



**Mental Math**  
In  
**Mathematics 8**



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# Introduction

## Definitions

It is important to clarify the definitions used around mental math. Mental math in Nova Scotia refers to the entire program of mental math and estimation across all strands. It is important to incorporate some aspect of mental math into your mathematics planning everyday, although the time spent each day may vary. While the *Time to Learn* document requires 5 minutes per day, there will be days, especially when introducing strategies, when more time will be needed. Other times, such as when reinforcing a strategy, it may not take 5 minutes to do the practice exercises and discuss the strategies and answers.

For the purpose of this booklet, fact learning will refer to the acquisition of the 100 number facts relating the single digits 0 to 9 for each of the four operations. When students know these facts, they can quickly retrieve them from memory (usually in 3 seconds or less). Ideally, through practice over time, students will achieve automaticity; that is, they will abandon the use of strategies and give instant recall. Computational estimation refers to using strategies to get approximate answers by doing calculations in one's head, while mental calculations refer to using strategies to get exact answers by doing all the calculations in one's head.

While we have defined each term separately, this does not suggest that the three terms are totally separable. Initially, students develop and use strategies to get quick recall of the facts. These strategies and the facts themselves are the foundations for the development of other mental calculation strategies. When the facts are automatic, students are no longer employing strategies to retrieve them from memory. In turn, the facts and mental calculation strategies are the foundations for estimation. Attempts at computational estimation are often thwarted by the lack of knowledge of the related facts and mental calculation strategies.

## Rationale

In modern society, the development of mental computation skills needs to be a major goal of any mathematical program for two major reasons. First of all, in their day-to-day activities, most people's calculation needs can be met by having well developed mental computational processes. Secondly, while technology has replaced paper-and-pencil as the major tool for complex computations, people need to have well developed mental strategies to be alert to the reasonableness of answers generated by technology.

Besides being the foundation of the development of number and operation sense, fact learning itself is critical to the overall development of mathematics. Mathematics is about patterns and relationships and many of these patterns and relationships are numerical. Without a command of the basic relationships among numbers (facts), it is very difficult to detect these patterns and relationships. As well, nothing empowers students with confidence and flexibility of thinking more than a command of the number facts.

It is important to establish a rationale for mental math. While it is true that many computations that require exact answers are now done on calculators, it is important that students have the necessary skills to judge the reasonableness of those answers. This is also true for computations students will do using pencil-and-paper strategies. Furthermore, many computations in their daily lives will not require exact answers. (e.g., If three pens each cost \$1.90, can I buy them if I have \$5.00?) Students will also encounter computations in their daily lives for which they can get exact answers quickly in their heads. (e.g., What is the cost of three pens that each cost \$3.00?)

# The Implementation of Mental Computational Strategies

## General Approach

In general, a strategy should be introduced in isolation from other strategies, a variety of different reinforcement activities should be provided until it is mastered, the strategy should be assessed in a variety of ways, and then it should be combined with other previously learned strategies.

## Introducing a Strategy

The approach to highlighting a mental computational strategy is to give the students an example of a computation for which the strategy would be useful to see if any of the students already can apply the strategy. If so, the student(s) can explain the strategy to the class with your help. If not, you could share the strategy yourself. The explanation of a strategy should include anything that will help students see the pattern and logic of the strategy, be that concrete materials, visuals, and/or contexts. The introduction should also include explicit modeling of the mental processes used to carry out the strategy, and explicit discussion of the situations for which the strategy is most appropriate and efficient. The logic of the strategy should be well understood before it is reinforced. (Often it would also be appropriate to show when the strategy would not be appropriate as well as when it would be appropriate.)

## Reinforcement

Each strategy for building mental computational skills should be practised in isolation until students can give correct solutions in a reasonable time frame. Students must understand the logic of the strategy, recognize when it is appropriate, and explain the strategy. The amount of time spent on each strategy should be determined by the students' abilities and previous experiences.

The reinforcement activities for a strategy should be varied in type and should focus as much on the discussion of how students obtained their answers as on the answers themselves. The reinforcement activities should be structured to insure maximum participation. Time frames should be generous at first and be narrowed as students internalize the strategy. Student participation should be monitored and their progress assessed in a variety of ways to help determine how long should be spent on a strategy.

After you are confident that most of the students have internalized the strategy, you need to help them integrate it with other strategies they have developed. You can do this by providing activities that includes a mix of number expressions, for which this strategy and others would apply. You should have the students complete the activities and discuss the strategy/strategies that could be used; or you should have students match the number expressions included in the activity to a list of strategies, and discuss the attributes of the number expressions that prompted them to make the matches.

## Assessment

Your assessments of mental math and estimation strategies should take a variety of forms. In addition to the traditional quizzes that involve students recording answers to questions that you give one-at-a-time in a certain time frame, you should also record any observations you make during the reinforcements, ask the students for oral responses and explanations, and have them explain strategies in writing. Individual interviews can provide you with many insights into a student's thinking, especially in situations where pencil-and-paper responses are weak.

Assessments, regardless of their form, should shed light on students' abilities to compute efficiently and accurately, to select appropriate strategies, and to explain their thinking.

## Response Time

Response time is an effective way for teachers to see if students can use the mental math and estimation strategies efficiently and to determine if students have automaticity of their facts.

For the facts, your goal is to get a response in 3-seconds or less. You would give students more time than this in the initial strategy reinforcement activities, and reduce the time as the students become more proficient applying the strategy until the 3-second goal is reached. In subsequent grades when the facts are extended to 10s, 100s and 1000s, a 3-second response should also be the expectation.

In early grades, the 3-second response goal is a guideline for the teacher and does not need to be shared with the students if it will cause undue anxiety.

With other mental computational strategies, you should allow 5 to 10 seconds, depending upon the complexity of the mental activity required. Again, in the initial application of the strategies, you would allow as much time as needed to insure success, and gradually decrease the wait time until students attain solutions in a reasonable time frame.

## Mental Math: Grade 8 Yearly Plan

In this yearly plan for mental math in grade 8, an attempt has been made to align specific activities with the related topic in the grade 8 text. In some areas, the mental math content is too broad to be covered in the time frame allotted for a single chapter. While it is desirable to match this content to the unit being taught, it is quite acceptable to complete some mental math topics when doing subsequent chapters that do not have obvious mental math connections. For example practice with operations on rational numbers could continue into the data management and geometry chapters.

	Skill	Example
Squares, Square root and, Pythagoras	<p><i>Review</i> multiplication and division facts through</p> <p>a) rearrangement/ decomposition</p> <p>b) multiplying by multiples of 10</p> <p>c) multiplication strategies such as doubles, double/double, double plus one, halve/double etc.</p> <p>d) applying the distributive property <i>(Intent is to practice facts through previously learned strategies)</i></p>	<p>a) <math>8 \times 7 \times 5 = 8 \times 5 \times 7</math> <math>12 \times 25 = 3 \times 4 \times 25 = 3 \times 100</math></p> <p>b) <math>70 \times 80 = 7 \times 8 \times 10 \times 10</math> <math>4200 \div 6 = 7 \times (600 \div 6)</math></p> <p>c) <math>12.5 \times 4 = 12.5 \times 2 \times 2 = 25 \times 2 = 50</math> <math>3 \times 12.5 = (2 \times 12.5) + (1 \times 12.5)</math> <math>= 25 + 12.5 = 37.5</math></p> <p>d) <math>3 \times 26 = (3 \times 20) + (3 \times 6)</math> <math>= 60 + 18</math> <math>= 78</math></p>
	<p>Mentally be able to determine perfect squares between 1 and 144 and the corresponding square roots. Daily practice should move to automaticity so students can use these facts in further work.</p> <p>a) Have students find the square root of larger numbers by looking at the factors of the numbers.</p> <p>b) Understanding powers of 10 smaller than 1 that are perfect squares, and how to use them in finding square roots. (<math>0.01 = 0.1 \times 0.1</math>, <math>0.0001 = 0.01 \times 0.01</math>, etc.)</p> <p>c) Find the square root of a quotient</p>	<p><math>8^2 = 64</math> so <math>\sqrt{64} = 8</math> <math>1^2 = 1</math> so <math>\sqrt{1} = 1</math> <math>5^2 = 25</math> so <math>\sqrt{25} = 5</math> <math>11^2 = 121</math> so <math>\sqrt{121} = 11</math></p> <p>a) <math>\sqrt{6400} = \sqrt{64 \times 100}</math> <math>= 8 \times 10 = 80</math> <math>\sqrt{900} = \sqrt{100 \times 9} = 10 \times 3 = 30</math></p> <p>b) <math>\sqrt{0.09} = \sqrt{\frac{9}{100}} = \frac{3}{10} = 0.3</math></p> <p>c) <math>\sqrt{\frac{144}{36}} = \frac{12}{6} = 2</math></p>

