Each strategy for building mental computational skills should be practised in isolation until students can give correct solutions in a reasonable time frame. Students must understand the logic of the strategy, recognize when it is appropriate, and explain the strategy. The amount of time spent on each strategy should be determined by the students' abilities and previous experiences.

The reinforcement activities for a strategy should be varied in type and should focus as much on the discussion of how students obtained their answers as on the answers themselves. The reinforcement activities should be structured to insure maximum participation. At first, time frames should be generous and then narrowed as students internalize the strategy. Student participation should be monitored and their progress assessed in a variety of ways to help determine how long should be spent on a strategy.

After you are confident that most of the students have internalized the strategy, you need to help them integrate it with other strategies they have developed. You can do this by providing activities that includes a mix of number expressions, for which this strategy and others would apply. You should have the students complete the activities and discuss the strategy/strategies that could be used; or you should have students match the number expressions included in the activity to a list of strategies, and discuss the attributes of the number expressions that prompted them to make the matches.

Language

Students should hear and see you use a variety of language associated with each operation, so they do not develop a single word-operation association. Through rich language usage students are able to quickly determine which operation and strategy they should employ. For example, when a student hears you say, "Six plus five", "Six and five", "The total of six and five", "The sum of six and five", or "Five more than six", they should be able to quickly determine that they must add 6 and 5, and that an appropriate strategy to do this is the *double-plus-one* strategy.

Context

You should present students with a variety of contexts for each operation in some of the reinforcement activities, so they are able to transfer the use of operations and strategies to situations found in their daily lives. By using contexts such as measurement, money, and food, the numbers become more real to the students. Contexts also provide you with opportunities to have students recall and apply other common knowledge that should be well known. For example, when a student hears you say, “How many days in two weeks?” they should be able to recall that there are seven days in a week and that double seven is 14 days.

Number Patterns

You can also use the recognition and extension of number patterns can to reinforce strategy development. For example, when a student is asked to extend the pattern “30, 60, 120, …,”, one possible extension is to double the previous term to get 240, 480, 960. Another possible extension, found by adding multiples of 30, would be 210, 330, 480. Both possibilities require students to mentally calculate numbers using a variety of strategies.

Examples of Reinforcement Activities

Reinforcement activities will be included with each strategy in order to provide you with a variety of examples. These are not intended to be exhaustive; rather, they are meant to clarify how language, contexts, common knowledge, and number patterns can provide novelty and variety as students engage in strategy development.

Assessment

Your assessments of computational strategies should take a variety of forms. In addition to the traditional quizzes that involve students recording answers to questions that you give one-at-a-time in a certain time frame, you should also record any observations you make during the reinforcements, ask the students for oral responses and explanations, and have them explain strategies in writing. Individual interviews can provide you with many insights into a student’s thinking, especially in situations where pencil-and-paper responses are weak.

Assessments, regardless of their form, should shed light on students’ abilities to compute efficiently and accurately, to select appropriate strategies, and to explain their thinking.

Response Time

Response time is an effective way for you to see if students can use the computational strategies efficiently and to determine if students have automaticity of their facts.

For the facts, your goal is to get a response in 3-seconds or less. You would certainly give students more time than this in the initial strategy reinforcement activities, and reduce the time as the students become more proficient applying the strategy until the 3-second goal is reached. In subsequent grades, when the facts are extended to 10s, 100s and 1000s, you should also ultimately expect a 3-second response.

In the early grades, the 3-second response goal is a guideline for you and does not need to be shared with your students if it will cause undue anxiety.

With mental calculation strategies and computational estimation, you should allow 5 to 10 seconds, depending upon the complexity of the mental activity required. Again, in the initial application of these strategies, you would allow as much time as needed to insure success and gradually decrease the wait time until students attain solutions in a reasonable time frame.

Integration of Strategies

After students have achieved competency using one strategy, you should provide opportunities for them to integrate it with other strategies they have learned. The ultimate goal is for students to have a network of mental strategies that they can flexibly and efficiently apply whenever a computational situation arises. This integration can be aided in a variety of ways, some of which are described below.

- You should give them a variety of questions, some of which could be done just as efficiently by two or more different strategies and some of which are most efficiently done by one specific strategy. It is important to have a follow-up discussion of the strategies and the reasons for the selection of specific strategies.
- You should take every opportunity that arises in regular math class time to reinforce the strategies learned in mental math time.
- You should include written questions in regular math time. This could be as journal entry, a quiz/test question, part of a portfolio, or other assessment for which students will get individual feedback. You might ask students to explain how they could mentally compute a given question in one or more ways, to comment on a student response that has an error in thinking, or to generate sample questions that would be efficiently done by a specified strategy.

A. Addition — Fact Learning

In grade 2, students were expected to recall addition facts to 18, with a three-second response. At the beginning of grade 3, it is important that students review these addition facts to 18 and the related fact-learning strategies. While some time may be spent reinforcing clusters of facts by one strategy, this review should concentrate on the reinforcement of all addition facts, with follow-up discussions of answers and strategies. One suggested reinforcement activity would be to post pictures of 10 articles with price tags from $0 to $9, to ask for the total cost of two items that you name, and to repeat by naming other combinations until students have practised a variety of facts. This activity could be used to reinforce particular strategies or all strategies depending upon the articles you name.

The following are the grade 2 fact strategies with examples:

Doubles Facts (Review)

The nine facts: 1 + 1, 2 + 2, 3 + 3 4 + 4, 5 + 5, 6 + 6, 7 + 7, 8 + 8, 9 + 9 These facts were learned in grade 1 through association.

Examples

For 5 + 5, think: The number of fingers on two hands is 10.

For 9 + 9, think: The number of tires on an 18-wheeler

Plus One Facts (Review)

The 16 facts: 2 + 1, 3 +1, 4 + 1, 5 + 1, 6 + 1, 7 + 1, 8 + 1, 9 + 1 and their commutative pairs 1 + 2, 1 + 3, 1 + 4, 1 + 4, 1 + 6, 1 + 7, 1 + 8, 1 + 9 (Note: 1 + 1 was already a double but could also be thought of as next number.) These were learned in grade 1 through association with a call for the next number.

Examples

For 6 + 1, think: The number after 6 is 7.

For 1 + 4, think: The number after 4 is 5.

1-Apart (Near Double) Facts (Review)

The 14 facts: 2 + 3, 3 + 4, 4 + 5, 5 + 6, 6 + 7, 7 +8, 8 + 9 and their commutative pairs 3 + 2, 4 + 3, 5 + 4, 6 + 5, 7 + 6, 8 + 7, 9 + 8. (Note: 1 + 2 and 2 + 1 were already included in the Plus 1 facts but could also be thought of as 1-Aparts.) The strategy is to double the smaller and add one, or to double the larger and subtract 1.

Examples

For 2 + 3, think: double 2 is 4 and next number is 5.

For 8 + 7, think: double 7 is 14 and next number is 15; or think: double 8 is 16 and subtract 1 is 15.

Plus Two Facts (Review)

The 12 facts: 4 + 2, 5 + 2, 6 + 2, 7 + 2, 8 + 2, 9 + 2 and their commutative pairs 2 + 4, 2 + 5, 2 + 6, 2 + 7, 2 + 8, 2 + 9. (Note: 1 + 2, 2 + 2, 3 + 2, 2 + 1, and 2 + 3 have already been included in other strategies but could also be thought of as next even or odd numbers.) The strategy is to associate adding 2 with skip counting by 2 to get the next even or odd number.

Examples

For 5 + 2, think: 5 is an odd number and the next odd number is 7.

For 6 + 2, think: 6 is an even number and the next even number is 8.

Plus Zero Facts (Review)

The 19 facts: 0 + 0. 1 + 0, 2 + 0, 3 + 0, 4 + 0, 5 + 0, 6 + 0, 7 + 0, 8 + 0, 9 + 0 and their commutative pairs 0 + 1, 0 + 2, 0 + 3, 0 + 4, 0 + 5, 0 + 6, 0 + 7, 0 + 8, 0 + 9. The strategy is to associate adding zero with making no change to the other addend.

Examples

For 5 + 0, think: adding zero will make no change to 5, so the answer is 5.

For 0 + 7, think: adding zero will make no change to 7, so the answer is 7.

Make-10 Facts (Review)

The 10 facts involving 9: 9 + 3, 9 + 4, 9 + 5, 9 + 6, 9 + 7 and their commutative pairs 3 + 9, 4 + 9, 5 + 9, 6 + 9, 7 + 9; and the 8 facts involving 8: 8 + 3, 8 + 4, 8 + 5, 8 + 6 and their commutative pairs 3 + 8, 4 + 8, 5 + 8, 6 + 8. The strategy is to take 1 or 2 from one addend to make the 9 or 8 addend a 10, and add this 10 to what was left from the other addend.

Examples

For 9 + 7, think: Take 1 from the 7 to make the 9 a 10, and then add 10 and 6 to get 16.

For 8 + 4, think: Take 2 from the 4 to make the 8 a 10, and then add 10 and 2 to get 12.

The Last 12 Facts (Review)

The facts include: 5 + 3, 6 + 3, 7 + 3 and their commutative pairs 3 + 5, 3 + 6, 3 + 7; 6 + 4, 7 + 4 and their commutative pairs 4 + 6, 4 + 7; 7 + 5 and 5 + 7.

These facts can be done by a number of strategies. Encourage your students to invent ways to do them. Some possible strategies with examples are below, but there are other viable ones:

1. 2-Apart Facts: *3 + 5, 4 + 6, 5 + 7, 5 + 3, 6 + 4, 7 + 5* One strategy is to double the number between the two given numbers; for example, for 3 + 5, think: Taking 1 from the 5 and giving it to the 3 makes 4 + 4, which is 8. Another strategy is to double the smaller and then add 2; for example, for 4 + 6, think: Double 4 is 8 plus 2 is 10.
2. Plus Three Facts: *6 + 3, 7 + 3, 3 + 6, 3 + 7* A strategy is to add 2 and then 1; for example, for 6 + 3, think: 6 + 2 is 8 and then add 1 is 9.
3. Make-10 with a 7: *7 + 3, 7 + 4, 7 + 5, 3 + 7, 4 + 7, 5 + 7* A strategy is to visualize the ten frame for a 7, note it is 3 more to make 10, and add on any leftovers.

