Mental Math

In

Mathematics 1

NOVA SCOTIA
Education
English Program Services

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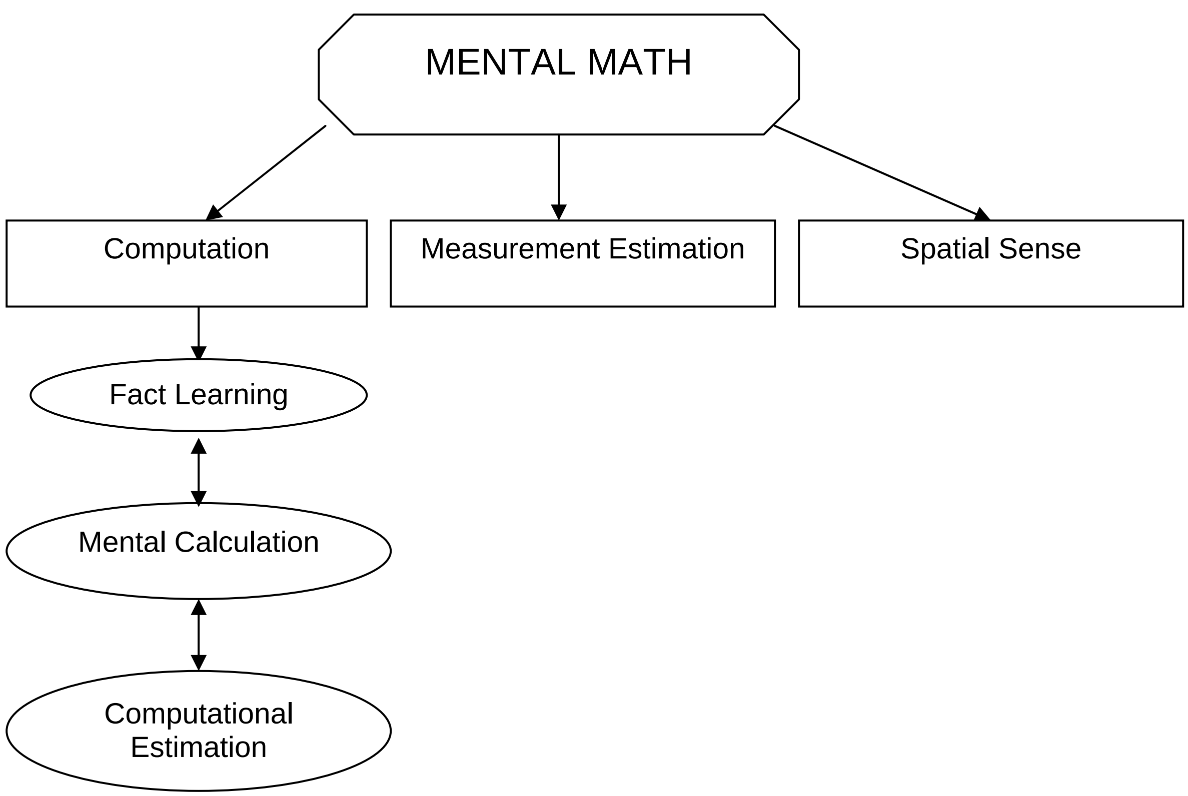
Contents

Introduction

Welcome to your grade-level mental math booklet. After the Department of Education released the *Time to Learn* document in which at least five minutes of mental math was required daily in grades 1 – 9, it was decided to support teachers by clarifying and outlining the specific mental math expectations at each grade. Therefore, grade-level booklets for computational aspects of mental math were prepared and released in draft form in the 2006–2007 school year. Building on these drafts, the current booklets describe the mental math expectations in computation, measurement, and geometry at each grade. These resources are supplements to, not replacements of, the Atlantic Canada mathematics curriculum. You should understand that the expectations for your grade are based on the full implementation of the expectations in the previous grades. Therefore, in the initial years of implementation, you may have to address some strategies from previous grades rather than all of those specified for your grade. It is critical that a school staff meets and plans on an on-going basis to ensure the complete implementation of mental math.

Definitions

In the mathematics education literature, there is not consensus on the usage of some of the words and expressions in mental math. In order to provide uniformity in communication in these booklets, it is important that some of these terms be defined. For example, the Department of Education in Nova Scotia uses the term *mental math* to encompass the whole range of mental processing in all strands of the mathematics curriculum. *Mental math* is broken into three categories in the grade-level booklets: *mental computation*, *measurement estimation*, and *spatial sense*. *Mental computation* is further broken down into *fact learning, mental calculation*, and *computational estimation*.



Fact learning refers to the acquisition of the 100 number facts related to the single digits 0 to 9 for each of the four operations. When students know these facts, they can quickly retrieve them from memory (usually in 3 seconds or less). Ideally, through practice over time, students will achieve automaticity; that is, they will have instant recall without using strategies. *Mental calculation* refers to getting exact answers by using strategies to do the calculations in one's head, while *computational estimation* refers to getting approximate answers by using strategies to do calculations in one's head.

While each category in computations has been defined separately, this does not suggest that the three categories are entirely separable. Initially, students develop and use strategies to get quick recall of the facts. These strategies and the facts themselves are the foundations for the development of other mental calculation strategies. When the facts are automatic, students are no longer employing strategies to retrieve them from memory. In turn, the facts and mental calculation strategies are the foundations for computational estimation strategies. Actually, attempts at computational estimation are often thwarted by the lack of knowledge of the related facts and mental calculation strategies.

Measurement estimation is the process of using internal and external visual (or tactile) information to get approximate measures, or to make comparisons of measures, without the use of measurement instruments.

Spatial sense is an intuition about shapes and their relationships, and an ability to manipulate shapes in one's mind. It includes being comfortable with geometric descriptions of shapes and positions.