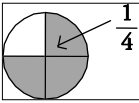



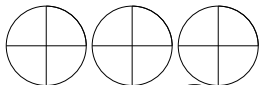
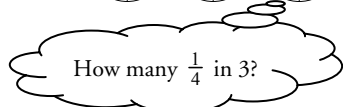

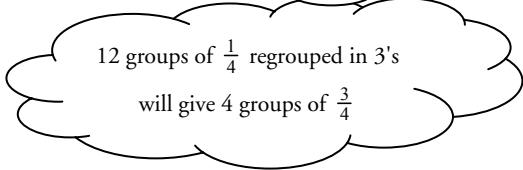
	<b>Skill</b>	<b>Example</b>
		$\sqrt{\frac{9}{100}} = \frac{3}{10} = 0.3$
	<p>Estimation: have students use boundaries to estimate square roots of numbers. (Students should already have the following facts committed to memory and can use them as a toolkit to help them do these problems</p> <p><math>2^2 = 4</math>  <math>3^2 = 9</math>  <math>\cdot</math>  <math>\cdot</math>  <math>\cdot</math>  <math>12^2 = 144</math>)</p> <p>It is important here that students learn to make judgments about what two whole numbers the square root is between and which whole number it is closer to.</p>	<p>a) <math>\sqrt{60}</math> is between 7 and 8 but much closer to 8</p> <p>b) <math>\sqrt{92}</math> is between 9 and 10 but slightly closer to 10</p>
<b>Fraction Operations</b>	<p>a) Learn common fractions and their decimal equivalents for thirds, fourths, fifths, eighths, tenths</p> <p>b) Use relationships between fractions and their decimal equivalents</p>	<p>a) Express as a decimal</p> <p>i. <math>\frac{1}{2}</math>      ii. <math>\frac{3}{4}</math>      iii. <math>\frac{1}{3}</math></p> <p>iv. <math>\frac{1}{8}</math>      v. <math>\frac{7}{8}</math>      vi. <math>\frac{3}{4}</math></p> <p>vii. <math>\frac{3}{4}</math></p> <p>- Express as a fraction</p> <p>i. 0.25      ii. 0.125      iii. 0.4</p> <p>iv. 0.44</p> <p>b) i. If <math>\frac{1}{5} = 0.2</math>, then <math>\frac{3}{5} = ?</math>    <math>\frac{4}{5} = ?</math></p> <p><math>\frac{3}{5} = 3 \times 0.2 = 0.6</math>, etc.</p> <p>ii. If <math>\frac{1}{8} = 0.125</math>, then <math>\frac{3}{8} = ?</math></p> <p>iii. If <math>\frac{1}{3} = 0.333</math>, then <math>\frac{1}{9} = ?</math></p> <p><math>\frac{1}{9} = \frac{0.333}{3} = 0.111</math></p>

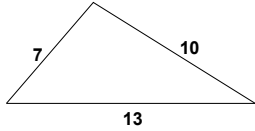
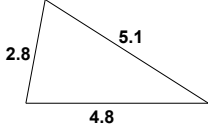
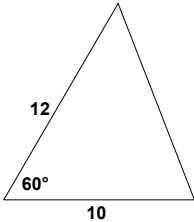
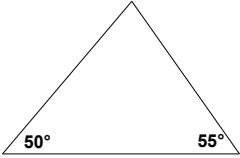
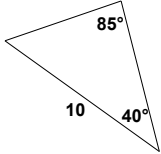
	Skill	Example
	<p>c) Review</p> <ul style="list-style-type: none"> <li>- equivalent fractions</li> <li>- converting from improper fractions to mixed numbers and vice versa</li> <li>- simplest form of a fraction</li> </ul>	<p>c) Find the missing number(s)</p> <p>i. <math>\frac{3}{5} = \frac{15}{\square}</math>      ii. <math>\frac{9}{11} = \frac{\square}{55}</math></p> <p>iii. <math>\frac{72}{81} = \frac{\square}{\square}</math>      iv. <math>3\frac{3}{4} = \frac{\square}{4}</math></p> <p>v. <math>\frac{21}{5} = \frac{\square}{\square}</math></p> <p>- Express in simplest form</p> <p>i. <math>\frac{21}{14}</math>      ii. <math>\frac{60}{36}</math>      iii. <math>\frac{125}{100}</math></p>
	<p>Compare and order fractions using</p> <p>a) benchmarks</p> <p>b) common denominators</p> <p>c) common numerators</p> <p>d) conversion to decimals</p> <p>Estimation: using friendly fractions to simplify</p>	<p>a) Use benchmarks to help you arrange these fractions in order</p> <p>i. <math>\frac{11}{12}, \frac{1}{3}, \frac{3}{7}, \frac{2}{5}</math></p> <p>ii. <math>\frac{4}{7}, \frac{25}{23}, \frac{2}{9}, \frac{14}{15}</math></p> <p>b) Use common denominators to help you arrange these fractions</p> <p><math>\frac{3}{4}, \frac{5}{8}, \frac{7}{12}</math></p> <p>c) Use common numerators to help you arrange these fractions <math>\frac{1}{3}, \frac{3}{7}, \frac{6}{16}</math></p> <p>d) Arrange these fractions in order by changing to decimal form <math>\frac{3}{5}, \frac{5}{8}</math></p> <p><math>\frac{1316}{3824} \rightarrow \frac{1300}{3900} = \frac{13}{39} = \frac{1}{3}</math></p>
	<p>Addition and Subtraction of Fractions – have students mentally add and subtract fractions using previously learned strategies and properties such as :</p>	

	Skill	Example
	<p>a) “make one” with like denominators</p> <p>b) “make one” with easy denominators</p> <p>c) “make one” using mixed numbers</p> <p>d) working with whole numbers and a fraction</p> <p>e) constant difference</p> <p>f) adding on</p> <p>Estimation: sums and differences using</p> <p>a) benchmarks</p>	<p>a) <math>\frac{3}{8} + \frac{7}{8} = \frac{3}{8} + \frac{5}{8} + \frac{2}{8} = 1 + \frac{2}{8} = 1\frac{2}{8}</math>  <math>\frac{2}{3} + \frac{2}{3}, \frac{1}{6} + \frac{5}{6},</math></p> <p>b) <math>\frac{3}{4} + \frac{5}{8} = \frac{6}{8} + \frac{5}{8} = \frac{6}{8} + \frac{2}{8} + \frac{3}{8} = 1\frac{3}{8}</math>  <math>\frac{1}{2} + \frac{7}{12}</math></p> <p>c) <math>2\frac{3}{4} + 1\frac{3}{4}, 4\frac{1}{2} + 3\frac{7}{8}</math></p> <p>d) <math>4 + \frac{2}{3}, 3 - 1\frac{2}{3}</math> (think <math>3 - 1 = 2</math> and <math>2 - \frac{2}{3} = 1\frac{1}{3}</math>)</p> <p>e) <math>4\frac{1}{3} - 2\frac{2}{3}</math> (add <math>\frac{1}{3}</math> to each part to get <math>4\frac{2}{3} - 3 = 1\frac{1}{3}</math>)</p> <p>f) <math>4\frac{1}{3} - 2\frac{2}{3} =</math>  start with <math>2\frac{2}{3}</math> and add on  <math>2\frac{2}{3} + \frac{1}{3} + 1 + \frac{1}{3} = 4\frac{1}{3}</math>  <math>\underbrace{\hspace{1.5cm}}_{1\frac{2}{3}}</math></p> <p>Estimation: sums and differences using</p> <p>a) benchmarks</p> <p>a) <math>\frac{3}{7} + \frac{7}{8} + \frac{1}{10} \rightarrow \frac{1}{2} + 1 + 0 = 1\frac{1}{2}</math>  or  <math>\frac{3}{7} + \frac{7}{8} + \frac{1}{10} \rightarrow \frac{3}{7} + \frac{7}{8} + \frac{1}{10} = 1\frac{3}{7}</math>  <math>\underbrace{\hspace{1.5cm}}_1</math></p>

