Creating an Effective Learning Environment

Flexible Grouping

In a learner-centred classroom, students should have the opportunity to work independently, but also with a variety of peers in different types of groups. These flexible groups may include:

- Large-group or whole-group instruction.
- Teacher-directed small-group instruction
- Small-group and co-operative learning groups.
- One-to-one teacher-student instruction.
- Independent work.
- Partner learning.
- Peer or cross-age tutoring.
- Computer work station instruction with teacher monitoring.

> The Learning Modalities—visual, auditory, and kinaesthetic—refer to different ways of learning.

Groupings:
The frequent use of heterogeneous groupings invites each student to bring his or her experiences and knowledge to the problem-solving situation. Learners benefit from hearing the thinking and language used by others. In a teacher-facilitated discussion, students realize that everyone has something to contribute.

There may be times when homogeneous groupings may be more appropriate. Teachers may identify a need to create a small group of students, who are struggling with the same concept, in order to clear up their misconceptions. The use of a guided group in mathematics is similar to a guided reading group in language arts. A small homogeneous group is brought together for explicit teaching. Another reason for grouping students of similar abilities together for a short period of time might be to provide an enrichment activity for students who are already comfortable with the concept being taught. Teachers who practise differentiation will develop the skill to know whether heterogeneous or homogeneous grouping is appropriate.

> Differentiation is the recognition of and commitment to plan for student differences. A differentiated classroom provides different avenues to acquire content, to process or make sense of information and ideas, and to develop products. The goals of a differentiated classroom are to maximize student growth and to promote individual student success.

The teacher should maintain flexibility in

- group membership,
- length of time the group stays together,
- the frequency of meetings.

This is one way of differentiating instruction and creating a learner-centred approach to learning mathematics. Overuse of one type of grouping can affect students' view of themselves and their classmates in learning mathematics. These attitudes can be counter-productive to a cooperative learning environment.

> Teachers may find it helpful to keep templates in a duo tang colour-coded for subjects.

The duration of the group will depend on the purpose and the productivity of the group. Some students may only be temporary group members in order to re-visit a concept or skill. Students need some control over choices of level of activity and degree of independence. Some students rarely choose to work independently. They may need encouragement to take risks, or they may need practice in concepts at a lower level.

Independent Work:
Students need time to work and think independently as they deal with the variety of tasks and questions during instruction and assessment. Teachers should be careful that students do not become too dependent on the teacher, calling on them to explain work before trying the task themselves, or on other students when working in groups. During the development of a concept there are questions and tasks that should be attempted individually before being discussed with a partner, a group, or the teacher. For example, asking students to record their ideas before sharing with a partner encourages all students to have something meaningful to share when they meet with a partner. **Think-Pair-Share** is a cooperative learning strategy that can promote and support higher level thinking. The teacher asks students to think about a math problem or question independently before pairing with another student to discuss their own thinking, and then sharing their ideas with the group.

Monitoring student work over a number of days allows the teacher to notice whether specific students need challenge or support. That is why documentation is so critical in a learner-centred classroom. Observations and assessment notes will inform day-to-day-planning, grouping, and conferences to provide appropriate and timely instruction.

To document formative assessment, teachers may select from a variety of tracking templates included at the end of this document. Additional templates can also be found in *Mathematics 7: A Teaching Resource or Mathematics 8: A Teaching Resource.* For example, if students are working on percent problems, the teacher can jot student names under the appropriate heading to indicate level of performance. This template may be as simple as the following heading indicates:

Activity/ assignment Date:	Meeting expectations (strong performance)	Meeting expectations	Not yet meeting expectations

The teacher may not be able to assess each student daily, but after a week or two, patterns of strengths and needs will appear. These patterns allow the teacher to plan appropriately for students who require extensions or support. This data will also inform a teacher's decision about grouping for differentiated instruction.