Rationale for Mental Math

In modern society, the development of mental skills needs to be a major goal of any mathematics program for two major reasons. First, in their day-to-day activities, most people's computational, measurement, and spatial needs can be met by having well developed mental strategies. Secondly, because technology has replaced paper-and-pencil as the major tool for complex tasks, people need to have well developed mental strategies to be alert to the reasonableness of the results generated by this technology.

PART 1

Computation

The Implementation of Mental Computation

General Approach

In general, a computational strategy should be introduced in isolation from other strategies, a variety of different reinforcement activities should be provided until it is mastered, the strategy should be assessed in a variety of ways, and then it should be combined with other previously learned strategies.

A. Introducing a Strategy

The approach to highlighting a computational strategy is to give the students an example of a computation for which the strategy would be useful to see if any of the students already can apply the strategy. If so, the student(s) can explain the strategy to the class with your help. If not, you could share the strategy yourself. The explanation of a strategy should include anything that will help students see the pattern and logic of the strategy, be that concrete materials, visuals, and/or contexts. The introduction should also include explicit modeling of the mental processes used to carry out the strategy, and explicit discussion of the situations for which the strategy is most appropriate and efficient. Discussion should also include situation for which the strategy would not be the most appropriate and efficient one. Most important is that the logic of the strategy should be well understood before it is reinforced; otherwise, it's long-term retention will be very limited.

B. Reinforcement

Each strategy for building mental computational skills should be practised in isolation until students can give correct solutions in a reasonable time frame. Students must understand the logic of the strategy, recognize when it is appropriate, and explain the strategy. The amount of time spent on each strategy should be determined by the students' abilities and previous experiences.

The reinforcement activities for a strategy should be varied in type and should focus as much on the discussion of how students obtained their answers as on the answers themselves. The reinforcement activities should be structured to insure maximum participation. At first, time frames should be generous and then narrowed as students internalize the strategy. Student participation should be monitored and their progress assessed in a variety of ways to help determine how long should be spent on a strategy.

After you are confident that most of the students have internalized the strategy, you need to help them integrate it with other strategies they have developed. You can do this by providing activities that includes a mix of number expressions, for which this strategy and others would apply. You should have the students complete the activities and discuss the strategy/strategies that could be used; or you should have students match the number expressions included in the activity to a list of strategies, and discuss the attributes of the number expressions that prompted them to make the matches.

Language

Students should hear and see you use a variety of language associated with each operation, so they do not develop a single word-operation association. Through rich language usage students are able to quickly determine which operation and strategy they should employ. For example, when a student hears you say, "Six plus five", "Six and five", "The total of six and five", "The sum of six and five", or "Five more than six", they should be able to quickly determine that they must add 6 and 5, and that an appropriate strategy to do this is the *double-plus-one* strategy.

Context

You should present students with a variety of contexts for each operation in some of the reinforcement activities, so they can apply operations and strategies to situations found in their daily lives. By using contexts such as measurement, money, and food, the numbers become more real to the students. Contexts also provide you with opportunities to have students recall and apply other common knowledge that should be well known. For example, when a student hears you say, "How many days in two weeks?" they should be able to recall that there are seven days in a week and that double seven is 14 days.

Number Patterns

You can also use the recognition and extension of number patterns can to reinforce strategy development. For example, when a student is asked to extend the pattern “30, 60, 120, …,”, one possible extension is to double the previous term to get 240, 480, 960. Another possible extension, found by adding multiples of 30, would be 210, 330, 480. Both possibilities require students to mentally calculate numbers using a variety of strategies.

Examples of Reinforcement Activities

Reinforcement activities will be included with each strategy in order to provide you with a variety of examples. These are not intended to be exhaustive; rather, they are meant to clarify how language, contexts, common knowledge, and number patterns can provide novelty and variety as students engage in strategy development.

C. Assessment

Your assessments of computational strategies should take a variety of forms. In addition to the traditional quizzes that involve students recording answers to questions that you give one-at-a-time in a certain time frame, you should also record any observations you make during the reinforcements, ask the students for oral responses and explanations, and have them explain strategies in writing. Individual interviews can provide you with many insights into a student’s thinking, especially in situations where pencil-and-paper responses are weak.

Assessments, regardless of their form, should shed light on students’ abilities to compute efficiently and accurately, to select appropriate strategies, and to explain their thinking.

Response Time

Response time is an effective way for you to see if students can use the computational strategies efficiently and to determine if students have automaticity of their facts.

For the facts, your goal is to get a response in 3-seconds or less. You would certainly give students more time than this in the initial strategy reinforcement activities, and reduce the time as the students become more proficient applying the strategy until the 3-second goal is reached. In subsequent grades, when the facts are extended to 10s, 100s and 1000s, you should also ultimately expect a 3-second response.

In the early grades, the 3-second response goal is a guideline for you and does not need to be shared with your students if it will cause undue anxiety.

With mental calculation strategies and computational estimation, you should allow 5 to 10 seconds, depending upon the complexity of the mental activity required. Again, in the initial application of these strategies, you would allow as much time as needed to insure success, and gradually decrease the wait time until students attain solutions in a reasonable time frame.

Integration of Strategies

After students have achieved competency using one strategy, you should provide opportunities for them to integrate it with other strategies they have learned. The ultimate goal is for students to have a network of mental strategies that they can flexibly and efficiently apply whenever a computational situation arises. This integration can be aided in a variety of ways, some of which are described below.

You should give them a variety of questions, some of which could be done just as efficiently by two or more different strategies and some of which are most efficiently done by one specific strategy. It is important to have a follow-up discussion of the strategies and the reasons for the selection of specific strategies.

You should take every opportunity that arises in regular math class time to reinforce the strategies learned in mental math time.