	Skill	Example
	<p>b) compensation</p> <p>c) replacements</p> <p>When estimating there can be more than one answer</p>	$3\frac{1}{3} + 4\frac{6}{7} + 9\frac{1}{8} \rightarrow 3\frac{1}{2} + 5 + 9 = 17\frac{1}{2}$ <p>or</p> $3\frac{1}{3} + 4\frac{6}{7} + 9\frac{1}{8} \rightarrow 3\frac{1}{2} + 4\frac{\overset{6}{\cancel{7}} + \overset{1}{\cancel{8}}}{1} = 17\frac{1}{3}$ <p>b) <math>4\frac{1}{10} + 2\frac{1}{6} + 3\frac{1}{4} \rightarrow 4 + 2 + 3\frac{1}{2} = 9\frac{1}{2}</math></p> $4\frac{2}{3} - 1\frac{5}{6} \rightarrow 4\frac{1}{2} - 2 = 2\frac{1}{2}$ <p>or</p> $4\frac{2}{3} - 1\frac{5}{6} \rightarrow 5 - 2 = 3$ $9\frac{1}{7} - 3\frac{1}{8} - 2\frac{1}{4} \rightarrow 9 - 3 - 2\frac{1}{2} = 3\frac{1}{2}$ <p>c) <math>\frac{1}{8} + \frac{1}{9} + \frac{1}{10} \rightarrow \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{3}{9} = \frac{1}{3}</math></p>
	<p>Multiplying Fractions:</p> <p>a) Multiply a whole number by a fraction</p> <p>b) Multiply a fraction by a whole number</p> <p>c) Using the commutative property to help get an answer to a multiplication problem</p>	<p>a) i. <math>\frac{2}{3} \times 12</math>, Think: <math>\frac{1}{3}</math> of 12 = 4, so <math>\frac{2}{3}</math> of 12 = 2(4) = 8</p> <p>ii. <math>\frac{5}{6} \times 36</math></p> <p>b) i. <math>8 \times \frac{3}{4} = \frac{24}{4} = 6</math></p> <p>ii. <math>16 \times \frac{1}{8}</math></p> <p>c) i. <math>72 \times \frac{7}{9}</math></p> <p>ii. <math>\frac{4}{5} \times 11</math></p>

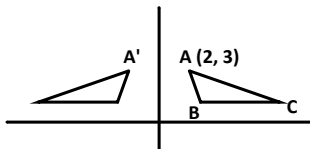
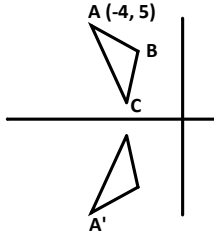

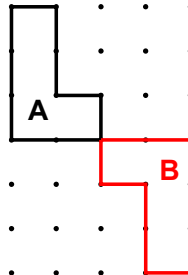
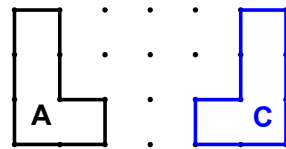
	Skill	Example
	<p><math>(48 \times \frac{7}{8})</math> can be thought of as <math>\frac{7}{8} \times 48</math>. Students can think that since <math>\frac{1}{8} \times 48 = 6</math>, then <math>\frac{7}{8} \times 48 = 7 \times 6 = 42</math>)</p> <p>d) Multiplying a mixed number by a whole number using the distributive law</p> $7 \times 3 \frac{1}{5}$ $= (7 \times 3) + (7 \times \frac{1}{5})$ $= 21 + \frac{7}{5}$ $= 21 + 1 \frac{2}{5}$ $= 22 \frac{2}{5}$	<p>d) i. <math>8 \times 3 \frac{1}{8}</math></p> <p>ii. <math>9 \times 3 \frac{1}{3}</math></p> <p>iii. <math>32 \times 2 \frac{1}{4}</math></p>
	<p>Using the halve/double strategy to multiply</p> $12 \times 2 \frac{1}{2}$ $= 6 \times 5$ $= 30$ <p>e) Multiplying a fraction by a fraction</p> <p>i) Visualize</p> $\frac{1}{3} \times \frac{3}{4}$ 	<p>e) i. <math>\frac{1}{5} \times \frac{5}{6}</math>, <math>\frac{1}{3} \times \frac{3}{8}</math>, <math>\frac{1}{5} \times \frac{5}{9}</math></p>

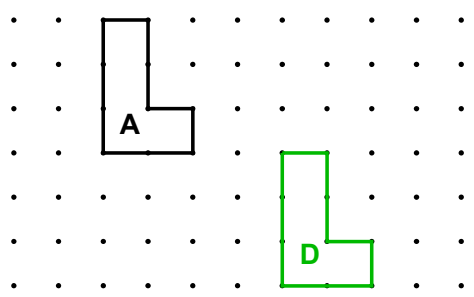
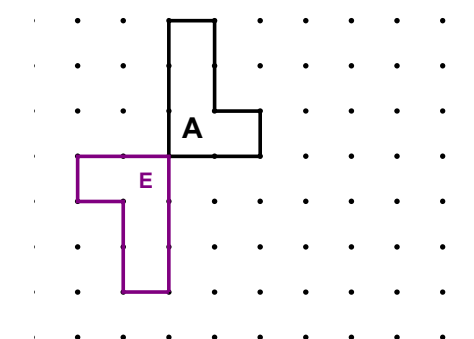
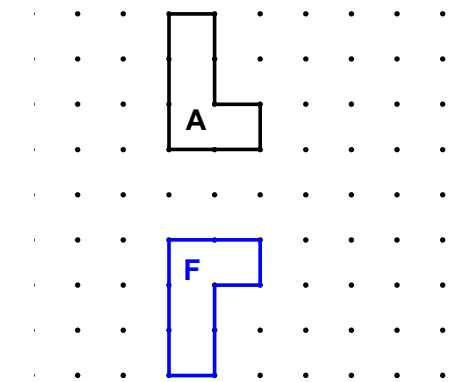
	Skill	Example
	<p>ii) Algorithm using simplification before multiplying</p> $\frac{3}{\cancel{14}} \times \frac{\cancel{14}}{5} = \frac{3}{5}$ <p>f) Estimate the product of fractions:</p> <p>i) using benchmarks</p> $5\frac{1}{7} \times 6\frac{9}{10} \rightarrow$ $5 \times 6 = 30$ <p>using friendly fractions and/or commutative property</p> $34 \times \frac{2}{7} \rightarrow$ $35 \times \frac{2}{7} = \frac{2}{7} \times 35 = 10$	<p>ii. <math>\frac{4}{9} \times \frac{9}{10}, \frac{2}{5} \times \frac{5}{9}, \frac{5}{6} \times \frac{6}{11}</math></p> <p>f) i. <math>3\frac{1}{10} \times 4\frac{1}{5}, 2\frac{7}{8} \times 9\frac{1}{7},</math>  <math>20\frac{1}{4} \times 4\frac{11}{12}</math></p> <p>ii. <math>\frac{7}{8} \times 65</math></p>
	<p>Dividing Fractions</p> <p>a) visualize a unit fraction divided by a whole number</p> <p>b) visualize a whole number divided by a unit fraction</p> <p>c) Estimate the quotient of 2 fractions</p> <p>d) whole number divided by a fraction using the common denominator method</p> <p>Check curriculum guide, Strand B, for additional questions</p>	<p>a) <math>\frac{1}{4} \div 3 =</math>  <math>= \frac{1}{12}</math></p> <p>b) i. <math>3 \div \frac{1}{4} = 12</math>    </p> <p>ii. Could also think:  How many <math>\frac{1}{4}</math> in 1? So how many <math>\frac{1}{4}</math> in 3?</p> <p>c) <math>\frac{5}{8} \div \frac{1}{8} = 5</math></p> <p>d) <math>3 \div \frac{3}{4} = \frac{12}{4} \div \frac{3}{4} = 4</math>    </p>