Examples of Some Practice Items

- $8 + $7
- 4 km + 7 km
- 3 greater than 8
- 1 more than 7
- 6 increased by 9
- 6 chocolate chip cookies and 2 oatmeal cookies. How many cookies?
- 3 days more than 1 week. How many days?
- How many days in two weeks?

B. Addition — Mental Calculation

Facts Applied to Multiples of 10 and 100 (Extension)

After the basic addition facts and the related strategies are reviewed, or at the same time, these facts should be applied to the related multiples of 10 and 100. For example, 4 + 3 would be related to 40 + 30 and 400 + 300. A simple strategy for these extensions is to combine the single non-zero digits as if they were single-digit addition facts and then attach the appropriate place-value name and symbol to the result. It would be beneficial to connect these sums to the addition of two groups of base-10 blocks. For example, for 5 small cubes and 6 small cubes or 5 rods and 6 rods or 5 flats and 6 flats, the results will all be 11 blocks, be they 11 ones, 11 tens, or 11 hundreds. The sums of 10s are a little more difficult than the sums of 100s because when the answer is more than ten 10s, students have to translate the number. For example, for 70 + 80, 7 tens and 8 tens are 15 tens, or one hundred fifty, while 700 +800 is 7 hundreds and 8 hundreds which is 15 hundreds, or fifteen hundred.

For each cluster of facts (below), there are other suggestions for extending the strategies that were learned for the single digit combinations to multiples of 10 and 100.

a) Double Facts

Examples

If you know that 6 + 6 = 12, then you know that 60 + 60 is *6 tens plus 6 tens* which equals 12 tens, or 120; you also know that 600 + 600 is *6 hundreds plus 6 hundreds* which makes 12 hundreds, or 1200.

If you know that 3 + 3 = 6, then you know 30 + 30 is *3 tens plus 3 tens* which equals 6 tens, or 60; you also know that 300 + 300 is *3 hundreds plus 3 hundreds* which equals 6 hundreds, or 600.

Examples of Some Practice Items

a) Examples of Some Practice Items for Multiples of 10:

- 70 + 70 =
- 30 minutes of Phys Ed on Tuesday and 30 minutes of Phys Ed on Friday. How much Phys Ed time do we get each week/ each cycle?
- Ten plus ten equals

b) Examples of Some Practice Items for Multiples of 100:

- 200 + 200 =
- Two glasses, each with a capacity of 400 mL. How much liquid will I have if I fill both glasses?
- 800 and 800 more =
- Seven hundred more than seven hundred =

b) *Plus One Facts*

When these are applied to multiples of 10 and 100, they are *Plus 10* and *Plus 100*, asking for the next decade (10) and the next century (100).

Examples

If 6 + 1 is a call for the next number 7, then 60 + 10 is a call for the next decade 70, and 600 + 100 is a call for the next century 700.

If 1 + 8 is a call for the next number 9, then 10 + 80 is a call for the next decade 90, and 100 + 800 is a call for the next century 900.

Examples of Some Practice Items

a) Examples of Some Practice Items for Multiples of 10:

- 10 more than 40 =
- 50 boys and 10 girls. How many children?
- What is a decade after 70?
- 60 m + 10 m =
- $10 + $30 + $10 =

b) Examples of Some Practice Items for Multiples of 100:

- $700 increased by $100 =
- The bank balance after a $300 withdrawal is $100. What was the previous balance?
- What is six hundred more than one hundred?

c) *Near-Doubles (1-Aparts) Facts*

When these are applied to multiples of 10 and 100, they become 10-Aparts and 100-Aparts, but can be done by only thinking about the single digits in the highest place and placing the result in the appropriate place value.

Examples

If you know that 6 + 7 = 13 because (6 + 6) + 1 = 13, then you also know that 60 +70 is (60 + 60) + 10 = 130, and that 600 + 700 = (600 + 600) + 100 = 1300.

If you know that 8 + 7 = 15 because (7 + 7) + 1 = 15, then you also know that 80 + 70 is (70 + 70) + 10 = 150, and that 800 + 700 = (700 + 700) + 100 = 1500

Examples of Some Practice Items

a) Examples of Some Practice Items for Multiples of 10:

- 50 more than 60 =
- 30 students in grade four and 20 students in grade three. How many students in the two grades?
- What is twenty greater than thirty?
- 70 cm + 80 cm =
- $90 + $80 =

b) Examples of Some Practice Items for Multiples of 100:

- 400 g + 500 g =
- 400 mL of tomatoes and 300 mL of kidney beans in the chili recipe. How many mL do these two ingredients account for?
- What is six hundred more than five hundred?

d) Plus Two Facts

Just as adding 2 is connected to skip counting and a call for the next even or odd number, adding 20 can be connected to a call for the next even or odd decade, and adding 200 can be connected to a call for the next even or odd century. Some students may see the pattern: The single-digits in the highest place value are added and the appropriate number of zeros attached.

Examples

For 50 + 20, think: 50 is odd and the next odd decade is 70; or think: 5 and 2 is 7 but these are tens, so the answer is 7 tens, or 70.

For 200 + 400, think: 400 is even and the next even century is 600; or think: 2 + 4 is 6 but these are hundreds, so the answer is 6 hundreds, or 600.

Examples of Some Practice Items

a) Examples of Some Practice Items for Multiples of 10:

- 70 + 20 =
- Twenty more than sixty =
- 30 and another 20 =
- $20 + $80 =

b) Examples of Some Practice Items for Multiples of 100:

- 200 more than 300 is
- 700 and 200 =
- Add 200 to 600
- Eight hundred plus swo hundred equals

e) Plus Zero Facts

Facts with zero do not require any strategy but a good understanding of the meaning of zero and addition. It will be obvious to most students in grade 3 that anytime zero is added to another number, there will be no change to that number. Therefore, it is suggested that you place some number phrases that include zeros in reinforcements to keep students alert, but not practice them in isolation.

f) Make 10 Facts

Just as these facts include 8 and 9, their extensions to multiples of 10 and 100 would include 80 and 90 or 800 and 900. The other addends are partitioned and re-distributed to make 100 or to make 1000*,* thus making the addition simpler. Just as the *Make-10* strategy can be extended to facts with 7s, so too can *Make 100* and *Make 1000* be extended to sums involving 70 and 700.

Examples

For 90 + 60, think: If 10 from the 60 is added to the 90, it will become 100 + 50, so the answer is 150. In symbols: 90 + 60 = (90 + 10) + 55 = 100 + 50 = 150

For 500 + 800, think: If 200 from the 500 is added to 800, it will become 1000 + 300, so the answer is 1300. In symbols: 500 + 800 = 300 + (200 + 800) = 300 + 1000 = 1300

For 70 + 40, think: If 30 from the 40 is added to the 70, it will become 100 + 10, so the answer is 110. In symbols: 70 + 40 = (70 + 30) + 10 = 100 + 10 = 110.

Examples of Some Practice Items

a) Examples of Some Practice Items for Multiples of 10:

- 50 + 90 =
- I have 80 cents in my pocket and I find 2 quarters. How much money do I have now?
- $90 + $30 =
- What is 70 greater than 50

b) Examples of Some Practice Items for Multiples of 100:

- 600 + 800 =
- 500 girls and 900 boys. How many children?
- 800 added to 400 =
- What is the sum of nine hundred and three hundred?
- $700 + $400

g) The Last 12 Facts

The variety of strategies for the last 12 facts (that were not included in the strategies already provided) can also be extended to the tens and hundreds.

i) *2-Aparts can be extended to 20-Aparts and 200-Aparts*

This strategy is used when two 2-digt numbers are 20 apart or when two 3-digit numbers are 200 apart. By taking 10 or 100 from the larger number and giving it to the smaller, the question becomes the double of the number between the two given numbers.

Examples

For 50 + 70, think: If 10 is taken from 70 and given to 50, the question becomes 60 + 60, which is 120.

For 500 + 300, think: If 100 is taken from 500 and given to 300, the question becomes 400 + 400, which is 800.

Examples of Some Practice Items

a) Examples of Some Practice Items for Multiples of 10:

- 50 + 30 =
- I have 70 cents in my pocket and I find 2 quarters. How much money do I have now?
- $60 + $40
- What is 30 greater than 50?
- What is 50 more than 70 kg?

b) Examples of Some Practice Items for Multiples of 100:

- 700 + 500
- The sum of $400 and $600
- 300 m more than 0.5 km
- 500 km increased by 700 kg

ii) *Plus 3 Facts can be extends to Plus 30 and Plus 300*

This extended strategy is used when 30 is added to a multiples of 10 or 300 to a multiple of 100. It involves adding 20 and then 10, or adding 200 and then 100.

Examples

For 50 + 30, think: 50 + 20 is 70 and 70 + 10 is 80.
In symbols: 50 + 30 = (50 + 20) + 10 = 70 + 10 = 80

For 300 + 400, think: 200 + 400 is 600 and 600 + 100 is 700.
In symbols: 300 + 400 = (200 + 400) + 100 = 600 + 100 = 700

Examples of Some Practice Items

a) Examples of Some Practice Items for Multiples of 10:

- 60 + 30 =
- I have 30 cents in my pocket and I find 70 cents. How much money do I have now?
- $50 + $30
- What is 30 greater than 60?
- What is 70 more than 30 kg?

b) Examples of Some Practice Items for Multiples of 100:

- 700 + 300
- The sum of $300 and $600
- 300 m more than 0.5 km
- 600 boys. 300 girls. How many children?
- 500 km increased by 300 kg

Front End Addition (New)

This mental strategy is applied to questions that involve two combinations of non-zero digits, one combination of which may require regrouping. The strategy involves first adding the digits in the highest place-value position, then adding the non-zero digits in another place-value position, and doing any needed regrouping. In grade 3, this strategy would be applied to all questions that involve the addition of two 2-digit numbers.