You should include written questions in regular math time. This could be as journal entry, a quiz/test question, part of a portfolio, or other assessment for which students will get individual feedback. You might ask students to explain how they could mentally compute a given question in one or more ways, to comment on a student response that has an error in thinking, or to generate sample questions that would be efficiently done by a specified strategy.

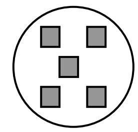
A. Pre-Operation Number Strategies

Set Recognition

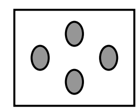
In grade 1, students should be able to recognize common configurations of sets of numbers, such as the dots on a standard die and dominoes. Most students should be able to instantly recognize up to sets of 6, and through experience they can extend this to sets of 10. Set recognition should be reinforced through flash math activities where students are presented with a number configuration for a few seconds, and are asked to identify the number after it is hidden. Sets of number configurations on overhead transparencies, on cards, and on paper plates would be good sources of materials for these reinforcements. Through playing dominoes and games with standard dice, students would have naturally developed set recognition.

Examples

For this paper plate displayed for about 2 seconds and then hidden, think: This looks like the 5 on a die.

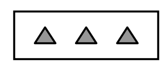


For this display shown for 2 seconds on an overhead, think: I see 4 dots.

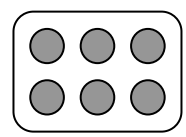


Examples of Some Practice Items

- Flash this card and ask, How many triangles did you see?



- Project this for 2 seconds on an overhead and ask, What number did you see?

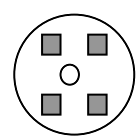


Part-Part-Whole

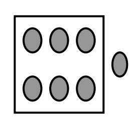
Set recognition can easily be extended to the recognition of two parts in a whole by presenting for a few seconds number configurations made up of two colours or two shapes, or number configurations in two parts. Ask students to identify the number and the two parts.

Examples

For this paper plate shown for about 3 seconds, think: I see 5. I see 4 squares and 1 circle.

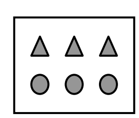


For this display shown for 3 seconds on an overhead, think: I see 6 dots inside and 1 dot outside, so there are 7 dots.

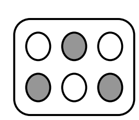


Examples of Some Practice Items

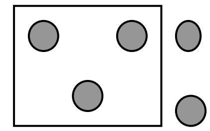
- Flash this card and then ask, How many shapes did you see? How many were triangles? How many were circles?



- Project this for 2 seconds on an overhead and then ask, What number did you see? How many were white? How many were gray?



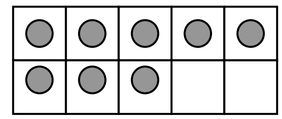
- Flash this card for 3 seconds and then ask, How many dots did you see? How many dots were inside the box? How many were outside?



Ten Frame Visualization

Students' work with ten frames should eventually lead to a mental math stage where students can visualize the standard ten-frame representations for numbers and answer questions from their visual memories.

Examples

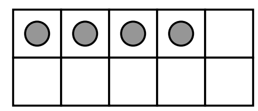


While this ten frame is displayed for a few seconds, think: I see 8 dots: 5 in the full top row and 3 in the bottom row. There are 2 empty cells to make 10.

When your teacher asks you to visualize 6 on a ten frame, think: The top row will be full to make 5 and 1 more in the bottom row to make 6.

Examples of Some Practice Items

- Ask students to visualize 7 on a ten frame, and ask, How many dots are in the first row? How many are in the second row? How many more dots are needed to make 10? What number would you have if you added one more dot? What number would you have if you removed 2 dots?



- Flash this ten frame for 3 seconds and then ask: What number did you see? How many dots would be added to make 5? How many to make 10? What number would be shown if you removed two dots?

Counting

Students' counting experiences should lead to a mental math stage where students, without concrete materials or pictures, can count on from a given number, count back from a given numbers, and skip count by 2, 5s, and 10s from a given number.

Examples

When asked to count by 10s starting at 8, think: I see 8 on a Hundreds Chart and counting by 10 will go vertically down the chart; or think: Counting by 10s starting from 8 means I'll say the number with an 8 in each decade.

When asked to count by 2s starting at 6 and ending at 20, think: These will be even numbers so it's 6, 8, 10, 12, 14, 16, 18, and 20.

When asked to count back starting at 8, think: I automatically say 8, 7, 6, 5, 4, 3, 2, 1.

Examples of Some Practice Items

- Ask students to count by 5s starting at 5 and ending at 45. Record their responses on the whiteboard. Then ask, Do you see a pattern? Describe it.
- Ask students to count on from 7 to 15.
- Ask students to write 12 on their individual whiteboards. Ask them to quickly write the numbers as they count back from 12 to 7.

Next Number

Another pre-operation skill that should be reinforced is the ability to immediately state the number after any given number. The emphasis here is on immediately: there should be no hesitation. This ability is essential for students to reach the counting-on-from-the-larger stage in addition, and will be the necessary skill to use later for the *Plus One* facts. Once students are able to give automatic responses to numbers from 1 to 9, this should be extended to the teen numbers and numbers in the 20s by an awareness of the pattern in the sequence of whole numbers.

Examples

When asked to state the number after 6, think: I automatically say 7.

When asked to state the number after 18, think: A 9 comes after an 8 so the answer is 19.

Examples of Practice Items

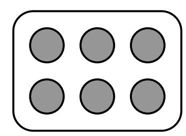
- Provide students with a set of numbers cards from 1 to 9, and have them turn them up in random order on their desktops. Explain that you will state a number and they should quickly find the number card for the number that comes after the one you state. When directed they should hold up their cards.
- Provide each student partnership with a calculator. Direct them to press +, 1, and =. Partner A chooses a number from 2 to 19, puts it in the calculator, and asks Partner B to quickly state the number that comes after this number. Partner A presses = to check. (Remind them not to press clear; rather, just enter the next number.) After 5 numbers, partners change roles.