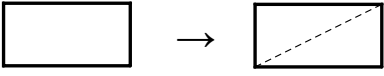
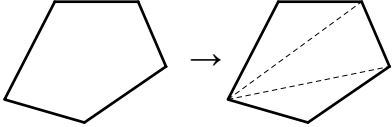
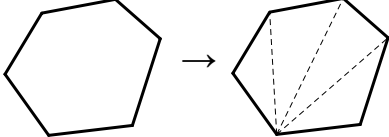
	Skill	Example
	<p>Order of Operations                      Create problems for students that encourage the use of previously learned strategies as well as the order of operations.                      (Some students may prefer to use a “quick calculation” where they record the intermediate steps.)</p>	<p>a) <math>3 - \frac{3}{5} \times \frac{5}{6}</math></p> <p>b) <math>1\frac{1}{3} \times \frac{3}{4} = \frac{4}{3} \times \frac{3}{4} = 1</math>                      or  <math>1\frac{1}{3} \times \frac{3}{4} = 1 \times \frac{3}{4} + \frac{1}{3} \times \frac{3}{4}</math></p> <p>c) <math>2\frac{1}{3} \times 3 \div \frac{1}{2}</math></p> <p>d) <math>3\frac{5}{6} + 1\frac{7}{10} + 1\frac{1}{6} + 2\frac{9}{10}</math></p> <p>e) <math>2\frac{1}{2} \times 8 - 4\frac{1}{2}</math></p>
<p>Geometry</p>	<p>a) Unique Triangles                      Show students a picture of a labeled triangle and ask whether or not it is unique.</p>	<p>a) Determine which triangle(s) is unique</p> <p>i. </p> <p>ii. </p> <p>iii. </p> <p>iv. </p> <p>v. </p>

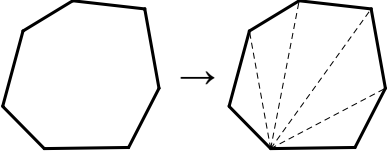
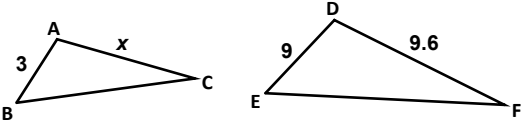
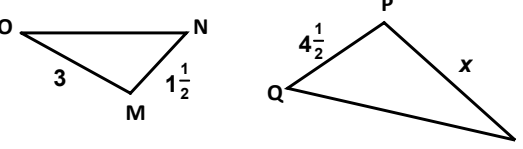
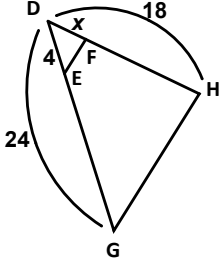
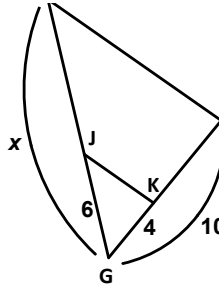


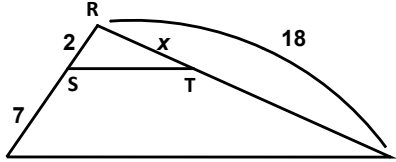
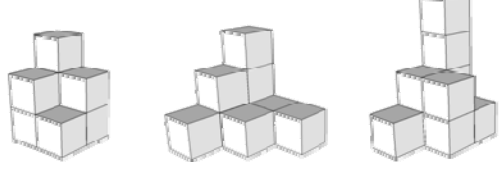
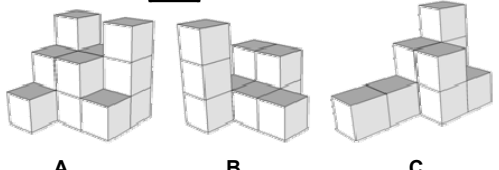
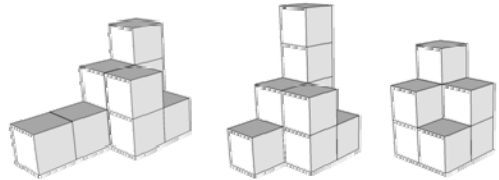


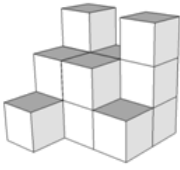
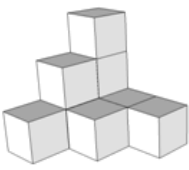
	Skill	Example
	<p>d) Use transformations for spatial sense activities. Give students a sheet with a pre-image. Then show them a diagram with the pre-image and the image for 5 seconds. Remove the diagram and ask students to reproduce the transformation.</p>	<p>iii. Determine the co-ordinates of A' if <math>\triangle ABC</math> is reflected in the <math>y</math>-axis</p>  <p>iv. Determine the co-ordinates of D' if <math>\triangle DEF</math> is reflected in the <math>x</math>-axis</p>  <p>d) Pre-image:</p>  <p>i. <math>A \rightarrow B</math></p>  <p>ii. <math>A \rightarrow C</math></p> 

	Skill	Example
		<p>iii. <math>A \rightarrow D</math></p>  <p>iv. <math>A \rightarrow E</math></p>  <p>v. <math>A \rightarrow F</math></p> 

	Skill	Example
	<p>Patterns in Regular Polygons</p> <p>e) Opportunities are available to practice fact learning strategies and the distributive property when determining angle measures in polygons.</p> <p>To determine the total angle measure in a polygon, subdivide into non-overlapping triangles.</p>	<p>e) Determine the total number of degrees in each polygon.</p> <p>i.  →</p> $  \begin{aligned}  2 \times 180^\circ &= 2 \times 100 + 2 \times 80 \\  &= 200 + 160 \\  &= 200 + 100 + 60 \\  &= 360^\circ  \end{aligned}  $ <p>ii.  →</p> $  \begin{aligned}  3 \times 180^\circ &= 3 \times 100 + 3 \times 80 \\  &= 300 + 240 \\  &= 300 + 200 + 40 \\  &= 540^\circ  \end{aligned}  $ <p>iii.  →</p> $  \begin{aligned}  4 \times 180^\circ &= 4 \times 100 + 4 \times 80 \\  &= 400 + 320 \\  &= 400 + 300 + 20 \\  &= 720^\circ  \end{aligned}  $ <p>or</p> $  \begin{aligned}  4 \times 180^\circ &= 180 \times 4 \\  &= (180 \times 2) \times 2 \\  &= 360 + 2 \\  &= 720^\circ  \end{aligned}  $

	Skill	Example
	<p>f) Similar triangles provide an opportunity to practice proportional thinking when looking for the length of one side. Create questions that can be done mentally and allow students to produce the "within the ratio" relationship as well as the "between the ratios" relationship in a proportion.</p> <p>Within the ratio:  <math>\frac{5}{15} = \frac{x}{21}</math>, <math>\frac{1\frac{1}{3}}{x} = \frac{10}{20}</math>, <math>\frac{3}{12} = \frac{x}{10}</math></p> <p>Between the ratios:  <math>\frac{6}{x} = \frac{18}{33}</math>, <math>\frac{2.1}{5} = \frac{x}{20}</math>,  <math>\frac{\frac{1}{3}}{2\frac{1}{2}} = \frac{\frac{2}{3}}{x}</math> or <math>\frac{21}{2\frac{1}{2}} = \frac{42}{x}</math></p>	<p>iv. </p> $5 \times 180^\circ = 5 \times 100 + 5 \times 80$ $= 500 + 400$ $= 900$ <p>or <math>5 \times 180^\circ = 10 \times 90</math>  <math>= 900</math></p> <p>f) In each pair of similar triangles, solve for <math>x</math></p> <p>i. <math>\triangle ABC \sim \triangle DEF</math></p>  <p>ii. <math>\triangle MNO \sim \triangle PQR</math></p>  <p>iii. <math>\triangle DEF \sim \triangle DGH</math></p>  <p>iv. <math>\triangle GJK \sim \triangle GHI</math></p> 