Examples

For 56 + 23, think: 50 plus 20 is 70, and 6 plus 3 is 9, so the answer is 79. In symbols: 56 + 23 = (50 + 20) + (6 + 3) = 70 + 9 = 79

For 45 + 17, think: 40 plus 10 is 50, 5 plus 7 is 12, and 50 plus 12 is 62. In symbols: 45 + 17 = (40 + 10) + (5 + 7) = 50 + 12 = 62

For 44 + 37 + 23, think: 40 plus 30 plus 20 is 90, 4 plus 7 plus 3 is 14, and 90 plus 14 is 104. In symbols: 44 + 37 + 23 = (40 + 30 + 20) + (4 + 7 + 3) = 90 + 14 = 104

Examples of Some Practice Items

- 15 minutes of recess and 46 minutes of language arts. How many minutes have elapsed?
- 74 cm longer than 19 cm
- 38 + 23 + 32 =
- Twenty-five plus eleven plus forty-three
- The sum of $41, $17 and $25

Quick Addition (Extension)

This strategy is actually the *Front-End* strategy applied to questions that involve more than two combinations and with no regrouping needed. As such, this strategy requires students to holistically examine the demands of each question as a first step in choosing a strategy: this habit of thinking needs to pervade all mental math lessons. (One suggestion for an activity during the discussion of this strategy is to present students with a list of 20 questions, some of which do require regrouping, and direct students to apply quick addition to the appropriate questions and leave out the other ones.) The questions are always presented visually, either horizontally or vertically, and students quickly record their answers on paper. While this is a pencil-and-paper strategy, it is included here as a mental math strategy because most students will do all the combinations in their heads starting at the front end.

Most likely, students will add the digits in corresponding place values of the two addends without consciously thinking about the names of the place values. Therefore, in the discussion of the questions, you should encourage students to read the numbers correctly and to use place-value names. This will reinforce place value concepts at the same time as addition.

This strategy would be reinforced in grade 3 with two 3-digit whole numbers. These are the only 3-digit questions that grade 3 students would be expected to get exact answers for in mental math time: all other strategies are applied to 1- and 2-digit numbers in grade 3.

Examples

For 562 + 123, think and record: Starting at the front end, 5 and 1 is 6, 6 and 2 is 8, 2 and 3 is 5, so, the answer is six hundred eighty-five (685).

For 643 + 256, think and record: Starting at the front end, 6 and 2 is 8, 4 and 5 is 9, 3 and 6 is 9, so the answer is eight hundred ninety-nine (899).

Examples of Some Practice Items

Examples of Some Practice Items for Numbers in the 100s:

- Add 231 to 657
- $\begin{array}{r} 673 \\ +\underline{413} \end{array}$ $\quad$ $\begin{array}{r} 245 \\ \underline{+312} \end{array}$
- The sum of 291 and 703
- $144 more than $333
- $532 + $301

Finding Compatibles (Extension)

This strategy for addition is used when there are lists of three, or more, numbers to be added. It involves looking for pairs of numbers that add to 10 or 100 to make the addition easier. Some examples of common compatible numbers are 1 and 9, 40 and 60, 30 and 70, and 75 and 25. (In some professional resources, these compatible numbers are referred to as *friendly* numbers or *nice* numbers.) You should be sure that students are convinced that the numbers in an addition expression can be combined in any order (the associative property of addition).

Examples

For 3 + 8 + 7, think: 3 and 7 is 10, and 10 plus 8 is 18.
In symbols: 3 + 8 + 7 = (3 + 7) + 8 = 18

For 20 + 46 + 80, think: 20 and 80 is 100, and 100 and 46 is 146.
In symbols: 20 + 46 + 80 = (20 + 80) + 46 = 100 + 46 = 146

Examples of Some Practice Items

a) Examples of Some Practice Items for Numbers in the 1s:

- Two more than the sum of eight and six
- $\begin{array}{r} 3 \\ 7 \\ \underline{+4} \end{array}$ $\quad$ $\begin{array}{r} 9 \\ 8 \\ \underline{+1} \end{array}$ $\quad$ $\begin{array}{r} 5 \\ 9 \\ \underline{+5} \end{array}$
- In bowling, I knock down 1 pin, then 9 pins, and then 5 pins. What is my score so far?
- Add six, five and five

b) Examples of Some Practice Items for Numbers in the 10s:

- $\begin{array}{r} 60 \\ 30 \\ \underline{+70} \end{array}$ $\quad$ $\begin{array}{r} 40 \\ 70 \\ \underline{+60} \end{array}$ $\quad$ $\begin{array}{r} 25 \\ 28 \\ \underline{+75} \end{array}$
- $60 + $30 + $40 =
- Start with eighty. Add seventy-nine and twenty to it.

Break Up and Bridge (New)

In grade 3, this strategy is applied to two 2-digit whole numbers. It involves starting with the one addend in its entirety and adding the place values of the second addend, one-at-a-time, starting with the tens.

The Hundreds Chart is an effect tool to teach this strategy because it is a natural way to add on this chart. For example, to add 36 and 27, go to 36 on the chart, go down two spaces vertically from 36 to get to 56, and then count on 7 spaces horizontally to get to 63. Similarly, to model addition on a number line, a natural approach is to start at the first addend, jump the number of tens in the second addend, and then jump the number of ones in the second addend.

Examples

For 45 + 38, think: 45 and 30 (the tens in 38) is 75, and 75 plus 8 (the ones in 38) is 83.
In symbols: 45 + 38 = (45 + 30) + 8 = 75 + 8 = 83

For 56 + 25, think: 56 and 20 (the tens in 25) is 76, and 76 plus 5 (the ones in 25) is 81.
In symbols: 56 + 25 = (56 + 20) + 5 = 76 + 5 = 81

Examples of Some Practice Items

Examples of Some Practice Items for Numbers in the 10s:

- 37 more than 45
- The sum of 72 and 28
- 25 girls and 76 boys. How many children?
- 38 cm longer than 43 cm
- Find the total of 59 + 15
- 29 eggs and a dozen more. How many eggs?

Compensation (New)

This strategy involves changing one addend in an addition question to a nearby multiple of ten, carrying out the addition using that multiple of ten, and adjusting the answer to compensate for the original change. Students should understand that the number is changed to make it more compatible, and that they have to hold in their memories the amount of the change. In the last step, it is helpful if they remind themselves that they added too much so they will have to take away that amount. Some students may have used this strategy when learning their facts involving 9s in grade 2; for example, for 9 + 7, they may have found 10 + 7 and then subtracted 1. This strategy is most often used when there is an 8 or 9 in the ones place of one addend, although some students are also comfortable using it when there is a 7 in the ones place.

Examples

For 52 + 9, think: The sum of 52 and 10 is 62, but 1 too many was added, so to compensate, subtract 1 from 62 to get 61. In symbols: 52 + 9 = (52 + 10) – 1 = 61.

For 65 + 28, think: The sum of 65 and 30 is 95, but 2 too many was added, so subtract 2 to get 93. In symbols: 65 + 28 = (65 + 30) – 2 = 93.

Examples of Some Practice Items

- 9 greater than 43
- 56 minutes and 8 more minutes
- 79 more than 8
- 5 kg added to 48 kg
- Sue had 33 holidays in 2007 and 38 holidays in 2008. How many holidays altogether?
- $49 and $27

Make Multiples of 10 (Extension)

As an extension of the *Make-10* strategy in fact learning, this strategy is similar to the *Compensation* strategy in that it makes use of the compatibility of multiples of 10 in addition and is most useful when the ones digit of one addend is an 8 or 9. This *Make-Multiples-of-10* strategy, however, involves changing both addends by distributing part of one addend to the other addend in order to make that addend a multiple of 10. Students should understand that this strategy centers on getting a more compatible addend, the multiple of 10.

A common error occurs when students forget that both addends have changed. This strategy needs to be compared to the compensation strategy to see how it is alike and how it is different. As well, the *Make-Multiples-of-10* strategy can be extended to addends with 7 in the ones place.

For sums involving the addition of a single-digit number to a 2-digit number, there is an interesting pattern that students should notice when using this strategy: the digit in the ones place of the answer is the part of the 8 or 9 that was left after its other part was distributed to make the multiple of 10. (See Examples below.)

Examples

For 52 + 9, think: If 1 is taken from 52 and given to 9, the question becomes 51 + 10, which is 61. In symbols: $52 + 9 = (52 – 1) + (9 + 1) = 51 + 10 = 61$.

For 48 + 6, think: If 2 is taken from the 6 and given to 48, the question becomes 50 + 4, which is 54. In symbols: $48 + 6 = (48 + 2) + (6 – 2) = 50 + 4 = 54$.

For 65 + 28, think: If 2 is taken from 65 and given to 28, the question becomes 63 + 30, which is 93. In symbols: $65 + 28 = (65 – 2) + (28 + 2) = 63 + 30 = 93$.