Differentiated Instruction

"At its most basic level, differentiating instruction means ... that students have multiple options for taking in information, making sense of ideas, and expressing what they learn. In other words, a differentiated classroom provides different avenues to acquiring content, to processing or making sense of ideas, and to developing products so that each student can learn effectively." (Tomlinson, 2001. p. 1)

Learner-centred classrooms benefit from the use of differentiated instruction. Differentiation means offering more than one assignment or activity or an open ended assignment to accommodate various interests and levels of achievement within the class. Students can select ways to learn that are meaningful to them and set at their instructional level. Some students may need teacher guidance in their selection. Differentiation is flexible. It may be as simple as allowing some students more time or the freedom for some students to demonstrate the concept by writing their own problems. Small groups may come together based on their choices of work. Students can move in and out of these groups after a day or several days. Differentiated tasks are designed to respond to different learning styles, interests, levels of thinking and achievement.

In order to differentiate instruction and assessment, it is important to first determine the key understandings for a concept. For example, when teaching percent, the key understandings are:

- A percent is a ratio or a comparison of a number to 100.
- The actual amount that a percent represents is based on the whole of which it is a percent.
- Comparing percents is as easy as comparing whole numbers or decimals.
- Percents are hundredths and are therefore a third way of writing both fractions and decimals. If a student can express common fractions and simple decimals as hundredths, the term "percent " can replace the term "hundredth".
- Percents can be as low as 0 but can exceed 100.
- Sometimes percents are used to describe change
- There are a variety of strategies used to solve percent problems.

Teachers also differentiate instruction and assessment in their classrooms by using information from student assessments and observations to guide their planning. In the Junior High Mathematics Program Assessment administered in 2004 and reported on in the document *Continuing a Coherent Mathematics Program: a Study Document for Educators at the Junior High Level*
http://plans.ednet.ns.ca/files/Program-Results/ContinuingCoherentMath.pdf,
The following questions on percent and student responses give teachers valuable information that can be used to help plan instruction.

#5/pg.14 Estimation:

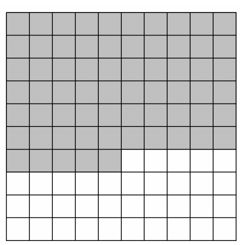
When practising basketball throws, Mary notes that she throws the basketball into the basket 25 times out of 35 tries. Approximately what percent did she throw the basketball into the basket? (Grade 7/SCO B8)

(A) 25 %
(B) 50 %
(C) 75 %
(D) 90%

Approximately 70 percent of the students correctly answered (C). Most students recognized that $\frac{25}{35}$ is more than one-half. A number of students incorrectly selected (D), indicating that they had difficulty recognizing whether $\frac{25}{35}$ was closer to 75 % or 90 %. Using benchmarks such as 0, $\frac{1}{2}$, and 1 as 0%, 50%, and 100 % helps students with their reasoning.

#2/pg.24 Number Concepts and Operations: Selected-Response Questions:

Which of the following is not true about this grid? (Grade 7/ SCO A10)



(A) O.65 is shaded
(B) 0.65 % is shaded
(C) $\frac{13}{20}$ is shaded
(D) 65 % is shaded

Approximately 44 % of the students selected the correct answer (B). Response (C) was selected by more than 33 percent of the students, indicating that they did not recognize that 65 % and $\frac{13}{20}$ represent the same amount.

#5 pg.20 Paper and Pencil Procedure Question:

120% of 43 (Grade 8/SCO A8)

Almost 33 percent of grade 8 students did the computation correctly. Students gave answers that were less than 43, therefore, not attending to the reasonableness of their answer. Students need to ask themselves if their answer makes sense. If 100% of 43 is 43, could 120% be less than 43?