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	<p><i>If work on proportions has not been completed yet, save these questions for the proportion unit.</i></p> <p>g) Use mat plans and isometric drawings for spatial sense activities.</p>	<p>v. <math>\triangle RST \sim \triangle RUV</math></p>  <p>g) i. Which of these buildings could be the result of this mat plan?</p> <table border="1" data-bbox="1031 661 1177 808"> <tr><td>3</td><td>1</td><td>1</td></tr> <tr><td>2</td><td>1</td><td></td></tr> <tr><td>1</td><td></td><td></td></tr> </table>  <p>A B C</p> <p>ii. Which of these buildings could be the result of this mat plan?</p> <table border="1" data-bbox="1015 1102 1161 1249"> <tr><td>3</td><td>2</td><td>3</td></tr> <tr><td>2</td><td>2</td><td>1</td></tr> <tr><td>1</td><td></td><td></td></tr> </table>  <p>A B C</p> <p>iii. Which of these buildings could be the result of this mat plan?</p> <table border="1" data-bbox="1047 1512 1161 1627"> <tr><td>3</td><td>2</td></tr> <tr><td>2</td><td>1</td></tr> </table>  <p>A B C</p>	3	1	1	2	1		1			3	2	3	2	2	1	1			3	2	2	1
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<p><b>Proportions</b></p>	<p><i>Review</i> fractions, decimals, and percent equivalencies. Students should have automaticity for</p> <ul style="list-style-type: none"> <li>- halves, fourths, eights</li> <li>- thirds, ninths</li> <li>- tenths, fifths</li> </ul> <p>The use of flashcards and numberlines will promote automaticity</p>	<p>Complete this chart</p> <table border="1" data-bbox="860 1318 1323 1690"> <thead> <tr> <th>Fraction</th> <th>Decimal</th> <th>Percent</th> </tr> </thead> <tbody> <tr> <td><math>\frac{1}{2}</math></td> <td></td> <td></td> </tr> <tr> <td></td> <td>0.2</td> <td></td> </tr> <tr> <td></td> <td></td> <td>90%</td> </tr> <tr> <td></td> <td><math>0.\bar{3}</math></td> <td></td> </tr> <tr> <td><math>\frac{2}{5}</math></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td>12.5%</td> </tr> </tbody> </table>	Fraction	Decimal	Percent	$\frac{1}{2}$				0.2				90%		$0.\bar{3}$		$\frac{2}{5}$					12.5%																																							
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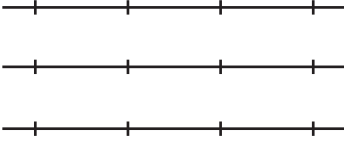
	Skill	Example
	<p>Fractions, Decimals and Percents:</p> <p>a) Find mentally what percentage one number is of another, using equivalent fractions.            12 is what % of 15  <math>\frac{12}{15} \rightarrow \frac{4}{5} = \frac{80}{100} = 80\%</math></p> <p>b) Estimate what percentage one number is of another using friendly fractions.            13 is what % of 19?  <math>\frac{13}{19} \rightarrow \frac{14}{20} = \frac{7}{10} = 70\%</math></p> <p>c) To mentally find the percentage of a given number, there are many possible strategies.            For: What is 28% of 1200?            Students might think</p> <p>i. 25% of 1200 + 3% of 1200            or            ii. 25% of 1200 + 1% of 1200 + 1% of 1200 + 1% of 1200            or            iii. 30% of 1200 – 2% of 1200            or</p> <p>a) Think:            since 28% of 100 = 28            then            28% of 1200 = <math>12 \times 28</math>  <math>= 10 \times 28 + 2 \times 28</math>  <math>= 280 + 56</math>  <math>= 336</math></p> <p>d) Estimate the percent of a given number by rounding.            28% of 1231, think 30% of 1200</p>	<p>a) i. 12 is what % of 55?            ii. 35 is what % of 68?</p> <p>b) i. 9 is what % of 12?            ii. 18 is what % of 30?            iii. 22 is what % of 66?</p> <p>c)</p> <p>i. 26% of 800            ii. 31% of 90            iii. 50.5% of 80</p> <p>d) i. 24% of 79            ii. 98% of 720            iii. 32% of 69</p>

	Skill	Example
	<p>e) Mentally find the whole when a percent is given using proportional thinking and/or converting to common fractions.</p> <p>If 5 is 10% of <input type="text"/></p> <p>what is 100% of <input type="text"/></p> <div style="border: 1px solid black; border-radius: 50%; padding: 10px; width: fit-content; margin: 10px auto;"> <p>Since 100% = 10 × 10% the answer must be 10 × 5 or 50.</p> </div> <p><math>\frac{5}{\square} = \frac{10}{100}</math> or <math>\frac{5}{10} = \frac{\square}{100}</math></p> <p>f) Estimate the mixed number equivalent for percents. Round to friendly fractions</p> <p>i) 149% → 150% = <math>\frac{150}{100} = \frac{15}{10} = \frac{3}{2} = 1\frac{1}{2}</math></p> <p>ii) 119% → 120% = <math>\frac{120}{100} = 1\frac{1}{5}</math></p> <p>iii) 249% → 250% = <math>2\frac{1}{2}</math></p> <p>g) Estimate percents equivalent for mixed numbers.</p> <p><math>3\frac{3}{7} \rightarrow 3\frac{1}{2} = 3.5 = 350\%</math></p>	<p>e) i. 5 is 10% of <input type="text"/></p> <p>ii. 12 is 33.3% of <input type="text"/></p> <p>iii. 8 is 25% of <input type="text"/></p> <p>iv. 30 is 20% of <input type="text"/></p> <p>v. 9 is 60% of <input type="text"/></p> <p>f) Estimate the mixed number equivalent for:</p> <p>i. 112%</p> <p>ii. 224%</p> <p>iii. 183%</p> <p>g) Estimate the % equivalent for:</p> <p>i. <math>2\frac{5}{8}</math></p> <p>ii. <math>3\frac{9}{11}</math></p> <p>iii. <math>1\frac{17}{18}</math></p>
	<p>Applying Proportions: There are many applications of proportions. Create problems using a context and numbers that encourage mental calculations. Give questions that use the relationship within a ration and between ratios.</p>	



	Skill	Example
	<p>Jenson drives 270 km in 3 hours. Travelling at the same speed, how far should he drive in 5 hours?</p> <p>Solving the question using a within the ratio relationship</p> $\times 90 \left( \frac{3 \text{ h}}{270 \text{ km}} = \frac{5 \text{ h}}{x} \right) \times 90$ <p>Solving the question using a between the ratios relationship</p> $\begin{array}{c} 90 \\ \curvearrowright \\ \frac{3 \text{ h}}{5 \text{ h}} = \frac{270 \text{ km}}{x} \\ \curvearrowleft \\ 90 \end{array}$ <p>a) Solving Proportions</p> <p>b) Unit Rates</p> <p>c) Comparison Shopping</p> <p>d) Distance, speed, and time</p>	<p>a) i. <math>7 : 9 = 63 : \square</math>            ii. <math>6 : 18 = 21 : \square</math>            iii. <math>\frac{5}{\square} = \frac{25}{30}</math>            iv. <math>\frac{\square}{8} = \frac{24}{64}</math>            v. <math>\frac{2.5}{\square} = \frac{\square}{12.4}</math></p> <p>b) i. <math>\frac{64\text{¢}}{8 \text{ candy}} = \frac{\square}{1 \text{ candy}}</math>            ii. <math>\frac{300 \text{ km}}{4 \text{ h}} = \square / \text{h}</math>            iii. <math>\frac{12 \text{ beads}}{15 \text{ sec}} = \square / \text{min}</math></p> <p>c) What is the better deal: a dozen oranges for \$2.40 or 3 oranges for 45 cents?</p> <p>d) If you travel 300km in 4 hrs, how long would it take you to travel 450 km?</p>

	<b>Skill</b>	<b>Example</b>
<b>Data Management And Probability</b>	<i>As a brief review</i> , create sets of data and have students mentally calculate the mean using previously learned strategies	Calculate the mean of each. a) $28 + 36 + 22 + 34$ (compatibles) b) $75 + 29 + 46 + 54$ (break up and bridge) c) $4.6 + 3.5 + 8.4 + 1.5 + 2$ (make one) d) $410 + 120 + 330 + 140$ (front end addition or break up and bridge) e) 43, 37, 46, 32, 47 (central value method – choose a central value, such as 40, and then find the mean of the positive and negative differences between the central value and the numbers and add to the central value. $+3 + (-3) + +6 + -8 + +7 = +1$ Add +1 to 40 to get 41 as the mean
	Estimation: a) Applying samples to population: – If 20% of sample voted yes, about how many in a population of 769 would vote yes? b) Working with samples for friendly fractions $\frac{27}{164} \rightarrow \frac{28}{164} =$ $\frac{7}{41} \rightarrow \frac{7}{42} = \frac{1}{6}$	
	Adding some number to all members of a set: Find new mean, medium, and mode.	If for the set {9, 9, 9, 14, 15, 17, 18} Mean = 13 Median = 14 Mode = 9 Find the new mean median, mode if - all numbers of the set are multiplied by 7 - all numbers of the set are divided by 2 - etc.