Other Number Relationships

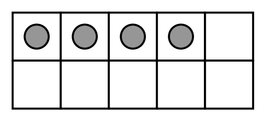
The work students do in grade 1 with one more/one less and two more/two less should lead to a mental math stage where students are presented with a number and asked for the number that is one more, one less, two more, or two less than this number. Reinforcements could use number configurations on paper plates, cards, and overhead acetates; digit cards; and ten frame cards.

Examples

When shown this paper plate configuration and you are asked for the number that is one less, think: I see 6 and one less than 6 is the number before, so it's 5.



When this ten fame is displayed on the overhead and you are asked for the number that is 2 more, think: I see 4 and two more dots would make 6.



Examples of Practice Items

Ask students to quickly write on their individual white boards the number that is one less than the number they will see when you roll the large floor die. Ask them to display their white boards when directed.
Explain to students that you will be turning over a card from a deck of number cards, you will wait 5 seconds while they think of the number that is two more than the number they see, and then you will ask them to chant their responses.
Ask students to quickly think of the number that is two less than each ten-frame configuration that you will display on the overhead for 3 seconds. When directed they should put up the number of fingers that shows their answers.

Explain that you will display a time on a digital clock. Ask students to write on their individual whiteboards the time that is two hours more than the time they see. Ask them to display their whiteboards after each time they are given.

B. Addition — Fact Learning

In grade 1, students are to know the addition facts to 10, and some facts to 18, such as the *Doubles Facts* and the *Plus One Facts*. The major goal is to have students realize and be convinced that they do not need to count to get answers to addition questions.

Other mental strategies for fact learning will be better understood and more easily done when students have reached the developmental stage where they are naturally solving addition questions by counting on from the larger, and can partition numbers to 10 and are comfortable working with the parts.

Facts to 10 Using Ten-Frame Visualization

Ten is the basis of our number system and plays a key role in many of the mental math strategies students will learn. A ten frame, by its very structure, provides students with visual anchors for 5 and 10. Therefore, students' work with ten-frames should reach the visualization stage so students can can use these visual images to respond to addition facts to 10.

In order to reinforce these facts to 10, you could group facts in clusters, practice each cluster until students attain quick recall, integrate each cluster with the others, and gradually build up to the full set. How these facts are visualized may certainly vary with the students, so you should given them facts and ask them to share how they thought about them. It is likely that some facts are visualized, others are thought about in a different way, and still others are just known (automatic). That being said, however, you still can reinforce them in clusters. One possible clustering of facts:

Facts that Make 5: *4 + 1 and 1 + 4, 3 + 2 and 2 + 3, 3 + 2* The strategy for these facts is to visualize the larger addend in a ten frame and see that the other addend is the number of empty cells in the top row, so the answer is 5.

Facts That Make Less Than 5: *1 + 1, 2 + 1 and 1 + 2, 3 + 1 and 1 + 3, 2 + 2* The strategy for these facts is to visualize the larger addend in a ten frame and then add a number of dots equal to the other addend to see the new number formed in the ten frame.

Facts Involving 5: *5 + 1 and 1 + 5, 5 + 2 and 2 + 5, 5 + 3 and 3 + 5, 5 + 4 and 4 + 5* The strategy for these facts is to visualize a full top row of a ten frame (5), and add a number of dots in the bottom row equal to the second addend and see the new number formed in the ten frame.

Facts That Make 10: *9 + 1 and 1 + 9, 8 + 2 and 2 + 8, 7 + 3 and 3 + 7, 6 + 4 and 4 + 6, and 5 + 5* The strategy for these facts is to visualize the larger addend in a ten frame and notice that the number of empty cells in the ten frame is the same as the second addend, so the answer is 10.

Facts That Make 7, 8 or 9: *6 + 1 and 1 + 6, 6 + 2 and 2 + 6, 6 + 3 and 3 + 6, 7 + 1 and 1 + 7, 7 + 2 and 2 + 7, 6 + 1 and 1 + 8 2* The strategy for these facts is to visualize the larger addend in a ten frame and then add a number of dots equal to the other addend to see the new number formed in the ten frame.

Facts with Zeros: *1 + 0, 0 + 1, 2 + 0, 0 + 2, 3 + 0, 0 + 3, 4 + 0, 0 + 4, 5 + 0. 0 + 5, 6 + 0, 0 + 6, 7 + 0, 0 + 7, 8 + 0, 0 + 8, 9 + 0, 0 + 9, 10 + 0, and 0 + 10*. Students often make errors with these facts because they believe that addition should make a change and treat them as if they were Plus 1 Facts; however, since the Plus 1 Facts have received a great deal of reinforcement before these ones, they can soon realize that they just need to visualize the non-zero number.

Another possible clustering of facts:

The Plus 1 Facts: *1 + 1, 2 + 1, 1 + 2, 3 + 1, 1 + 3, …, 9 + 1, and 1 + 9.* The strategy for these facts is to visualize the larger number on the ten frame and then *one more* or *next number*.