#5 pg.30 Constructed Response Question:

Janet was given a worked problem to solve. She made the calculations shown below in order to find the final answer of $6.68. Examine Janet's calculations and create a reasonable word problem that she might have been solving. (Grade 8/SCO B13)

2 x $3.99 + 5 x $1.89 = $17.43
15% x $17.43 = $2.61

$$\frac{(\$17.43 + \$2.61)}{3} = \$6.68$$

3

This question was marked out of three points. Approximately 26 percent of students received full value. Some of the errors that students made were simply rewriting the problem in words and not giving their answer in the form of a question. Students need practice in writing story problems requiring three-step computations.
(You can find the rubric used to score this in the document)

In the Elementary Mathematics Literacy Assessment (EMLA) written by Grade 6 students in June 2010, students scored less that 50 percent on questions that asked students to

- compare and order fractions using benchmarks of 0, ½ and 1, whether placing numbers on a number line or identifying which number , from a group, was closest to a certain benchmark.
- use percent benchmarks of 100% and 50 %.

The above information is connected to several of the bullets in the key understandings for percent and will help teachers plan instruction.

There are many effective best practices that can be used to differentiate instruction in a mathematics classroom. They include

- providing students with multiple entry points
- using multiple variations of a common task
- using well formulated open-ended questions

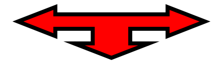
Teachers can use the key understandings for a concept, information from student work, and the best practices for differentiated instruction that follow to help plan assessments, units, and lessons.

An important note is that not all instruction needs to be or even should be differentiated. By varying whole class instruction, small group instruction and individual work, teachers seek a balance that allows all students to be actively engaged in rich mathematical learning.

Differentiation Using Multiple Entry Points

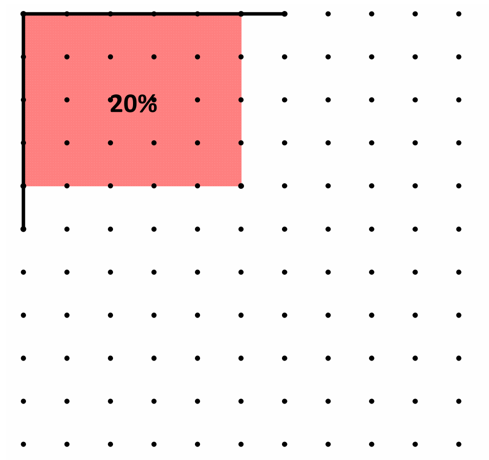
Van de Walle (2006) recommends using multiple entry points, so that all students are able to gain access to a given concept. These entry points are based on the five representations, which are considered to be tangible ways of representing knowledge.

- concrete – materials are used to model a concept
- contextual – situations to engage student interest
- pictorial – diagrams represent a problem or record understanding
- verbal – students talk and write about their learning
- symbolic – students use symbols to record understanding



Multiple entry points—diverse tasks that tap students' particular inclinations and favoured way of representing knowledge.

For example, if students were asked to determine the whole region that the shaded region below is a part of, they might approach it in the following ways:



- Concrete – students might use cube-a-links and model the 20% as 1 cube and bring other cubes into the model until they have 100%. They might see other percents such as 1%, 5%, or 10% and work with the percent they see, bringing in more cubes until they reach 100%. (See note below)
- Contextual– "I wanted to buy a new sweater but the clerk told me that I only had 20 % of the price of the sweater. What is the cost of the sweater?
- Pictorial – students work with the above diagram and extend it to determine the region that represents 100%
- Verbal – Journal Entries could go like these: "If this rectangle represents 20%, I need a rectangle that is 5 times as big as this rectangle" OR they might also say, "I see 20 blocks in the shaded region so each block represents 1%. The entire region would be 100 of these blocks" OR, they might say, "I can subdivide this rectangle into 2 equal parts and each will represent 10% so the whole region needs to be 10 of these sections that is each 10%"
- Symbolic – Depending on the percent they are working with, students might record expressions such as 5 x 20%, 100 x 1%, 10 x 10%, 20 x 5%, 20% + 20% + 20% + 20% + 20% (See note below)