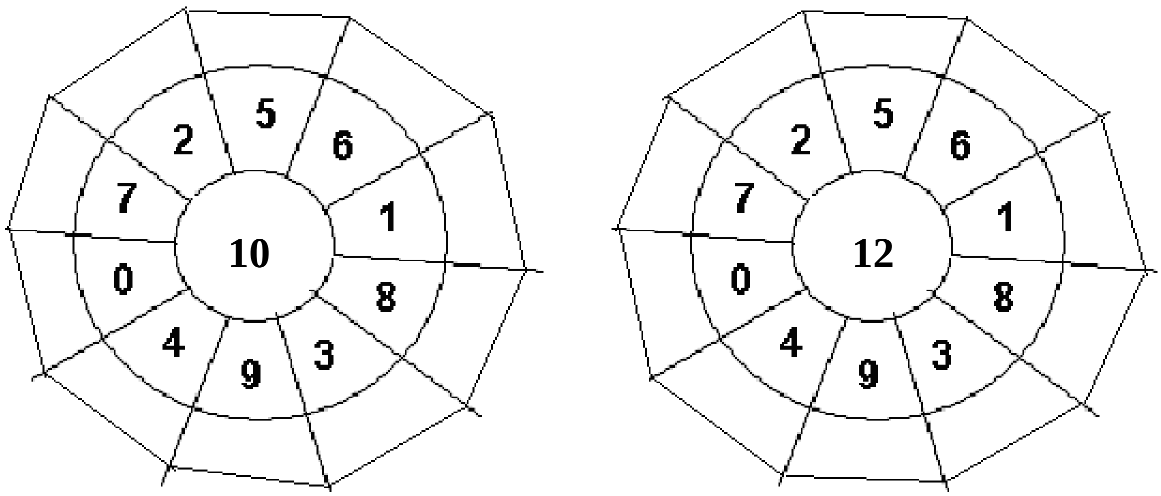
Examples of Some Practice Items:

- 8 greater than 34
- 68 kg increased by 7 kg
- $57 + $38
- 19 more than 45
- 69 added to 15
- The sum of 32 and 68

C. Subtraction — Fact Learning

As a first step in abandoning counting back on their fingers or by using materials, students in grades 1 and 2 used visualization of ten frames to help them get answers to subtraction facts with minuends of 10 or less. The principal mental strategy for the subtraction facts, however, should be *Think Addition*. This strategy should have been introduced in the development of subtraction in regular math class, but with no expectation of quick responses. In mental math time, however, it is important that students have achieved a reasonable response time for the addition facts before trying to use *Think Addition* to get a quick response time for the subtraction facts.

As a get-ready activity, select a sum and ask students a series of questions related to that sum. For example, select 12 as a sum and ask students: 6 and what make 12? 9 and what make 12? 7 and what make 12? 4 and what make 12? You could also ask questions such as: If 7 and 5 make 12, what two subtraction facts do you know? If 9 and 3 make 12, what two subtraction facts do you know? You could also prepare webs by placing different sums in the middle and getting students to complete them; for example, if you place 10 and 12 in the middle of webs (see below), ask students to complete the webs by thinking of the other numbers that would add to the given numbers to get the sum in the middle.



Think Addition (New)

This strategy involves students using their knowledge of addition facts to find the answers to subtraction facts since addition and subtraction are inverse operations. Students would have experience with fact families to draw upon. For example, they would know that 4 + 1 = 5, 1 + 4 = 5, 5 – 4 = 1, and 5 – 1 = 4 are a family of facts. If students are given one of these number sentences, they should readily state the other three related number sentences.

The strategy involves asking oneself what number would be added to the subtrahend to get the minuend.

Examples

For 9 – 4, think: What number would be added to 4 to get 9? Recall that 4 + 5 = 9, so 9 – 4 = 5

For 14 – 6, think: What number would be added to 6 to get 14? Recall that 6 + 8 = 14, so 14 – 6 = 8.

Examples of Some Practice Items

- 10 – 4 =

- It is 4 o'clock. How many more hours until 12 o'clock?
- 8 less 6 =
- 12 minus 5
- 9-L container with 3 L missing. What's left?
- 6 pencils. 4 of them are dull. How many are sharp?
- What is the difference between $14 and $9?

If your students are very competent with their addition facts, you could start applying the *Think-Addition* strategy to all the subtraction facts. However, you may want to apply this strategy to clusters of subtraction facts that are related to the addition fact clusters. Two examples of such clusters are provided; however, you can refer to the addition facts and create the clusters of corresponding subtraction facts.

Examples of Clusters of Subtraction Facts

a) Subtraction Facts Related to Double Facts in Addition

This strategy uses the addition double facts to help find the answers to related subtraction facts.

Examples

For 12 – 6, think: 6 plus what number makes 12? 6 and 6 are 12, so 12 – 6 = 6.

For 18 – 9, think: 9 plus what number makes 18? 9 and 9 are 18, so 18 – 9 = 9.

Examples of Some Practice Items

- 8 – 4
- $6 – $3 =
- 14 days less a week of days
- The difference between a score of 16 and 8
- 10 points minus 5 points

b) Subtraction Facts Related to 1-Apart (Near Doubles) Facts in Addition

This strategy also uses the addition double facts and near-double facts to help find the answers to related subtraction combinations. When the part being subtracted is close to half of the total, we can think of an addition double fact and make the necessary adjustments.

Example

For 9 – 4, think: 4 plus what number makes 9? Since 4 and 4 is 8, 4 and 5 must be 9, so 9 – 4 = 5.

For 13 – 7, think: 7 plus what number makes 13? Since 7 and 7 is 14, 7 and 6 must be 13, so 13 – 7 = 6

Examples of Some Practice Items

- 9 kg of potatoes and 4 kg of them eaten. How many kg of potatoes left now?
- 13 – 6

- Five less than nine
- $15 – ? = $7
- 17 children. 8 boys. How many girls?

Quick Counting Back (New)

While the *Counting-Back* strategy, as a direct modelling of a "take away" meaning for subtraction, was a prevalent strategy used throughout the development of the concept of subtraction, it is very inefficient and cumbersome, especially if more than 1, 2, or 3 have to be subtracted. Therefore, it should not be encouraged as a mental strategy except, perhaps, for the subtraction of 1, 2, or 3. Even then, the counting back needs to be done quickly and in one's head, without any reliance on fingers or other external props. Students may also associate "subtract 1" with a call for the number before, and "subtract 2" with a call for the even/odd number before: if so, they abandon any direct counting back.

A very common error that students make when they count back is to start counting from the given number rather than the number before it. For example, if they are to subtract 2 from 6, they say, 6, 5, and give the answer as 5. Modelling on the number line can help students understand the logic of starting to count from the number before the one given.

Examples

For 6 – 2, think: 5, 4, so the answer is 4.

For 7 – 1, think: 6, so the answer is 6.

For 8 – 3, think: 7, 6, 5, so the answer is 5.

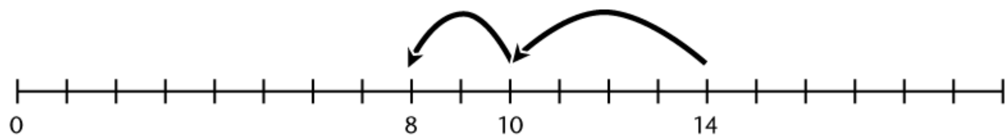
Examples of Some Practice Items

- 9 – 1
- $10 – $2
- One less than 6
- The difference between 3 km and 8 km
- 7 kg reduced by 1 kg

Back Through 10 (New)

This strategy is a good one for students to have in their repertoire as an alternative to *Think Addition* when the minuend is greater than 10. It involves subtracting in two steps: one part of the subtrahend is subtracted to get to ten and the other part of the subtrahend is then subtracted from 10. This strategy is most effective when only 1 or 2 has to be subtracted from 10 in the second step.

Modelling this strategy on a number line as a "take-away" would help students visualize the two steps: Mark the minuend, show the leap from the minuend to 10, and then the final leap to the answer. (14 – 6 is shown on the number line below.)



Examples

For 15 – 7, think: To take 7 from 15, first take 5 from 15 to get to 10, and then take 2 (the rest of the 7) from 10 to get 8.
In symbols: 15 – 7 = (15 – 5) – 2 = 10 – 2 = 8

For 14 – 5, think: To take 5 from 14, first take 4 from 14 to get to 10, and then take 1 (the rest of 5) from 10 to get 9.
In symbols: 14 – 5 = (14 – 4) – 1 = 10 – 1 = 9

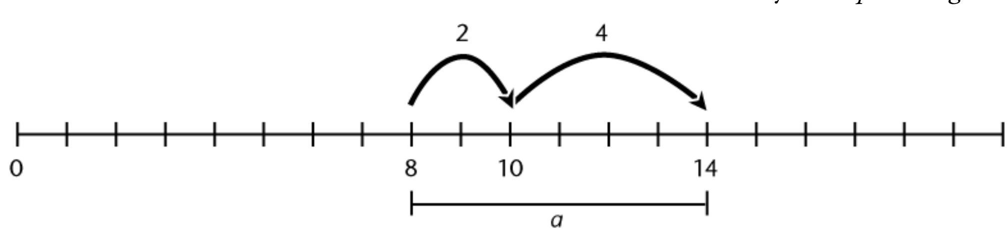
Examples of Some Practice Items

- 15 – 6
- 4 less than 12
- The difference between $5 and $13
- 15 questions with 7 correct. How many incorrect?
- 16 minus 8
- Subtract 3 from 11

Up Through 10 (New)

This strategy, like the *Back-Though-10* strategy, is another alternative to *Think Addition* when the minuend is greater than 10. This strategy, however, involves finding the difference between the two numbers in two steps: first, the difference between the subtrahend and 10 is found, then the difference between 10 and the minuend is found, and finally these two differences are added to give the total difference. This strategy is most effective when the subtrahend is 8 or 9.

Modelling this strategy on a number line will help students visualize the two steps: Mark both the minuend and the subtrahend on the number line, show the leap from the subtrahend to 10, show the leap from 10 to the minuend, and see that the two leaps together represent the total difference between the two numbers. The number line below models 14 – 8 by the *Up-Through-10* strategy.



Examples

For 12 – 9, think: From 9 to 10 is 1, and from 10 to 12 is 2, so the total difference is 1 plus 2, or 3. In symbols: 12 – 9 = (10 – 9) + (12 – 10) = 1 + 2 = 3

For 14 – 8, think: From 8 to 10 is 2, and from 10 to 14 is 4, so the total difference is 2 plus 4, or 6. In symbols: 14 – 8 = (10 – 8) + (14 – 10) = 2 + 4 = 6

Examples of Some Practice Items

- 14 – 9
- $11 minus $8
- The difference between 9 and 15
- How much less is 8 km than 12 km?
- 8 subtracted from 13
- 16 children, 9 of them boys. How many girls?