	Skill	Example
	<p>Circle Graphs: Find percents of 360° by using proportional thinking</p>	<p>a) 100% of 360° b) 50% of 360° c) 25% of 360° d) 12.5% of 360° e) <math>33.\bar{3}</math> % of 360° f) <math>66.\bar{6}</math> % of 360° g) 10% of 360° h) 20% of 360° i. 5% of 360° j) 15% of 360° k) 27.5% of 360° l) 16% of 360°</p>
	<p>Reinforce equivalency among common fractions, decimals, and percent.</p> <p>Develop automaticity for conversions as well as associations with the words Never, Seldom, About half of the time, Often, and Always.</p> <p><i>Refer to the grade 7 Mental Math Yearly Plan.</i></p>	<p>Use three stacked number lines: Show 0, <math>\frac{1}{4}</math>, <math>\frac{1}{2}</math>, <math>\frac{3}{4}</math> and 1 on the first, the equivalent decimals on the second line and percents on the third line.</p> 
	<p>Experimental Probability:</p> <p>a) use context to set up experimental probability questions</p> <p>b) use “friendly fractions” to estimate experimental probability</p>	<p>a) There are 3600 fish in the pond and 889 are speckled trout. Estimate the probability that you will catch a trout when you go fishing. (Students should see that 889 is close to 900 and so the probability is about <math>\frac{1}{4}</math> or 0.25 or 25%)</p> <p>b) Estimate the experimental probability of:</p> $\frac{124}{506} \Rightarrow \frac{125}{500} \xrightarrow{+25} = \frac{5}{20} = \frac{1}{4}$ $\frac{31}{46} \Rightarrow \frac{30}{45} \xrightarrow{+15} = \frac{2}{3}$

	<b>Skill</b>	<b>Example</b>
	<p>Complementary Probabilities: Create problems that give students a chance to practice strategies such as subtraction by parts, finding compatibles. Having them use the idea of money when working with decimals is also beneficial.</p>	<p>Find the complementary probability If <math>P(\text{drawing a red ball}) = 0.47</math> Then <math>P(\text{not drawing a red ball}) = 1 - 0.47 = 0.53</math></p> <p>If <math>P(\text{drawing a blue ball}) = 0.112</math> <math>P(\text{not drawing a blue ball}) = 1 - 0.112 = 0.888</math></p>
<b>Rational Numbers</b>	<p><i>Review</i> the comparing and ordering of</p> <p>a) Integers</p> <p>b) Fractions</p> <p>c) Decimals</p> <p><i>Review</i> previously learned strategies for integer, fractions, and decimal operations</p> <p>d) Integers</p> <p>e) Fractions</p> <p>f) Decimals</p> <p>g) Exponents</p>	<p>Order from smallest to largest</p> <p>a) <math>-1, 0, -100, +3, -3</math></p> <p>b) <math>\frac{1}{10}, \frac{3}{4}, \frac{16}{29}, \frac{5}{4}, \frac{12}{25}</math></p> <p>c) <math>2.7, 2.71, 2.17, 2.017, 0.2017</math></p> <p>Perform the indicated operation</p> <p>d) i) <math>-3 + (-5) + (+2)</math> ii. <math>-3 - (-5)</math> iii. <math>(+7) \times (-1) \times (-2)</math> iv. <math>(-64) \div (+8)</math> v. <math>-10 - 3 \times (+4)</math></p> <p>e) i. <math>\frac{3}{4} + \frac{7}{8}</math> ii. <math>\frac{9}{10} - \frac{1}{2}</math> iii. <math>1\frac{3}{4} \times 8</math> iv. <math>\frac{2}{3} \div \frac{1}{2}</math></p> <p>f) i. <math>2.9 + 3.4 + 1.1</math> ii. <math>6 - 1.27</math> iii. <math>0.25 \times 80</math> iv. <math>9 \div 0.1</math> v. <math>3.5 - 0.5 \times 4</math></p>

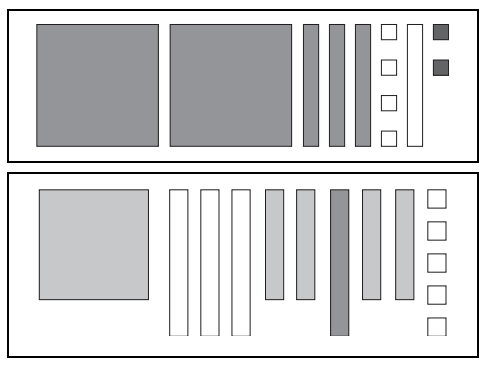
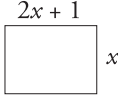


	Skill	Example
	b) Arrange in order from smallest to largest	Express in standard form v. $3.4 \times 10^5 =$ vi. $6.25 \times 10^{-4} =$ v. $9.0 \times 10^{-6} =$ b) Arrange from smallest to largest i. 420, $4.2 \times 10^{-3}$ , 0.42 ii. $2.5 \times 10^{-3}$ , $0.25 \times 10^{-4}$ , $25 \times 10^{-2}$
	Compare and Order Rational Numbers Students should be encouraged to use the previously learned strategies listed below and apply them to an extended number set.  a) a negative is always less than a positive  b) use benchmarks $(-1, -\frac{1}{2}, 0, +\frac{1}{2}, +1)$  c) change to common denominators  d) change to common numerators  e) convert to decimals	Arrange from least to greatest  a) i. $-3.1, 2, +2.42, -1.6, -1.75$ ii. $+\frac{1}{3}, -\frac{9}{10}, +\frac{3}{7}, -\frac{4}{9}, -\frac{4}{3}$  b) $+\frac{7}{8}, +\frac{3}{4}, +\frac{3}{7}, +\frac{1}{10}, 0.99$  c) $-\frac{2}{3}, -\frac{1}{6}, -\frac{1}{2}, -\frac{5}{6}, -\frac{4}{3}$  d) $-\frac{3}{4}, -\frac{2}{7}, -\frac{1}{3}, -\frac{6}{11}$  e) $+\frac{5}{8}, +\frac{2}{3}, -\frac{3}{5}, -\frac{5}{9}, -0.64$
	Operations with Rational Numbers: <i>Review</i> operations with rational numbers using strategies and properties such as: <ul style="list-style-type: none"> <li>• Front End Addition</li> <li>• Compatible Addends</li> <li>• Front End Multiplication</li> <li>• Compatible Factors</li> <li>• Making Compatible Numbers</li> <li>• Halve/ Double</li> <li>• Compensate:</li> </ul>	Use the properties of numbers (Associative, Commutative, Distributive, and Identity) to assist mental calculation: <ul style="list-style-type: none"> <li>a) <math>2 \times 24 \times 50</math></li> <li>b) <math>2 \times 3.4 \times 5</math></li> <li>c) <math>4 \times \frac{3}{10} \times 2.5</math></li> <li>d) <math>50 \times 14</math></li> <li>e) <math>2.5 \times 16</math></li> <li>f) <math>7 \times \frac{3}{4} \times 12</math></li> </ul>