The Plus 2 Facts: *2 + 2, 3 + 2, 2 + 3, …, 8 + 2, and 2 + 8*. The strategy for these facts is to visualize the larger number on the ten frame and then *two more*.

a) The Plus 3 Facts: *3 + 3, 4 + 3, 3 + 4, 5 + 3, 3 + 5, 6 + 3, 3 + 6, 7 + 3, and 3 + 7.* The strategies for these facts may vary. For example, 5 + 3 or 3 + 5 is how students visualize making 8 on the ten frame. 7 + 3 or 3 + 7 is 7 and the empty cells on a ten frame. For the others, students visualize the larger number and then the addition of 3 more dots, perhaps as *2 more* and then *1 more*.

b) The Plus 4 Facts : *4 + 4, 5 + 4, 4 + 5, 6 + 4, and 4 + 6.* Again, the strategies may vary for these facts. For example, 5 + 4 or 4 + 5 is how students visualize 9 on a ten frame. 6 + 4 or 4 + 6 is 6 and the empty cells on a ten frame. This leaves 4 + 4 that might be visualized as 4 with 1 more to make 5 and then 3 more to make 8. Some students might already know that 4 + 4 is a double and that it is 8.

c) Facts with Zeros: *1 + 0, 0 + 1, 2 + 0, 0 + 2, 3 + 0, 0 + 3, 4 + 0, 0 + 4, 5 + 0. 0 + 5, 6 + 0, 0 + 6, 7 + 0, 0 + 7, 8 + 0, 0 + 8, 9 + 0, 0 + 9, 10 + 0, and 0 + 10*. Students often make errors with these facts because they believe that addition should make a change and treat them as if they were Plus 1 Facts; however, since the Plus 1 Facts have received a great deal of reinforcement before these ones, they can soon realize that they just need to visualize the non-zero number.

Doubles Facts

There are only nine doubles from 1 + 1 to 9 + 9. Many students seem to have a strong affinity for these doubles facts, often knowing them with little, or no, teaching. These doubles facts are easily learned, however, through an association strategy. That is, each fact is associated with a real-life context and students associate the answers to these doubles facts with these contexts, without the need to count.

A set of flash cards with a fact on one side and a picture of the associated context on the other side would be a good material to use to introduce and reinforce these double facts. The following are some possible associations:

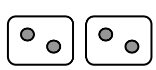
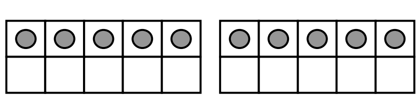
Examples

For 5 + 5, think:
The number of fingers on two hands is 10.

For 9 + 9, think:
The number of tires on an 18-wheeler is 18.

Double Fact	Possible Association
1 + 1	Number of tires on two unicycles
2 + 2	Number of tires on two bicycles
3 + 3	Number of tires on two tricycles; Number of sides on two triangles; Six pack of pop
4 + 4	Number of tires on two cars; Number of sides on two squares
5 + 5	Number of fingers on two hands
6 + 6	Dozen eggs in a carton
7 + 7	Number of days in two weeks
8 + 8	Number of crayons in two rows in a box; Number of legs on two octopuses
9 + 9	Number of tires on an 18-wheeler truck

Examples of Some Practice Items

- 4 + 4
- 5 and 5
- 8 pencils in your desk. 8 pencils in your backpack. How many pencils?
- 7¢ plus 7¢
- 
- $3 more than $3
- Double 9 minutes
- 

Plus One Facts

Students should associate any fact involving 1 as a call for the next number. This can be modelled with unifix cubes by having students build a row of towers for the numbers 2 to 9. If they add one unifix cube to any of these towers, they can easily see that they get the next tower. This would also be true if each of these towers were added to one unifix cube. If they have the ability to state the next number to any given number with no hesitation, they can quickly learn that adding 1 to a number, or adding a number to 1, will be the next number.

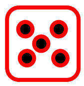
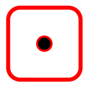
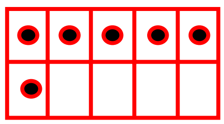
Example

For 7 + 1, think: The number after 7 is 8.

For 1 + 5, think: The number after 5 is 6.

Examples of Some Practice Items

Some practice examples for numbers in the 1s are:

- 2 + 1
- 1 + 6
-  and 
- 

C. Addition — Mental Computation

Adding 10 to a Number

Students in grade 1 should also experience how the addition of 10 to a single-digit number does not require any counting. This will coincide with students' early development of place value as they begin to understand that the 1 in a teen number is 1 ten or 10 ones.

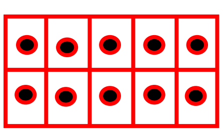
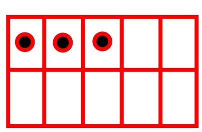
Example

For 3 + 10, they should think, three and ten are thirteen.

Examples of Some Practice Items

Some practice examples for numbers in the 1s are:

- 10 + 2 =
- 3 + 10 =

D. Subtraction — Fact Learning

In their regular mathematics class time, grade 1 students learn the subtractions facts with minuends of 10 or less, using a variety of learning strategies, and relate subtraction facts to the corresponding addition facts. During mental math time, you should restrict the subtraction activities to the visualization strategy described below. Even then, you should expect to allow more response time than for addition facts.

Ten Frame Visualization Strategy

Students should be able to visualize many of the subtraction facts to 10, by visualizing the first number on a ten-frame (minuend) and removing the number of dots (subtrahend) to get the result.

A good start might be to deal with the facts that subtract from 5, so students will visualize the first row of a ten-frame filled and remove the necessary dots to see the result.

Example

For 5 – 2, students would visualize the five dots in the first row of the ten-frame, remove 2 of the dots, and see 3 as a result.

Examples of Some Practice Items

- 5 – 2
- 4 – 1
- 5 take away 4

This could be followed by facts that subtract from 10 and finally other facts with minuends to 9.

Example

For 10 – 2, students would visualize the ten dots in the ten-frame, remove 2 of the dots, and see 8 as a result.

Examples of Some Practice Items

- 10 – 6
- 10 – 1
- 10 subtract 4

PART 2

Measurement Estimation

The Implementation of Measurement Estimation

General Approach

A measurement estimation strategy would normally be reinforced and assessed during mental math time in the grades following its initial introduction. The goal in mental math is to increase a student's *competency* with the strategy. It is expected that measurement estimation strategies would be introduced as part of the general development of measurement concepts at the appropriate grade levels. Each strategy would be explored, modeled, and reinforced through a variety of class activities that promote understanding, the use of estimation language, and the refinement of the strategy. As well, the assessment of the strategy during the instructional and practice stages would be ongoing and varied.