D. Subtraction — Mental Calculation

In contrast to the expectation for addition of two 2-digit numbers, it is not expected that students in grade 3 would mentally find every possible difference of two 2-digit numbers. Students should be able to subtract any two 2-digit numbers that would not require regrouping, a 2-digit and 1-digit number that would require regrouping, and some two 2-digit numbers that are in consecutive decades. The full scope of two 2-digit subtraction questions will be a mental math expectation in grade 4.

Facts Applied to Multiples of 10 and 100 (New)

In grade 3, some basic subtraction facts (with minuends of 10, or less) should be applied to the related multiples of 10 and 100 for quick response times. For example, 9 – 6 would be applied to 90 – 60 and 900 – 600. A simple strategy for these related questions is to combine the front-end digits as if they were subtraction facts and then attach the appropriate place-value name to the results. These applications should be connected to the subtraction of two groups of base-10 blocks. For example, if 3 small cubes are taken away from 7 small cubes, or 3 rods are taken away from 7 rods, or 3 flats are taken away from 7 flats, the results will all be 4 blocks, be they 4 ones, 4 tens, or 4 hundreds.

Examples

For 90 – 60, think: 9 tens subtract 6 tens is 3 tens, or 30; or think: 9 subtract 6 is 3 but these are tens, so the answer is 30.

For 800 – 500, think: 8 hundreds subtract 5 hundreds is 3 hundreds, or 300; or think: 8 subtract 5 is 3 but these are hundreds, so the answer is 300.

For 1000 – 200, think: 10 hundreds subtract 2 hundreds is 8 hundreds, or 800; or think: 10 subtract 2 is 8 but these are hundreds, so the answer is 800.

Examples of Some Practice Items

a) Examples of Some Practice Items for Multiples of 10:

- Of 100 pencils, 70 are sharpened. How many are not sharpened?
- $60 – $30 =
- 50 minus 40
- The difference between 90 km and 60 km
- 20 less than 70
- 40 subtract 10
- $20 subtracted from $50
- Have 10 bottles but need 60. How many more do I need to get?

b) Examples of Some Practice Items for Multiples of 100:

- 800 – 200
- The difference between 500 cm and 900 cm
- 700 taken from 1000
- 200 km less than 400 km
- $400 – $100
- 300 subtracted from 600
- 800 attended on Saturday and 500 on Sunday. How many more attended on Saturday?
- 500 mL of milk in a carton. How much is left after I drink 200 mL?

Quick Subtraction (Extension)

This strategy is used when two 2- or 3-digit numbers are to be subtracted and there is no regrouping needed. Starting at the highest place value, simply subtract and record each place value's digits. Because this strategy only applies to questions with no regrouping, students must holistically examine the demands of each question as a first step: this habit of thinking needs to pervade all mental math lessons.

Students should be able to apply this strategy mentally to the differences between two 2-digit numbers and state the answer; however, most students will likely need to record each place-value difference for two 3-digit numbers and read their answers.

One suggestion for an activity during the discussion of this strategy is to present students with a list of 20 questions, some of which require regrouping, and direct students to apply quick subtraction to the appropriate questions, leaving out the ones for which this strategy cannot be used.

Examples

For 56 – 12, think about each place-value difference: Starting at the front end, 5 minus 1 is 4, 6 minus 2 is 4, so, the answer is 44 (forty-four).

For 657 – 241, think and record each place-value difference: Starting at the front end, 6 minus 2 is 4, 5 minus 4 is 1, 7 minus 1 is 6, so the answer is 416 (four hundred sixteen).

Examples of Some Practice Items

Examples of Some Practice Items for Numbers in the 10s:

- 76 – 25
- The difference between 18 and 39
- $97 – $22
- A mass of 46 kg reduced by 15 kg
- 35 children. 14 girls. How many boys?
- 69 eggs. 1 dozen removed. How many eggs left?

Examples of Some Practice Items for Numbers in the 100s:

- 357 – 123
- 899 – 666
- $123 less than $487
- 312 m decreased by 211 m
- 578 subtract 125
- The difference between 566 kg and 879 kg

Back-Through Multiples of 10 (New)

This strategy involves subtracting a part of the subtrahend to get to the nearest multiple of 10 and then subtracting the rest of the subtrahend. In grade 3, this is an effective strategy to subtract a single-digit number from a 2-digit number when there would be regrouping.

Examples

For 74 – 6, think: To take 6 from 74, first take 4 from 74 to get to 70, and then take 2 (the rest of the 6) from 70 to get 68.
In symbols: 74 – 6 = (74 – 4) – 2 = 70 – 2 = 68

For 43 – 8, think: To take 8 from 43, first take 3 from 43 to get to 40, and then take 5 (the rest of the 8) from 40 to get 35.
In symbols: 43 – 8 = (43 – 3) – 5 = 40 – 5 = 35

Examples of Some Practice Items

- 35 – 6
- The difference between $8 and $56
- There are 42 students in the gym. 7 of them leave when the bell rings. How many students remain?
- 4 less than 73
- 85 minus 7
- 92 kg reduced by 5 kg

Up-Through Multiples of 10 (New)

This strategy involves finding the difference between two numbers in two steps: first, the difference between the subtrahend and the next multiple of 10 is found, then the difference from this multiple of 10 and the minuend is found, and finally the two differences are added to give the total difference. This strategy is most effective when the two numbers are in consecutive decades and when the subtrahend has an 8 or 9 in the ones place.

Examples

For 82 – 79, think: From 79 to 80 is 1, and from 80 to 82 is 2, so the total difference is 1 plus 2, or 3. In symbols: 82 – 79 = (80 – 79) + (82 – 80) = 1 + 2 = 3

For 54 – 48, think: From 48 to 50 is 2, and from 50 to 54 is 4, so the total difference is 2 plus 4, or 6. In symbols: 54 – 48 = (50 – 48) + (54 – 50) = 2 + 4 = 6

Examples of Some Practice Items

- 34 – 29
- The difference between 38 m and 42 m
- $75 – $68
- 45 children. 39 girls. How many boys?
- 48 yesterday but 54 today. What is the increase?
- How much more is 93 than 88?

E. Addition and Subtraction — Computational Estimation

The ability to estimate computations is a major goal of any modern computational program. For most people in their daily lives, an estimate is all that is needed to make decisions and to be alert to the reasonableness of numerical claims and to answers generated by others and by technology.

Before attempting pencil-and-paper or calculator computations, students must find "ball park" estimates, so they are alert to the reasonableness of those pencil-and-paper or calculator answers. You should always model this estimating before you personally do any calculations in front of the class, and you should constantly remind your students to estimate before calculating.

While teaching estimation strategies, it is important to use the language of estimation. Some of the common words and phrases are: *about, just about, between, a little more than, a little less than, close, close to,* and *near.*

Note: For the most part, addition estimation questions in grade 3 mental math time will involve more than two 2–digit numbers and 3-digit numbers that would require regrouping and therefore cannot be done by *Quick Addition*. After all, grade 3 students are expected to exact answers mentally for the sums of two 2-digit numbers and have been introduced to a variety of strategies to achieve this expectation. The same level of competency in subtraction is not expected; therefore, subtraction estimation questions in grade 3 mental math time will include both 2- and 3-digit numbers. However, before they learn any strategies to mentally find the sum of two 2-digit numbers, students should also estimate these numbers prior to pencil-and-paper or calculator calculations. At all times, however, if three, or more, 2-digit numbers are to be added, estimation should be done before other forms of calculating.

Front-End Estimation (Extension)

This strategy involves adding or subtracting the values in the highest place-value position to get a "ball-park" figure. These "ball-park" estimates are usually good enough to be alert to the reasonableness of pencil-and-paper and technology-generated answers.

Because the other place values are not considered, the front-end estimates for addition questions will always be less than the actual answers. Therefore, you can always use the phrase *more than* in describing your estimate. For subtraction questions, however, without considering the other place values, you can only use the word *about* in describing your estimate.

Examples

To estimate 213 + 397, think: 200 + 300 = 500, so the estimate is more than 5 hundred.

To estimate 392 – 153, think: 300 – 100 = 200, so the estimate is about 2 hundred.

To estimate 437 + 548, think: 400 + 500 = 900, so the estimate is more than 9 hundred

To estimate 534 – 254, think: 500 – 200 = 300, so the estimate is about 3 hundred.

Examples of Some Practice Items

a) Examples of Some Practice Items for Estimating Differences of Numbers in the 10s:

- Estimate the difference between $92 and $68.
- Approximately how much is 81 kg more than 42 kg?
- 64 birds in the flock. 27 fly away. About how many are left?
- 43 boxes of tissues in the class in September. 16 boxes left now. Approximately how many did we use?
- 81 girls and 56 boys attended the dance. About how many more girls attended than boys?

b) Examples of Some Practice Items for Estimating Sums of Numbers in the 100s:

- Estimate the sum of 234 ml and 439 ml
- Approximately what is 746 added to 178?
- Estimate: 348 + 609
- It is 697 km to Ottawa from home and 315 km farther to Toronto. About how far is it from home to Toronto?
- Approximately how many days is 238 more than the days in a year?

c) Examples of Some Practice Items for Estimating Differences of Numbers in the 100s:

- Estimate: 327 – 142
- About what is the difference between 928 and 741?
- $804 saved. $537 left. Approximately how much was spent?
- 516 birds to start, 234 fly away. About how many are left?
- Estimate 456 fewer than 639

Adjusted Front-End Estimation (New)

This strategy for addition estimation begins by getting a Front-End estimate and then adjusting that estimate to get a better, or closer, estimate. Two possible adjustment strategies are: (a) considering the second highest place values or (b) by clustering all the values in the other place values to "eyeball" whether there would be enough together to account for an adjustment. These two adjustment strategies will not always result in the same final estimates.