	Skill	Example
	<p>(140 – 69 can be thought of as 140 – 70 then <i>compensate</i> by adding 1. )</p> <p>(12.5 – 4.7 can be thought of as 12.5 – 4.5 then <i>compensate</i> by subtracting 0.2.)</p>	<p>g) <math>0.\overline{33} \times 21 \times 5</math>            h) <math>0.25 \times 25 \times 16</math>            i) <math>299 \times 15</math></p> <p>Use your compensation strategies for mentally calculating the following:            j) <math>167 - 38</math>            k) <math>3\frac{1}{2} - 2\frac{7}{8}</math>            l) <math>6.7 - 7.8</math></p>
	<p>a) Associative property</p> <p>b) Identity Property</p> <p>c) Commutative Property</p> <p>d) Distributive Property</p> <p><i>Review</i> the four operations on integers, fractions, and decimals. Students should be able to mentally perform the sum, difference, product, or quotient of two “friendly” rational numbers using strategies from prior grades as well as those learned in grade 8. Bring in such things as the zero principle as well as the product of a number and its reciprocal is one.</p>	<p>a) i. <math>3\frac{3}{4} + 3\frac{3}{4} + 2\frac{1}{4}</math>            ii. <math>-4.9 + (-6.3) + (-5.1)</math>            b) i. <math>+6.1 + (-18) + (-6.1)</math>            ii. <math>\frac{1}{2} \times \frac{3}{4} \times \left(-\frac{4}{3}\right)</math>            c) <math>\frac{3}{4} \times 7 = 7 \times \frac{3}{4}</math>            d) i. <math>8 \times \left(-3\frac{1}{4}\right) \rightarrow 8 \times -3 + 8 \times \left(-\frac{1}{4}\right)</math>            ii. <math>-4 \times (1.13) - 4 \times (0.87)</math>  <math>= -4 \times (1.13 + 0.87)</math>  <math>= -4 \times 2</math>  <math>= -8</math></p>
	<p>Order of Operations:            Create problems using “friendly numbers” that practice using the properties and strategies previously learned.</p>	<p>a) <math>7 - \frac{2}{3} \times \frac{3}{2}</math>            b) <math>4 + \frac{4}{5} \div \frac{1}{10}</math>            c) <math>\left(-\frac{1}{2}\right)^2 - \frac{1}{4}</math></p>

	Skill	Example
		d) $-\frac{1^2}{2} - \frac{1}{4}$ e) $\frac{1}{2} \times \frac{2}{5} + \frac{1}{2} \times \frac{2}{5}$ f) $3 \times 1.3 + 10.1$ g) $2\frac{1}{2} \times 8 \div \frac{1}{2}$ h) $\sqrt{6.25 \times 16}$
	Estimation: Revisit the ideas developed in the unit on fractions. Problems can now be extended to include negative rational numbers.	Estimate: a) sums and differences using benchmarks i. $-\frac{3}{7} + \frac{7}{8} \rightarrow -\frac{1}{2} + 1 = \frac{1}{2}$ ii. $9\frac{1}{7} + \left(-2\frac{3}{4}\right) \rightarrow 9 + (-3) = 6$ iii. $10\frac{11}{12} - 8\frac{4}{7} \rightarrow 11 - 8\frac{1}{2} = 11 - 8 - \frac{1}{2}$ $= 3 - \frac{1}{2} = 2\frac{1}{2}$ b) the product of fractions using benchmarks i. $-2\frac{1}{8} \times 4\frac{5}{6} \rightarrow$ ii. $10\frac{1}{2} \times \left(-4\frac{1}{3}\right) \rightarrow 10\frac{1}{2} \times (-4)$ $= 10 \times (-4) + \frac{1}{2} \times (-4)$ $= -40 + (-2)$ $= -42$
Algebraic Expressions and Solving Equations	An important topic from grade 7 that can be practiced mentally at the start of this unit is operations on integers. Students will need to see that operations on algebraic expressions require the same approach as operations on integers.	Calculate a) $(+2) + (-3)$ b) $(+2a) + (-3a)$ c) $(+2) - (-3)$ d) $0 - (+4)$ e) $2 - 7$ f) $2c - 7c$ e) $(-1) + (-3) - (5)$ f) $(-2) \times (-3)$ g) $(+4) \times (-8)$ h) $(-2)^2$ i. $-2^2$

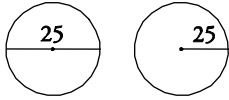
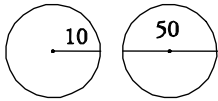
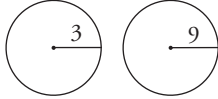
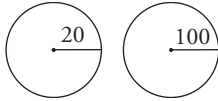
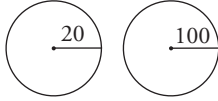
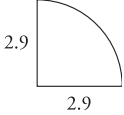
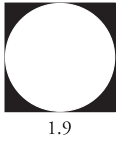
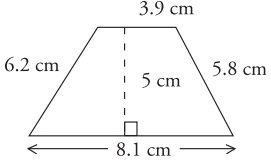


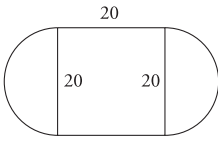
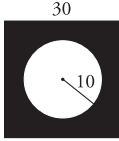
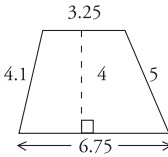
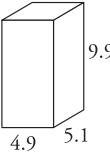
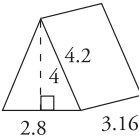
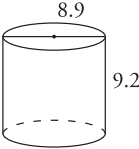
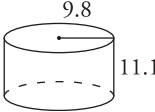
	Skill	Example
		j) $(-72) \div (-8)$ k) $(-64) \div (+8)$ l) $(-3) \times (+4) \times (-2)$ m) $(+32) \div (-8)$
<b>Addition and Subtraction of Algebraic Terms</b>	Students should be able to a) evaluate algebraic expressions  b) be given a model and ask for the expression in simplest form (The overhead or magnetic white board tiles work well)  c) symbolically simplify expressions by combining like terms (show the connection to operations on integers)	a) If $x = +2$ and $y = -5$ i. $x^2$ ii. $3y$ iii. $x - y$ iv. $y - x$ v. $2y - 1$ vi. $2x - 1$ vii. $x + y + 15$ viii. $-2x^2$ ix. $(-3y)^2$  b) State in simplest form the expressions illustrated below:  c) Combine Like Terms i. $2x + (-3y) + 6x + (-5y) + 2$ ii. $3x + 4 - 6x - 10$ iii. $3x - 2y - y - 4$ iv. $(3x + 4) + (8 - x)$ v. $(8x + 4) + (18 - 2x)$ vi. Find an expression for the perimeter  vii. $3x - 4 - 2x - 1$ viii. $y - 2x - 3y - x$ ix. $1 - 4x - y + 7x - 3y$ x. $(2x^2 - x - 1) - (x^2 - 2x + 3)$
	Students should be able to use the distributive property to mentally multiply an expression by a scalar.	<b>Multiply</b> a) $4 \times 23 = 4 \times 20 + 4 \times 3$ b) $5 \times 4.3 = 5 \times 4 + 5 \times 0.3$ c) $2(x + 2)$ d) $4(3 - 3x)$ e) $3(2x - 1)$