Competency means that a student reaches a reasonable estimate using an appropriate strategy in a reasonable time frame. While what is considered a reasonable estimate will vary with the size of the unit and the size of the object, a general rule would be to aim for an estimate that is within 10% of the actual measure. A reasonable time frame will vary with the strategy being used; for example, using the *benchmark strategy* to get an estimate in metres might take 5 to 10 seconds, while using the *chunking strategy* might take 10 to 30 seconds, depending upon the complexity of the task.

Introducing a Strategy

Measurement estimation strategies should be introduced when appropriate as part of a rich and carefully sequenced development of measurement concepts. For example, in grade 3, the distance from the floor to most door handles is employed as a *benchmark* for a metre so students can use a *benchmark strategy* to estimate lengths of objects in metres. This has followed many other experiences with linear measurement in earlier grades: in grade primary, students compared and ordered lengths of objects concretely and visually; in grade 1, students estimated lengths of objects using non-standard units such as paper clips; in grade 2, students developed the standard unit of a metre and were merely introduced to the idea of a *benchmark* for a metre.

The introduction of a measurement estimation strategy should include a variety of activities that use concrete materials, visuals, and meaningful contexts, in order to have students thoroughly understand the strategy. For example, a meaningful context for the *chunking strategy* might be to estimate the area available for bookshelves in the classroom. Explicit modeling of the mental processes used to carry out a strategy should also be part of this introduction. For example, when demonstrating the *subdivision strategy* to get an estimate for the area of a wall, you would orally describe the process of dividing the wall into fractional parts, the process of determining the area of one part in square metres, and the multiplication process to get the estimate of the area of the entire wall in square metres. During this introductory phase, you should also lead discussions of situations for which a particular strategy is most appropriate and efficient, and of situations for which it would not be appropriate nor efficient.

Reinforcement

Each strategy for building measurement estimation skills should be practised in isolation until students can give reasonable estimates in reasonable time frames. Students must understand the logic of the strategy, recognize when it is appropriate, and be able to explain the strategy. The amount of time spent on each strategy should be determined by the students' abilities, progress, and previous experiences.

The reinforcement activities for a strategy should be varied in type and should focus as much on the discussion of how students obtained their answers, as on the answers themselves. The reinforcement activities should be structured to ensure maximum participation. At first, time frames should be generous and then narrowed as students internalize the strategy and become more efficient. Student participation should be monitored and their progress assessed in a variety of ways. This will help determine the length of time that should be spent on a strategy.

During the reinforcement activities, the actual measures should not be determined every time an estimate is made. You do not want your students to think that an estimate is always followed by measurement with an instrument:

there are many instances where an estimate is all that is required. When students are first introduced to an activity, it is helpful to follow their first few estimates with a determination of the actual measurement in order to help them refine their estimation abilities. Afterwards, however, you should just confirm the reasonable estimates, having determined them in advance.

Most of the reinforcement activities in measurement will require the availability of many objects and materials because students will be using some objects and materials as benchmarks and will be estimating the attributes of others. To do this, they must see and/or touch those objects and materials.

After you are confident that most students have achieved a reasonable competency with the strategy, you need to help them integrate it with other strategies that have been developed. You should do this by providing activities that include a mix of attributes to be estimated in different units using a variety of contexts. You should have the students complete the activities and discuss the strategy/strategies that were, or could have been, used. You should also have students match measurement estimation tasks to a list of strategies, and have them discuss the reasoning for their matches.

Assessment

Your assessments of measurement estimation strategies should take a variety of forms. Assessment opportunities include making and noting observations during the reinforcements, as well as students' oral and written responses and explanations. Individual interviews can provide you with many insights into a student's thinking about measurement tasks. As well, traditional quizzes that involve students recording answers to estimation questions that you give one-at-a-time in a certain time frame can be used.

Assessments, regardless of their form, should shed light on students' abilities to estimate measurements efficiently and accurately, to select appropriate strategies, and to explain their thinking.

Measurement Estimation Strategies

The Benchmark Strategy
This strategy involves mentally matching the attribute of the object being estimated to a known measurement. For example, to estimate the width of a large white board, students might mentally compare its width to the distance from the doorknob to the floor. This distance that is known to be about 1 metre is a *benchmark*. When students mentally match the width of the white board to this benchmark, they may estimate that the width would be about two of these benchmarks; therefore, their estimate would be 2 metres. In mathematics education literature you will often see reference made to *personal referents.* These are benchmarks that individuals establish using their own bodies; for example, the width of a little finger might be a personal referent for 1 cm, a hand span a referent for 20 cm, and a hand width a referent for 1 dm. These benchmarks have the advantage of being portable and always present whenever and wherever an estimate is needed.

The Chunking Strategy (Starting in Grade 5)
This strategy involves mentally dividing the attribute of an object into a number of chunks, estimating each of these chunks, and adding the estimates of all the chunks to get the total estimate. For example, to estimate the length of a wall, a student might estimate the distance from one corner to the door, estimate the width of the door, estimate the distance from the door to the bookcase, and estimate the length of the bookcase which extends to the other corner. All these individual estimates would be added to get the estimate of the length of the wall.

The Unitizing Strategy (Starting in Grade 5)
This strategy involves establishing an object (that is smaller than the object being estimated) as a unit, estimating the size of the attribute of this unit, and multiplying by the number of these units it would take to make the object being measured. For example, students might get an estimate for the length of a wall by noticing it is about five bulletin boards long. The bulletin board becomes the unit. Students estimate the length of the bulletin board and multiply this estimate by five to get an estimate of the length of the wall.