Examples

To estimate 437 + 545 by (a), think: 400 plus 500 is 900, but this can be adjusted by thinking 30 and 40 is 70; so, the adjusted estimate would be about 970.

OR

To estimate 437 + 545 by (b), think: 400 plus 500 is 900, but this can be adjusted by "eyeballing" that 37 and 45 would be close to another 100; so, the adjusted estimate would be about 1000.

To estimate 324 + 228 by (a), think: 300 + 200 is 500, and 20 + 20 is 40; so, the adjusted estimate is about 540.

OR

To estimate 324 + 238 by (b), think: 300 + 200 = 500, and "eyeballing" 24 + 38 suggests about another about another 50 would be added to give an estimate of about 550.

Examples of Some Practice Items

Examples of Some Practice Items for Sums of Numbers:

- What is the approximate cost of two books that are $123 and $389?
- Estimate the total distance if you travel 243 km one day and a 108 km the next day.
- If 435 people came to Tuesday night's performance and 385 people came to Wednesday night's performance, about how many people attended the two performances?
- If you deposit $429 to your bank account that has a balance of $822, approximately how much do you then have in your account?
- If the television you want costs $699 and the VCR costs $109, about how much would it cost to buy both?

Rounding (Extension)

In grade 3, this strategy involves rounding each number to the highest place value and adding the rounded numbers and would be mostly applied to sums and differences of two 3-digit numbers, and to sums of three, or more, 2-digit numbers.

When the digit 5 is involved in the rounding procedure for numbers in the 10s and 100s, the number can be rounded up or down. However, the decision to round up or to round down should be based upon the effect the rounding will have in the overall result. For example, if both numbers to be added have a 5 or a 50, rounding one number *up* and one number *down* will minimize the effect the rounding will have on the estimation. Also, if both numbers are close to 5 or 50, it will be better to round one up and one down. For subtraction estimation, however, if both numbers have a 5 or 50, or if both numbers are close to 5 or 50, both numbers should be rounded up, or down, because you are finding the difference between the numbers and that difference will be greater if one is rounded up and the other rounded down1

Examples

To estimate 85 + 65 + 98, think: Round 98 to 100. Since 85 and 65 both have 5s, it would be best to round one up and one down to get 80 and 70, and add to get 150 and then add the 100 to get 250.

To estimate 378 + 230, think: 378 rounds to 400 and 230 rounds to 200; so, the estimate is 400 plus 200 or 600.

To estimate 348 + 249, think: Both these numbers are close to 50s, so round 348 to 400 and 249 to 200, so 400 plus 200 is 600.

To estimate 594 – 203, think: 594 rounds to 600 and 203 rounds to 200, so the estimate is 600 subtract 200 or 400.

To estimate 854 – 448, think: Both these numbers are close to 50s, so round both up to get 900 minus 500, or 400, as the estimate.

Examples of Some Practice Items

a) Examples of Some Practice Items for Estimating Sums of Numbers:

- Estimate: $326 + $593
- About how much more than 392 bottles do you have if you gather an additional 219?
- If $525 is increased by $608, approximately how much is the total?
- Estimate how much ribbon you have if the three pieces are 55 cm, 78 cm, and 43 cm.
- Estimate the sum of 188 kg and 579 kg

b) Examples of Some Practice Items for Estimating Difference of Numbers in the 10s:

- If there are 87 students in Grade 3 and 58 students in Grade 4, approximately how many fewer students are in Grade 4?
- Estimate how much more 43 is than18.
- You went out with $65 to buy gifts for your family. About how much do you have left if you spent $28 for a gift for your mother?
- If 39 cm of ribbon is cut off a 76-cm length of ribbon, about how much ribbon is left?
- Estimate the difference between 86 kg and 28 kg.

c) Examples of Some Practice Items for Estimating Differences of Numbers in the 100s:

- Grade 3 collected 583 bread tags while Grade 4 only collected 119. About how many more bread tags did Grade 3 collect than Grade 4?
- What is the approximate difference between a TV that costs $843 and one that costs $715?
- If 830 people attended the game on Friday and only 580 on Saturday, approximately how many fewer attended on Saturday?
- If you had $945 before you spent $648, estimate how much you have left.
- At the start of the trip, your odometer reading was 285 km, and at the end it was 776 km. About how far did you travel?

PART 2

Measurement Estimation

The Implementation of Measurement Estimation

General Approach

For the most part, a measurement estimation strategy would be reinforced and assessed during mental math time in the grades following its initial introduction. The goal in mental math is to increase a student's *competency* with the strategy. It is expected that measurement estimation strategies would be introduced as part of the general development of measurement concepts at the appropriate grade levels. Each strategy would be explored, modeled, and reinforced through a variety of class activities that promote understanding, the use of estimation language, and the refinement of the strategy. As well, the assessment of the strategy during the instructional and practice stages would be ongoing and varied.

Competency means that a student reaches a reasonable estimate using an appropriate strategy in a reasonable time frame. While what is considered a reasonable estimate will vary with the size of the unit and the size of the object, a general rule would be to aim for an estimate that is within 10% of the actual measure. A reasonable time frame will vary with the strategy being used; for example, using the *benchmark strategy* to get an estimate in metres might take 5 to 10 seconds, while using the *chunking strategy* might take 10 to 30 seconds, depending upon the complexity of the task.

Introducing a Strategy in Regular Classroom Time

Measurement estimation strategies should be introduced when appropriate as part of a rich and carefully sequenced development of measurement concepts. For example, in grade 3, the distance from the floor to most door handles is employed as a *benchmark* for a metre so students can use a *benchmark strategy* to estimate lengths of objects in metres. This has followed many other experiences with linear measurement in earlier grades: in grade primary, students compared and ordered lengths of objects concretely and visually; in grade 1, students estimated lengths of objects using non-standard units such as paper clips; in grade 2, students developed the standard unit of a metre and were merely introduced to the idea of a *benchmark* for a metre.

The introduction of a measurement estimation strategy should include a variety of activities that use concrete materials, visuals, and meaningful contexts, in order to have students thoroughly understand the strategy. For example, a meaningful context for the *chunking strategy* might be to estimate the area available for bookshelves in the classroom. Explicit modeling of the mental processes used to carry out a strategy should also be part of this introduction. For example, when demonstrating the *subdivision strategy* to get an estimate for the area of a wall, you would orally describe the process of dividing the wall into fractional parts, the process of determining the area of one part in square metres, and the multiplication process to get the estimate of the area of the entire wall in square metres. During this introductory phase, you should also lead discussions of situations for which a particular strategy is most appropriate and efficient, and of situations for which it would not be appropriate, or efficient.

Reinforcement in Mental Math Time

Each strategy for building measurement estimation skills should be practised in isolation until students can give reasonable estimates in reasonable time frames. Students must understand the logic of the strategy, recognize when it is appropriate, and be able to explain the strategy. The amount of time spent on each strategy should be determined by the students' abilities, progress, and previous experiences.

The reinforcement activities for a strategy should be varied in type and should focus as much on the discussion of how students obtained their answers, as on the answers themselves. The reinforcement activities should be structured to ensure maximum participation. At first, time frames should be generous and then narrowed as students internalize the strategy and become more efficient. Student participation should be monitored and their progress assessed in a variety of ways. This will help determine the length of time that should be spent on a strategy.

During the reinforcement activities, the actual measures should not be determined every time an estimate is made. You do not want your students to think that an estimate is always followed by measurement with an instrument: there are many instances where an estimate is all that is required. When students are first introduced to an activity, it is helpful to follow their first few estimates with a determination of the actual measurement in order to help them refine their estimation abilities. Afterwards, however, you should just confirm the reasonable estimates, having determined them in advance

Most of the reinforcement activities in measurement will require the availability of many objects and materials because students will be using some objects and materials as benchmarks and will be estimating the attributes of others. To do this, they must see and/or touch those objects and materials.

After you are confident that most students have achieved a reasonable competency with the strategy, you need to help them integrate it with other strategies that have been developed. You should do this by providing activities that include a mix of attributes to be estimated in different units using a variety of contexts. You should have the students complete the activities and discuss the strategy/strategies that were, or could have been, used. You should also have students match measurement estimation tasks to a list of strategies, and have them discuss the reasoning for their matches.

Assessment

Your assessments of measurement estimation strategies should take a variety of forms. Assessment opportunities include making and noting observations during the reinforcements, as well as students' oral and written responses and explanations. Individual interviews can provide you with many insights into a student's thinking about measurement tasks. As well, traditional quizzes that involve students recording answers to estimation questions that you give one-at-a-time in a certain time frame can be used.

Assessments, regardless of their form, should shed light on students' abilities to estimate measurements efficiently and accurately, to select appropriate strategies, and to explain their thinking.

Measurement Estimation Strategies

The Benchmark Strategy

This strategy involves mentally matching the attribute of the object being estimated to a known measurement. For example, to estimate the width of a large white board, students might mentally compare its width to the distance from the doorknob to the floor. This distance that is known to be about 1 metre is a *benchmark*. When students mentally match the width of the white board to this benchmark, they may estimate that the width would be about two of these benchmarks; therefore, their estimate would be 2 metres. In mathematics education literature you will often see reference made to *personal referents.* These are benchmarks that individuals establish using their own bodies; for example, the width of a little finger might be a personal referent for 1 cm, a hand span a referent for 20 cm, and a hand width a referent for 1 dm. These benchmarks have the advantage of being portable and always present whenever and wherever an estimate is needed.

The Chunking Strategy (Starting in Grade 5)

This strategy involves mentally dividing the attribute of an object into a number of chunks, estimating each of these chunks, and adding the estimates of all the chunks to get the total estimate. For example, to estimate the length of a wall, a student might estimate the distance from one corner to the door, estimate the width of the door, estimate the distance from the door to the bookcase, and estimate the length of the bookcase which extends to the other corner. All these individual estimates would be added to get the estimate of the length of the wall.