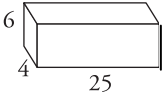
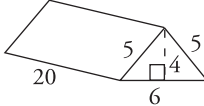
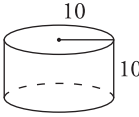
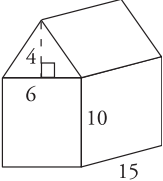
	<b>Skill</b>	<b>Example</b>
<b>Solving Linear Equations:</b>	<p>In grade 7, students are to mainly use concrete materials to solve simple linear equations. Before students can do anything mentally with solving equations, they need a lot of practice moving from the concrete and being confident and competent as they solve equations symbolically. Therefore the type of equations they should be asked to solve mentally at this stage should be fairly simple.</p>	<p>Mentally solve for x:</p> <p>a) <math>x - 4 = 6</math>                      b) <math>x + 6 = -1</math>                      c) <math>3x = -9</math>                      d) <math>\frac{1}{2}x = -8</math>                      e) <math>\frac{1}{2}x = 8</math>                      f) <math>2x - 1 = 5</math>                      g) <math>\frac{1}{3}x = 4</math></p> <div style="border: 1px solid black; border-radius: 50%; padding: 10px; width: fit-content; margin-left: auto; margin-right: auto;"> <p>Think <math>\frac{1}{3}x = 4</math>                      so <math>\frac{3}{3}x = 12</math></p> </div>
<b>Patterns and Relations</b>	<p>You may wish to use some of the mental math time for this chapter to finish some of the suggestions from other chapters</p>	
	<p>Evaluate a single variable expression (start with whole numbers, then fractions and decimals). Progression of the types of expressions is also important (e.g. <math>2x + 4</math>, <math>4 + 2x</math>, <math>2x - 4</math>, <math>-2x + 4</math>, <math>4 - 2x</math>)</p>	<p>Evaluate the following expressions for the given value : (Do each evaluation separately)</p> <p>a) <math>3x + 1</math>    <math>x = 2</math>;    <math>x = -6</math>;    <math>x = \frac{1}{3}</math>;  <math>x = -\frac{4}{3}</math>    <math>x = 0.3</math>    <math>x = -0.5</math></p> <p>b) <math>5 + 4x</math>    <math>x = 10</math>;    <math>x = -4</math>;    <math>x = -\frac{1}{4}</math>  <math>x = \frac{3}{2}</math>;    <math>x = 0.75</math>;    <math>x = 1.25</math></p> <p>c) <math>6x - 8</math>    <math>x = 0</math>;    <math>x = -2</math>;    <math>x = \frac{1}{3}</math>;  <math>x = -\frac{1}{12}</math>;    <math>x = 1.5</math>;    <math>x = -2.5</math></p> <p>d) <math>\frac{1}{2}x + 10</math>    <math>x = 6</math>;    <math>x = -8</math>;    <math>x = \frac{4}{7}</math>  <math>x = 4\frac{2}{3}</math>;    <math>x = -12.6</math></p> <p>e) <math>4 - x^2</math>    <math>x = 3</math>;    <math>x = -2</math>;    <math>x = \frac{1}{3}</math>  <math>x = 0.3</math>;    <math>x = 0.5</math></p> <p>f) <math>2^x + 3</math>    <math>x = 0</math>;    <math>x = 3</math>;    <math>x = -2</math></p> <p>g) <math>(4x) \div 3</math>    <math>x = 6</math>;    <math>x = 33</math>,    <math>x = -15</math>  <math>x = \frac{3}{4}</math>;    <math>x = 1.5</math></p>

	Skill	Example																																																												
	<p>Have students determine the pattern in a table and complete the missing part(s). Include asking students to determine the <math>n^{\text{th}}</math> term</p>	<p>Study the table, determine the pattern and use it to complete the missing parts.</p> <table border="1" style="margin-bottom: 10px;"> <tr> <td>n</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>10</td><td>20</td><td>n</td> </tr> <tr> <td>term</td><td>4</td><td>7</td><td>10</td><td>13</td><td></td><td></td><td></td><td></td><td>76</td> </tr> </table> <table border="1" style="margin-bottom: 10px;"> <tr> <td>n</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>10</td><td>20</td><td>n</td> </tr> <tr> <td>term</td><td>100</td><td>99</td><td>98</td><td>97</td><td></td><td></td><td></td><td></td><td>32</td> </tr> </table> <table border="1"> <tr> <td>n</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>10</td><td>20</td><td>n</td> </tr> <tr> <td>term</td><td>-1</td><td>3</td><td>7</td><td>11</td><td></td><td></td><td></td><td></td><td>395</td> </tr> </table>	n	1	2	3	4	5	6	10	20	n	term	4	7	10	13					76	n	1	2	3	4	5	6	10	20	n	term	100	99	98	97					32	n	1	2	3	4	5	6	10	20	n	term	-1	3	7	11					395
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<p>Measurement</p>	<p>Topics to <i>briefly review</i> at the start of the measurement unit are:</p> <p>a) <i>review</i> the metric units for area and volume</p> <p>b) <i>review</i> the connections between</p> <ul style="list-style-type: none"> <li>– <math>1 \text{ cm}^3</math>, 1 ml, and 1 g</li> <li>– <math>1 \text{ dm}^3</math>, 1L, and 1kg;</li> <li>– <math>1 \text{ m}^3</math>, 1 metric ton and 1 Kl</li> </ul> <p><i>For all of the above use models and visuals to reinforce the connections</i></p> <p>c) <i>review</i> all the metric prefixes – post visuals for students to use. Avoid using the metric chart for the memorization of prefixes.</p> <p>d) practice SI conversions</p>	<p>a) i. Draw rectangles with these areas and state the dimensions: <math>20 \text{ cm}^2</math>, <math>0.5 \text{ cm}^2</math>, <math>0.25 \text{ dm}^2</math>, <math>0.2 \text{ dm}^2</math>.</p> <p>ii. Estimate these areas: classroom door, your thumbprint, a soccer field, your room, your home.</p> <p>iii. Which of these areas would fit on a scribbler page: <math>20\,000 \text{ mm}^2</math>, <math>180 \text{ cm}^2</math>, <math>0.8 \text{ dm}^2</math>, <math>0.02 \text{ m}^2</math>?</p> <p>b) Show various shapes and ask for a measurement estimate of the volumes (can extend this later to surface area)</p> <p>d) Convert each of the following:</p> <ul style="list-style-type: none"> <li>i. <math>250 \text{ cm} = \underline{\hspace{2cm}} \text{ m}</math></li> <li>ii. <math>2.5 \text{ km} = \underline{\hspace{2cm}} \text{ m}</math></li> <li>iii. <math>0.4 \text{ km} = \underline{\hspace{2cm}} \text{ m}</math></li> <li>iv. <math>0.5 \text{ m}^2 = \underline{\hspace{2cm}} \text{ cm}^2</math></li> </ul>																																																												



	Skill	Example
	<p>f) Circumference and Area of Circles:</p> <ul style="list-style-type: none"> <li>– Estimate the circumference of circles using 3 to approximate <math>\pi</math></li> <li>– Mentally calculate the circumference using 3.14 for <math>\pi</math> and having the radius or diameter to be a multiple or power of 10</li> <li>– Estimate the area of circles by squaring the diameter (rough estimate) by squaring the radius and multiplying by 3</li> <li>– Mentally calculate the area using: <math>A = \pi r^2</math></li> </ul>	<p>f) Estimate the circumference of these circles.</p>  <p>Calculate the circumference of these circles.</p>  <p>Estimate the area of these circles.</p>   <p>Calculate the area of these circles.</p> 
	<p>Composite Figures</p> <p>a) Estimate the area of these figures (A good chance to practice rounding and compensating. Use 3 for <math>\pi</math>.)</p>	<p>a) Estimate the area of these figures</p> <p>i. </p> <p>ii.  (Shaded part)</p> <p>iii. </p>

	Skill	Example
	<p>b) Mentally calculate the area of these figures (Create problems with “friendly numbers” allowing students to practice previously learned strategies.)</p>	<p>b) Calculate the area of each figure</p> <p>i. </p> <p>ii.  (Shaded Part)</p> <p>iii. </p>
	<p>Have students estimate the solution to problems involving volume and surface area using appropriate estimation strategies. <i>Suitable models and diagrams for estimation of surface area are rectangular and triangular prisms, cylinders and composite 3-D figures. For estimating volume, suitable models and diagrams may include rectangular and triangular prisms, cylinders and composite 3-D figures.</i></p>	<p>Estimate the surface area and volume of each figure</p> <p>i. </p> <p>ii. </p> <p>iii. </p> <p>iv. </p>

	Skill	Example
	<p>Make decisions on what volume and surface area problems can be solved mentally. Recognize those that can use previously learned strategies (front-end, compatible factors, halve/double etc). Solve volume and surface area problems for prisms, cylinders and composite figures mentally when appropriate.</p> <p><i>Suitable models and diagrams are rectangular and triangular prisms, cylinders, and composite figures.</i></p>	<p>Calculate the volume and surface area of each figure.</p> <p>i. </p> <p>ii. </p> <p>iii. </p> <p>iv. </p>