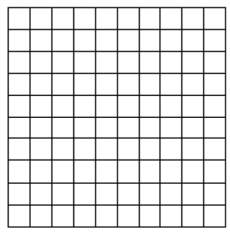
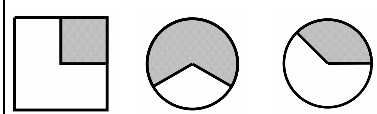
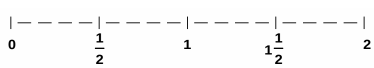
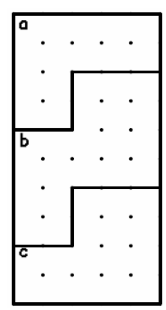
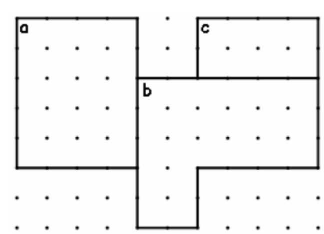
NOTE: While it is correct for students to use additive thinking (20% + 20% + 20% + 20% + 20%) to get the whole region, multiplicative thinking should be encouraged when having a conversation with a student or group of students or in whole group instruction. Use questions such as, “The whole region would be how many times the region given above?” As students move into junior high, they should use more efficient strategies for problem solving.

Tasks that Illustrate Multiple Entry Points:

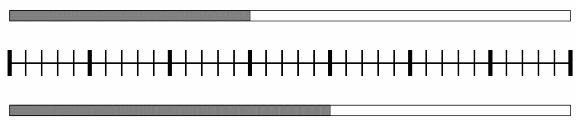
Think about how the following tasks demonstrate multiple entry points for students (tasks are taken from Atlantic Canada mathematics curriculum for Grade 7 and 8, *Mathematics 7: Focus on Understanding* and M*athematics 8: Focus on Understa*nding and other supporting resources). Each of the tasks allows students to explore the concept of percent. Some students may spend a great deal of time on one activity while others explore several entry points. Providing a range of choices allows all students to engage in meaningful learning.

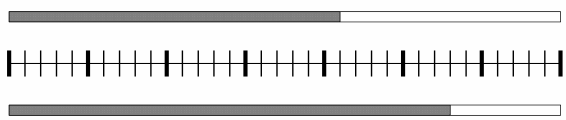
Grade Seven Questions:

1. Complete this story using the numbers given below: There was a sale at the sporting goods store. For one week, everything was _____ % off the original price. Cindy bought some running socks that usually sell for _______ a pair. With the sale price, the price of each pair was reduced by ____, so Cindy paid only _______ per pair. 20 $1.40 $7.00 $5.60 Contextual Verbal Symbolic **Outcome: 7A11/7B10**	2a. Using pattern blocks, create a design in which 60% of the area is yellow. Record your design and explain how you know that 60% of the area is yellow. 2b. Using patterns blocks and fraction blocks, create a design where 30% of the area is pink. Record your design and explain how you know that 30% of the area is yellow. Concrete Pictorial Verbal **Outcome: 7A11/7B8**	3. Have the students draw a design on a hundredths grid (or partially cover a flat) and describe the percentage of the grid covered. Ask further questions such as: How many more squares would you have to shade in (or cover) to cover 80 % of the squares? Pictorial Verbal **Outcome: 7A11/B78**
4. Ask students, Which of the following is the least? The most? Explain your answers. 1/20 20% 0.020 Concrete Pictorial Verbal **Outcome: 7A10/7B8**	5. Ask students to estimate a percent that is a close approximation of each of the following and to indicate why their estimate is larger or smaller than the exact value. $\frac{7}{11}$ 4:9 Symbolic Verbal **Outcome: 7A10/7B8**	6. The highway department is responsible for 600 km of two-lane roads and 300 km of four-lane roads. What percent of the roads are two-lane? Contextual Symbolic **Outcome: 7B8**