The Unitizing Strategy (Starting in Grade 5)

This strategy involves establishing an object (that is smaller than the object being estimated) as a unit, estimating the size of the attribute of this unit, and multiplying by the number of these units it would take to make the object being measured. For example, students might get an estimate for the length of a wall by noticing it is about five bulletin boards long. The bulletin board becomes the unit. Students estimate the length of the bulletin board and multiply this estimate by five to get an estimate of the length of the wall.

The Subdivision Strategy (Starting in Grade 6)

This strategy involves mentally dividing an object repeatedly in half until a more manageable part of the object can be estimated and then this estimate is multiplied by the appropriate factor to get an estimate for the whole object. For example, to estimate the area of a table top, students might mentally divide the table top into fourths, estimate the area of that one-fourth in square decimeters, and then multiply that estimate by four to get the estimate of the area of the entire table top.

F. Measurement Estimation — Length

In grade 2, students developed understanding of the benchmarks for centimetre and metre. They are now ready to use these benchmarks to develop estimation competency.

If students have established benchmarks against which they are visually comparing other lengths, they are less likely to give 'wild guesses' or to 'pull numbers out of the air'. During estimation activities, you should use a variety of answer formats to help students overcome any hesitation they might have in estimating and to help them refine their estimates. For example, questions might ask students to select a range for the estimate from a given set of ranges; to decide if the estimate is *more than, less than, or about* a given measure; or to select the estimate from a list of given possibilities. Eventually, through experiences, they should be able to determine a reasonable single-number estimate on their own. In the initial reinforcements, after students arrive at their estimates, you should have them get the actual measure using a ruler or metre stick, and discuss the strategies used to get the estimates and the reasonableness of their estimates. This will help them refine their estimating abilities. However, with continued practice, they should not find the actual measure. You want students to understand that they will often estimate in real life when an estimate is all that is needed. At other times, they will estimate to be alert to the reasonableness of measures they find or are given.

Benchmark – Centimetre

Students should use personal referents, such as the width of their little fingers for a 1-centimetre benchmark and the width of their hands for a 10-centimetre benchmark. They could also measure their individual hand spans to establish another personal referent for centimetres. Since a ruler is a very common tool in the classroom, it can serve as a benchmark for 30 centimetres. Using these benchmarks, students should estimate lengths of objects from 1 cm to 50 cm.

Examples of Some Practice Items

- Assemble a number of objects and prepare a list of ranges for estimates of the lengths you will ask. (8 – 10 questions at a time would be effective.) For example, as you display a wastebasket, ask: Will the height of a wastebasket be about 35 – 45 cm or 20 – 30 cm? Will the length of a piece of lined paper be about 10 – 20 cm or 20 – 30 cm? After each question, get students to share how they decided the most reasonable range.
- Assemble a number of objects that will all have lengths within a reasonable range of values, so you can ask students if the lengths are *less than, more than, or about* a given measure. For example, ask students to decide if each of the following is *less than, more than, or about* 10 cm as you display each one: The length of this calculator; the width of this box; the distance from my nose to my chin; the width of my foot; the length of this marker; the length of my ear; the distance between the Kleenex box and the pencil sharpener on my desk.
- Select 8 to 10 objects in the room. For each object, prepare three possible estimates from which you will ask students to select the one they think is the most reasonable. For example, ask: Which is the most reasonable estimate for this eraser: 5 cm, 7 cm, or 9 cm? Which is the most reasonable estimate for the length of this textbook: 20 cm, 30 cm, or 40 cm? After each question, have students discuss how they made their decisions.
- Select 4 or 5 objects that you will display one-at-a-time as you ask students to estimate some length. For example, as you display a board eraser, ask: What is the approximate length of this eraser? What is its approximate width? As you display a piece of ribbon, ask: What is a reasonable estimate for the length of this ribbon? The width of the ribbon?

Benchmark – Metre

Since most door handles are about 1 metre above the floor, students could use this distance as a benchmark for 1 metre. Measure the length of the classroom in metres and use it as a benchmark to estimate larger lengths. You could have students stretch out their arms sideways and measure with a metre stick to establish a 1-metre length as a personal referent. (For many people, the distance from their necks to the end of their out-stretched fingertips is about 1 m.) They should estimate lengths that are from 1 – 20 m.

Examples of Some Practice Items

- On the whiteboard, print three headings: more, less, and about the same. Explain to students that you will give them a length of an object to estimate. They are to decide if they think the length is more, less, or about the measure you describe, and to print the word on the post-it you will provide. When directed they will come to the whiteboard and post their answer under the appropriate heading. For example, ask: Will the width of the window be *more, less, or about* 2m? Will the length of the classroom whiteboard be *more than, less than, or about* 3m? After the answers are displayed, lead a discussion of strategies used to decide, and perhaps measure the whiteboard if there were too many incorrect estimates. Repeat by asking about the lengths of other objects.
- Prepare a number of questions that will ask students to select an estimate from a list of possibilities, such as: Will the width of the classroom be about 5m, 7m or 10m? Will the height of the door be 1 m, 2m, or 3 m? Students could write their choice on individual whiteboards to display when directed. After each question, lead a discussion of strategies used to decide.
- Present students with estimation questions involving lengths or distances in the classroom. Ask them to select the interval in which they think the estimate would most likely lie: 1 – 2 m, 3 – 4 m, 5 – 6 m.
- Prepare a list of estimation questions involving lengths/distances from 1 to 6 metres. You measure the lengths/distances ahead of time. Explain to students that their estimate should be a single number between 1 and 6 m. When directed they should display the number of fingers that corresponds to their choice for the estimate. After each question, indicate which is the best estimate and discuss how their decisions were made.
- In the school corridor, establish how far 20 m is from your classroom. Then get students to estimate the distances to other sites that you know will be between 10 and 20 metres. Decide how you want the students to respond and discuss how they made their decisions.

G. Measurement Estimation — Capacity

In grade 2, students developed an understanding of the benchmark for the litre. In grade 3, they should use this benchmark to develop estimation competency in capacity (in litres).

Benchmark for Litre

Students could use the capacity of a large cube in the Base-10 Blocks, or that of a 1-L milk carton, as a benchmark for 1 L. They should estimate capacities of objects that are between 1 – 10 L.

Examples of Some Practice items

- Present students with a variety of containers and ask them to choose if the capacity of each is more than, less than, or about a specified amount. For example, ask: Is the capacity of this glass jar more than, less than, or about 1 L? Is the capacity of this garbage can more than, less than, or about 10 L? After each estimate, discuss the strategies for making decisions.
- Present students with a variety of containers and ask them to choose the estimate of the capacity of each within specified ranges. For example, ask: Will the capacity of this cardboard box be between 5 L and 10 L or 10 L and 29 L?
- Save a variety of sizes of containers, such as boxes and plastic jugs/tubs. Line them up so all students can see them. Select the smallest container and ask students to estimate its capacity. Check their estimates by filling the container with a liquid or popcorn, and measuring the amount used with a known 1 L measure. Now ask students to estimate the capacities of the other containers using this found capacity as a benchmark.
- Present students with a large container, such as a large cooking pot, and ask them, working as partners, to estimate its capacity. When all partnerships have estimates, record them on the whiteboard to see the range of estimates. Ask some partners to explain how they arrived at their estimates. After the discussion, invite students to change their estimates if they wish. Did the range of estimates change? Check the actual capacity by measuring the number of litres of water or popcorn it holds.
- Ask students to estimate how many more litres one container holds than another. For example, ask: About how many more litres does this garbage hold than this ice cream container? How many ice cream containers full of water would you need to fill this electric kettle?

H. Measurement Estimation — Mass

In grade 2, students developed understanding of the benchmark for the kilogram. They are now ready to use this benchmark to develop estimation competency in mass (in kilograms).

Mass is one of the more difficult concepts in measurement because it utilizes the sense of touch rather than sight. In fact, students often go astray in making comparisons of mass because they are influenced by the visual appearance of objects. When students are making comparisons of the masses of two object, they should pick them up one-after-the- other using the same hand.

Benchmark for Kilogram

Students could use the mass of a 1 kg bag of sugar or another personally developed referent as a benchmark for 1 kg. Select two objects in the classroom that could be explored to develop a sense of touch for 5 kg and for 10 kg. These objects can then serve as further benchmarks to help students refine their estimates of the mass of objects from 1 – 10 kg.

Examples of Some Practice Items

- Ask the students about the masses of some well-known objects. For example, ask: Will the mass of two math texts be closer to 2 or 3 kg? Will the mass of 1 package of copier paper be more than 8 kg, less than 8 kg, or about 8 kg?
- Will the mass of a nickel be a little more, a little less, or just about 5 g? Will 10, 15, or 20 nickels have a mass of about 75g? Will the mass of a nickel be more than or less than 4 Centicubes? Will a honeybee have a mass of about 1g, 10 g, or 30 g?
- Ask students to write their estimate on their individual whiteboards and show you when they are directed to do so. Lead a discussion of how students decided.
- Present students with a variety of well-known objects and ask them to select an estimate range from a given list of ranges. For example, ask: Will the mass of a banana be about
100 - 150g or 500 - 550g or 700 – 750g?
- Display several objects of varying masses. Have the students, working as partners, lift each object and, using the benchmark of their choice, estimate the mass of each object in grams. When all partnerships have their estimates, select some partnerships to determine the actual masses using the classroom balance. Discuss the students' estimates, which objects were easier to estimate, any surprises they had, and which benchmark was most helpful.
- Place a 100-gram mass on one side of a ban balance. Have a variety of objects that are between 70 g and 130 g available for each group to handle. Focussing on one at a time, ask: Do you think this ___ has a mass greater or less than 100 g? Get each group's estimate before you place the object on the other pan to check.
- Establish the mass of an object, such as a jellybean. Ask: About how many jellybeans would have the same mass as 5 pennies? As a hexagon in the Pattern Blocks?

PART 3

Spatial Sense

The Development of Spatial Sense

What is Spatial Sense?

Spatial sense is an intuition about shapes and their relationships, and an ability to manipulate shapes in one's mind. It includes being comfortable with geometric descriptions of shapes and positions.