The Subdivision Strategy (Starting in Grade 6)

This strategy involves mentally dividing an object repeatedly in half until a more manageable part of the object can be estimated and then this estimate is multiplied by the appropriate factor to get an estimate for the whole object. For example, to estimate the area of a table top, students might mentally divide the table top into fourths, estimate the area of that one-fourth in square decimeters, and then multiply that estimate by four to get the estimate of the area of the entire table top.

Measurement Estimation

In grade primary, students developed understanding of what it means to measure, and used related measurement vocabulary, not involving units. Although students in grade 1 are developmentally working with numbers from 1 to 100, when estimating, you should limit their experiences to situations that will require them to visualize fewer than 5 or 6 units.

You are encouraged to use a variety of questioning formats during estimation activities to help students make reasonable estimates, rather than 'wild guesses' or 'pulling a number out of the air'. For example, questions might lead students to respond with a single number as in multiple choice; numbers within a given range; or to use comparison language such as *more than*, *less than* and *about*.

E. Measurement Estimation – Length

Grade 1 students should estimate and compare lengths using non-standard units, such as hand spans, eraser lengths, and paper clips

Examples of Some Practice Items

- Will the length of your math scribbler in hand spans be *more than* or *less than* 5?
  (On the whiteboard the teacher can print headings 'more' and 'less'. Students can then place a check mark or post-it note under their estimate. As a class, check and discuss their estimates.)
- Will the width of your desk in hand spans be closer to ?
- Will the height of a small paper-recycling bin be about 1, 3 or 5 hand spans?
- About how wide is a piece of lined chart paper in hand spans?

Additional activities

- Identify an item in the room that is between 1 and 5 paper clips long.
- Identify an item in your desk that is about 3 paper clips long.
- About how many paper clips long is your pencil?
- Will the width of your foot be *more than* or *less than* 4 paper clips?

F. Measurement Estimation – Capacity

In grade 1, students should estimate capacity using non-standard units, such as marbles, cups of rice, or styrofoam packing peanuts

Examples of Some Practice Items

- Will an egg cup hold *more than*, *less than*, or *about* 4 marbles?
  On the whiteboard the teacher can print headings more than, less than and about. Students can then place a check mark or post-it note under their estimate. As a class, check and discuss their estimates.
- Will an egg cup hold more marbles or less marbles than a small yogurt container?
- The teacher showed the class a pill bottle filled with rice. Jacques estimated that a small milk container would hold 3 of these pill bottles of rice. Madeline estimated the same container would hold 5 pill bottles of rice, and Winifred estimated that it would hold 7 pill bottles of rice. Whose estimate do you agree with? Why?
- If a glass container holds about 2 cups of rice, estimate how many cups of rice a large yogurt container could hold. Would it be between 2 and 4 cups of rice or between 5 and 7 cups of rice?
- Estimate the number of cups of rice you would need to fill this mixing bowl.

PART 3

Spatial Sense

The Development of Spatial Sense

What is spatial sense?

Spatial sense is an intuition about shapes and their relationships, and an ability to manipulate shapes in one's mind. It includes being comfortable with geometric descriptions of shapes and positions.

There are seven abilities in spatial sense that need to be addressed in the classroom:

- **Eye-motor co-ordination**. This is the ability to co-ordinate vision with body movement. In the mathematics classroom, this involves children learning to write and draw using pencils, crayons, and markers; to cut with scissors; and to handle materials.
- **Visual memory**. This is the ability to recall objects no longer in view.
- **Position-in-space perception**. This is the ability to determine the relationship between one object and another, and an object's relationship to the observer. This includes the development of associated language (over, under, beside, on top of, right, left, etc.) and the transformations (translations, reflections, and rotations) that change an object's position.
- **Visual discrimination**. This is the ability to identify the similarities and differences between, or among, objects.
- **Figure-ground perception.** This is the ability to focus on a specific object within a picture or within a group of objects, while treating the rest of the picture or the objects as background.
- **Perceptual constancy**. This is the ability to recognize a shape when it is seen from a different viewpoint, or from a different distance. This is the perception at play when students recognize similar shapes (enlargements/reductions), and when they perceive as squares and rectangles, the rhombi and parallelograms in isometric drawings.
- **Perception of spatial relationships.** This is the ability to see the relationship between/among two or more objects. This perception is central when students assemble materials to create an object or when they solve puzzles, such as tangram, pattern block, and jigsaw puzzles.

These abilities develop naturally through many experiences in life; therefore, most students come to school with varying degrees of ability in each of these seven aspects. However, students who have been identified as having a learning disability are likely to have deficits in one or more of these abilities. Classroom activities should be planned to allow most students to further develop these perceptions. Most spatial sense activities involve many, if not most, of the spatial abilities, although often one of them is highlighted.

Classroom Context

Spatial abilities are essential to the development of geometric concepts, and the development of geometric concepts provides the opportunity for further development of spatial abilities. This mutually supportive development can be achieved through consistent and ongoing strategic planning of rich experiences with shapes and spatial relationships. Classroom experiences should also include activities specifically designed for the development of spatial abilities. These activities should focus on shapes new to the grade, as well as shapes from previous grades. As the shapes become more complex, students' spatial senses should be further developed through activities aimed at the discovery of relationships between and among properties of these shapes. Ultimately, students should be able to visualize shapes and their various transformations, as well as sub-divisions and composites of these shapes.

Spatial Sense in Mental Math Time

Following the exploration and development of spatial abilities during geometry instruction, mental math time can provide the opportunity to revisit and reinforce spatial abilities periodically throughout the school year.