There are seven abilities in spatial sense that need to be addressed in the classroom:

- **Eye-motor co-ordination**. This is the ability to co-ordinate vision with body movement. In the mathematics classroom, this involves children learning to write and draw using pencils, crayons, and markers; to cut with scissors; and to handle materials.
- **Visual memory**. This is the ability to recall objects no longer in view.
- **Position-in-space perception**. This is the ability to determine the relationship between one object and another, and an object's relationship to the observer. This includes the development of associated language (over, under, beside, on top of, right, left, etc.) and the transformations (translations, reflections, and rotations) that change an object's position.
- **Visual discrimination**. This is the ability to identify the similarities and differences between, or among, objects.
- **Figure-ground perception.** This is the ability to focus on a specific object within a picture or within a group of objects, while treating the rest of the picture or the objects as background.
- **Perceptual constancy**. This is the ability to recognize a shape when it is seen from a different viewpoint, or from a different distance. This is the perception at play when students recognize similar shapes (enlargements/reductions), and when they perceive as squares and rectangles, the rhombi and parallelograms in isometric drawings.
- **Perception of spatial relationships.** This is the ability to see the relationship between/among two or more objects. This perception is central when students assemble materials to create an object or when they solve puzzles, such as tangram, pattern block, and jigsaw puzzles.

These abilities develop naturally through many experiences in life; therefore, most students come to school with varying degrees of ability in each of these seven aspects. However, students who have been identified as having a learning disability are likely to have deficits in one or more of these abilities. Classroom activities should be planned to allow most students to further develop these perceptions. Most spatial sense activities involve many, if not most, of the spatial abilities, although often one of them is highlighted.

Classroom Context

Spatial abilities are essential to the development of geometric concepts, and the development of geometric concepts provides the opportunity for further development of spatial abilities. This mutually supportive development can be achieved through consistent and ongoing strategic planning of rich experiences with shapes and spatial relationships. Classroom experiences should also include activities specifically designed for the development of spatial abilities. These activities should focus on shapes new to the grade, as well as shapes from previous grades. As the shapes become more complex, students' spatial senses should be further developed through activities aimed at the discovery of relationships between and among properties of these shapes. Ultimately, students should be able to visualize shapes and their various transformations, as well as sub-divisions and composites of these shapes.

Spatial Sense in Mental Math Time

Following the exploration and development of spatial abilities during geometry instruction, mental math time can provide the opportunity to revisit and reinforce spatial abilities periodically throughout the school year.

Assessment

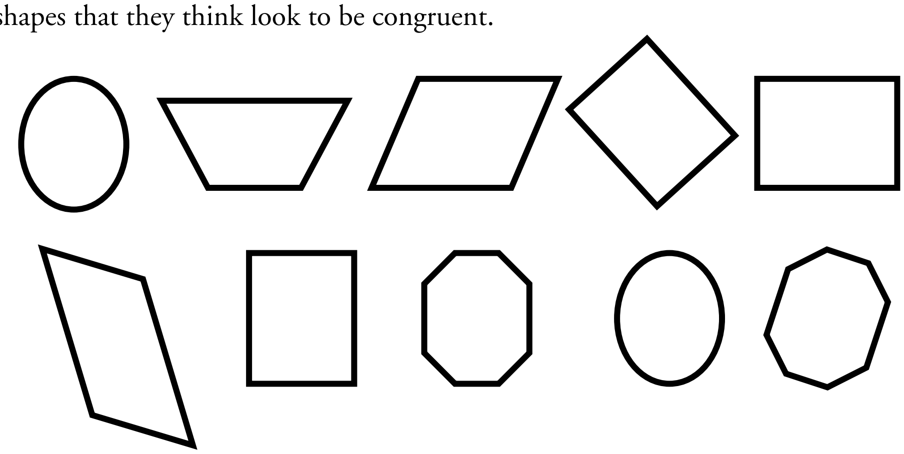
Assessment of spatial sense development should take a variety of forms. The focus in this aspect of mental math is on individual growth and development in spatial sense, rather than on an arbitrary level of competency to be achieved. You should record any observations of growth students make during the reinforcements, as well as noting students' oral and written responses and explanations. For spatial sense, traditional quizzes that involve students recording answers to questions that you give one-at-a-time in a certain time frame should play a very minor role.

I. Spatial Sense in 2-D Geometry

In grade 3, students recognize, name, describe and represent the kite. The kite is the last of the quadrilateral family of shapes that the students have learned and it should be integrated with the other family members from previous grades: squares, rectangles, trapezoids, rhombi, and parallelograms. They are introduced to the concept and meaning of the terms regular, concave and convex in relation to polygons. They recognize and represent angles that are less than/more than right angles and develop understanding of congruent angles and congruent polygons. They continue to find the lines of reflective symmetry in polygons and can recognize, name, describe, and represent, half and quarter turns of 2-D figures.

Examples of Spatial Sense Activities

- What might I be?
  Display pictures of top views of familiar object that students would not normally see from that point of view. Explain to students that these are top views of objects. Ask what objects they might be.
  (Blackline master needed of the top view of objects. See guide page 3-61 for examples.)
- Show students a picture on the overhead of two adjoining rectangles with the outline of a triangle attached to the side of one rectangle. (BLM needed. The base of the triangle should not be drawn.) Ask students how many of each kind of angle-right angle, greater than right angle, less than right angle- they can find in the picture. Have them record their responses under the appropriate headings on the whiteboard. Discuss and check responses, referring to the picture on the overhead.
- Show students pictures of ten shapes, labeled A to J, three pairs of which are congruent figures. (BLM needed.) Ask students to find the pairs of congruent figures and to list them on paper. Share and discuss answers.
- Show students a folded piece of paper with a “half design” of a rectangle drawn on it (Graphic needed). Ask students to predict by showing on their individual geoboards what the shape will look like when you cut it on the drawn lines and unfold the cutout. Discuss how the original shape is like cut-and-unfolded shape and how the two shapes differ. Repeat this activity with various shapes and simple designs. [Navigating through Geometry in Pre-kindergarten-Grade 2, NCTM 2001 page 74.
- Display on an overhead an acetate picture of 8 – 10 shapes (see example), some pairs of which are congruent. (You could also have three that are congruent.) Scatter the pictures of the shapes on the page, and be sure to place the congruent ones in different positions as rotated or reflected images. Number the shapes. Ask students to decide if there look to be congruent shapes on display. Ask them to record the numbers of the shapes that they think look to be congruent.



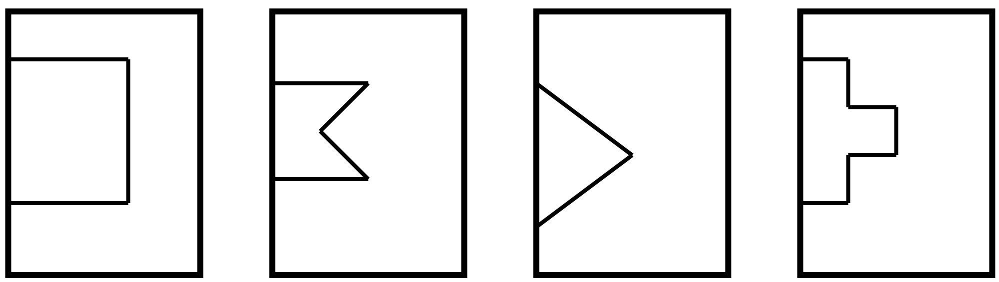
- Give each student a sheet of folded paper with a design drawn against the folded edge. (See below.) Have students draw pictures of what they think the shapes will look like when they are cut out and unfolded. Ask questions:

  How many sides will your cut-out have? How do you know?

  Which sides of your unfolded cut-out will be the same length as a side of your folded cut-out? Which will be longer? Will any be shorter?

  How can you predict the number of angles your cut-out will have just by looking at the drawing of the folded shape?

  Pass out scissors and have students cut out their shapes to test their predictions.



J. Spatial Sense in 3-D Geometry

In grade 3, students recognize, name, describe and represent trapezoidal, pentagonal, hexagonal and octagonal prisms and pyramids and continue to use rectangular, triangular and square pyramids and rectangular, triangular and square prisms. They explore net patterns for, and build skeletons of, pentagonal and hexagonal prisms and pyramids. They describe these shapes by their faces, edges and vertices.

Examples of Spatial Sense Activities

- Select a rectangular prism, a triangular prism and a cube. On an overhead acetate, display nets of each of these solids and one for a solid that is not on display. These should be labelled A, B, C, and D. Show one of the solids and ask students to select the net for this solid. Have students record their responses (the letter that matches the net they choose) on their individual whiteboards before sharing. Repeat using the other solids.
- Explain to students that you will give them a name of a 3-D shape. They are to visualize that shape. You will then ask them a question about the number of vertices, edges, or faces that it has. You will wait 5 or 6 seconds. When you direct them, they are to display the number of fingers that correspond to their answer. Example 1: Visualize a hexagonal prism. (Wait). How many faces does this prism have? (Wait 5 or 6 seconds.) Show me the number with your fingers. Discuss. Example 2: Visualize a square pyramid. (Wait.) How many vertices does this pyramid have? (Wait 5 or 6 seconds.) Show me the number with your fingers. Discuss. Example 3: Visualize a triangular prism. (Wait.) How many edges does this prism have? (Wait 5 or 6 seconds.) Show me. Discuss.
- Explain to students that in the past they have built 3-D shapes using toothpicks and marshmallows. You are going to tell them how many toothpicks and/or marshmallows they have. They have to decide which 3-D shape they can make to use all the materials. They are to write the name of the 3-D shape on their individual whiteboards. You will wait and ask them to show their answers. Example 1: Which 3-D shape can you make if you have a quantity of marshmallows but only 6 toothpicks? Example 2: Which 3-D shape can you make with 6 marshmallows and 9 toothpicks? Example 3: Which 3-D shape can you make with 12 toothpicks and 7 marshmallows?