Assessment

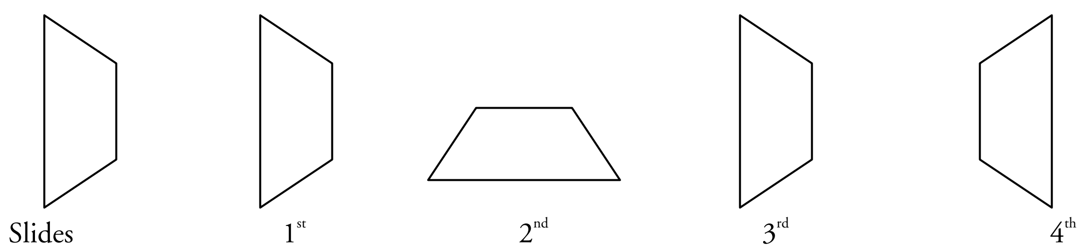
Assessment of spatial sense development should take a variety of forms. The focus in this aspect of mental math is on individual growth and development in spatial sense, rather than on an arbitrary level of competency to be achieved. You should record any observations of growth students make during the reinforcements, as well as noting students' oral and written responses and explanations. For spatial sense, traditional quizzes that involve students recording answers to questions that you give one-at-a-time in a certain time frame should play a very minor role.

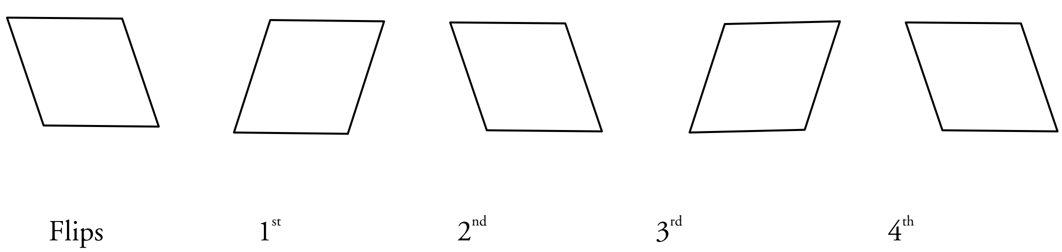
G. 2-Dimentional Geometry

In grade 1, students recognize, name, describe, and represent rhombi, trapezoids, and hexagons, and continue to work with squares, triangles, circles and rectangles. They build, divide, and change these 2-D shapes. They describe angles in shapes as “sharp corners”, “square corners”, and “wide corners”. They recognize, name, describe and represent slides and reflections of 2-D shapes, continuing to use positional language.

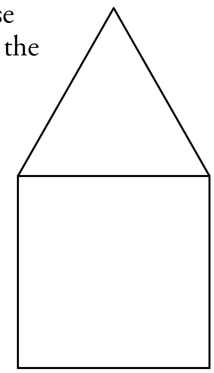
Examples of Spatial Sense Activities

1. Using overhead tangram puzzle pieces create a square with the two small triangles. Students should watch as you do this. Ask students to use their tangram puzzle pieces to recreate the square. This activity can be repeated having students use the large triangles to make a large square; use the square and the two small triangles to make a trapezoid; use the square and two small triangles to make a rectangle. Watch that students choose the correct shapes and put them in the correct positions. Eventually just display a shape composed of tangram pieces and ask students to create it.
2. Show a trapezoid from the pattern blocks on the overhead. Beside it, show four identical trapezoids, labeled first, second, third, fourth (using numbers) two of which show a slide of the original trapezoid. Ask students to identify, by ordinal number, the trapezoids that show a slide of the original, recording their responses on their individual whiteboards. Repeat this activity using different shapes, such as a rectangle, a rhombus, or a triangle. This activity may also be done using flips of the above shapes.

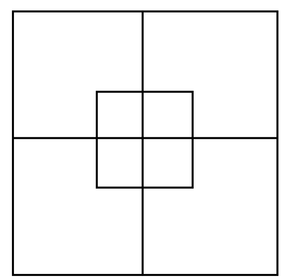




3. Show students a picture made up of two shapes e.g., the outline of a house made from a rectangle and a triangle, for a few seconds. Have them draw the design from memory. Show the picture again and encourage students to compare their drawings with the design. Repeat the activity with other simple two-shape designs.



4. Show students a picture such as graphic of a square within a square, with the larger square divided with vertical and horizontal lines into 4 equal parts and ask them how many squares they can find. After a reasonable time, ask them to show with their fingers how many squares they see. Repeat this activity, using other similarly challenging composite drawings of rectangles and triangles.



H. 3-Dimensional Geometry

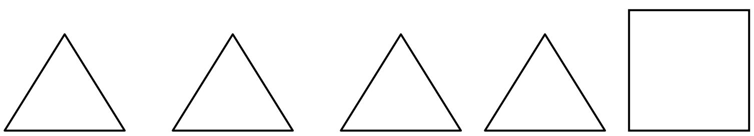
In grade 1, students recognize, name, describe, and represent square prisms, triangular prisms, rectangular prisms, triangular pyramids, and square pyramids and continue to use spheres, cylinders, cubes, and cones. They name the 2-D faces of 3-D shapes.

Examples of Spatial Sense Activities

1. Display a triangular prism (solid not picture). On the overhead show sets of faces for two solids - a square pyramid and a triangular prism, labeled A and B. Ask students to find the set of faces that matches the triangular prism and to record their response on their whiteboard. Repeat this activity, using any of the 3-D shapes explored in primary and grade one.

Find the set of faces that match the triangluar prism that I am holding.

A. 

B. 

2. Display pairs of 3-D shapes and ask students how the shapes are alike and how they are different. For example, show a cylinder and a cone, and students might observe that they both would make a circular footprint and have a smooth rounded surface, but the cone comes up to a point while the cylinder does not, and the cylinder has two circles while the cone has one.

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