7. Using the geo-blocks, create a design where the triangular prism (T11) is 20% of the volume of the design. Record your design and explain how you know that 20% of the volume is the T11 piece. Concrete Pictorial Verbal **Outcome: 7A11/B8**	8. Give a situation where 1% would be a lot and where 1% would be a little. Contextual **Outcome: 7A11**	9. Put > or < or = between the following a) 100% of 30 20% of 1 000 b) 50 % of 2.5 1/10 of 16.3 Symbolic **Outcome: 7B9**
10. Complete a Frayer Model for the word "percent". Verbal Symbolic Pictorial **Outcome: 7A11**	11. Shade in 20% on a hundredth grid 5 different ways?  Pictorial **Outcome: 7A11**	12. Estimate the percent that the shaded portion of each of the following diagrams represents.  Pictorial Symbolic **Outcome: 7A11**
13. Louisa had to put a list of fractions, decimals, and percents in order from least to greatest. Her method was to translate all her numbers into percents and put the percents in order from least to greatest. But she then forgot to write the numbers in their original forms. What might the original numbers have been? 62.5 % < 75 % < 80 % < 100 % Verbal Symbolic **Outcome: 7A10**	14. Place these fractions, decimals, and percents on a number line. Use a different line for each set of numbers. Explain the reasoning of your placement of each number.  25%, 1.55, 2/3, 75% ¼, 1.49, 88%, 45% 0.05, 0.5, 50%, 0.55 Pictorial Verbal Symbolic **Outcome: 7A10**	15. Determine the percent of area each section is of the whole region.   Pictorial Symbolic **Outcome: 7A11/B8**

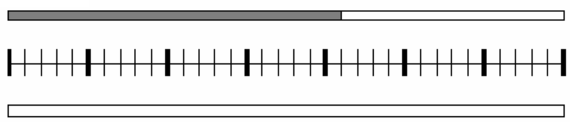
16. a) Tell what percent the top line is of the bottom line.





b) Draw the 100% line on the bottom so that it makes the top line the correct percent.

60%



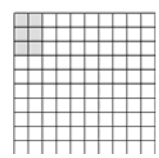
Pictorial
symbolic
Outcome: 7B8

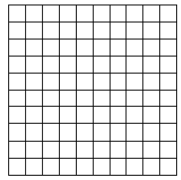
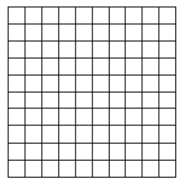
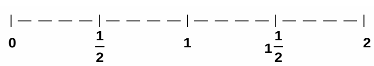
17. a) Use newspapers, advertisements etc. to create percent problems.

b) Keep track of calories consumed over a week or month and create a percent graph that shows the percent in various food groups.

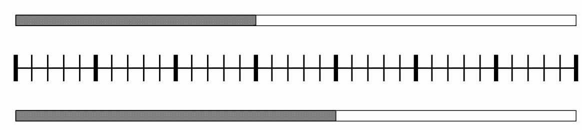
Verbal
Pictorial
Contextual
Outcome: 7B10

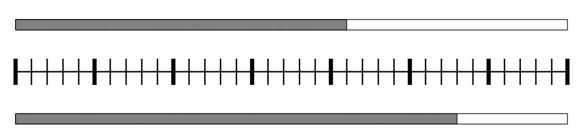
Grade Eight Questions:

1. CD's are on sale at two stores. Future Shop advertises "Buy 1 get 1 free. Wal-mart says "50% off all CDs". Where would you buy your CDs? Explain your reasoning. Contextual Verbal Symbolic **Outcome: 8B3**	2. Zach bought a skateboard for $120. Eight months later he sold it for $75. What percent decrease does the new price represent? Contextual Symbolic **Outcome: 8B4**	3. Draw a design on a 10 x 10 grid (or partially cover a flat) and determine the percentage of the grid covered. How many more squares would you have to shade in (or cover) to increase the area of your design by 110% ? 150%? 300%? Pictorial Verbal **Outcome: 8A8**
4. Jill was asked to shade each of the following numbers on 10 by 10 grids. If one 10 by 10 grid represents one whole or 100%, which of the following numbers would have the least number of squares shaded? The most? Explain your answers. $\frac{3}{2}$ 0.195 120% Concrete Pictorial Verbal **Outcome: 8A8**	5. John's father said "In my youth I could buy a chocolate bar and a pop for 20 cents. What would be a typical cost for these items today? Estimate the percent increase this represents. Symbolic Verbal Contextual **Outcome: 8B4**	6. If the entire grid below represents $300, fill in this percent sentence to represent the shaded region. ___% of $300 = ______  Contextual PIctorial Symbolic **Outcome: 8B3**
7.a. Using geo-blocks, create a design where the triangular prism (T11) is 20% of the volume of the design. Record your design and explain how you know that 20% of the volume is the T11 piece. b. Create a second figure that would represent 120% of your design. Explain how you know this new figure is 120% of your original design. Concrete Pictorial Verbal **Outcome: 8A8**	8. *Gaming Galaxy* purchased 3D Video Game Systems for $129 per unit and is planning to sell them for $195.99. They purchased 150 units. a) What is the percentage of increase (mark-up) per unit? b) How much can *Gaming Galaxy* expect to make if they sell all the units? c) After one month, *Gaming Galaxy* realizes that the 3D Video Game Systems are not selling as fast as they hoped, so they put them on sale for 20% off. i) Will there still be a profit? ii) If they sell 56 units during the sale, how much money will they make on the items sold? Contextual Symbolic **Outcome: 8B4**	9. Put > or < or = between the following a) 110% of 30 3.4% of 1 000 b)125% of 64 140% of 50 Symbolic **Outcome: 8B3**

10. Complete a Frayer Model for the word "percent". Verbal Symbolic Pictorial **Outcome: 8A8**	11. a) If each hundreds grid represents 100%, find three different ways to shade in 120%. .   Pictorial **Outcome: 8 A8**	12. 420 students in the school own a cell phone. If this was 60% of the population, how many students attend the school? b. Explain how you found your answer. Contextual Pictorial Symbolic Verbal **Outcome: 8B3**
13. Jack spent between \$100 and \$200 on a new Snowboard. Kate spent 150% of this amount on her new Snowboard, Explain what you can conclude about the cost of Kate's Snowboard? Verbal Contextual Symbolic **Outcome: 8B3**	14. Place these fractions, decimals, and percents on a number line. Use a different line for each set of numbers. Explain the reasoning of your placement of each number.  125%, 1.55, 2/3, 75% $\frac{5}{4}$, 1.49, 88%, 45% 0.05, 0.5, 150%, 0.55 Pictorial Verbal Symbolic **Outcome: 8A8**	15. Edible Gardens Inc. design and plant rectangular vegetable gardens for their customers. Draw a model of a rectangular garden with dimensions of 12cm by 24 cm on 1 cm grid paper, and calculate the following: a) Find the area and the perimeter of your rectangle. b) Increase the dimensions of your rectangle by 25% and find the new perimeter and area. c) Decrease the dimensions of your original rectangle by 75% and find the new perimeter and area. d) Find the ratio of the new perimeter to original perimeter and the ratio of new area to original area of each of part a) and b). What do you notice? Contextual Pictorial Symbolic **Outcome: 8B4**

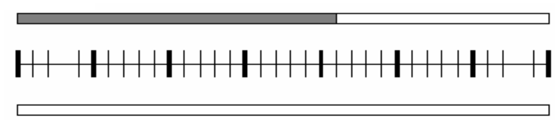
16. a) Tell what percent the bottom line is of the top line.





b) Draw the 70% line on the bottom so that it makes the top line the correct percent.

210%



Pictorial
symbolic
Outcome: 8A8

17. Every month Statistics Canada calculates the Consumer Price Index (CPI), which is a means of measuring changes in retail prices.
Visit www.statcan.gc.ca to research the CPI to determine what it is used for. Choose three examples to discuss.

Verbal
Contextual
Outcome: 8B4

A suggestion for managing student choice of the tasks in the grid:

Introduce some tasks to students being careful to select a range of entry points. Ask students to choose a small number of tasks. Other tasks can be used for reinforcement or assessment tasks.

Differentiation Using a Common Task with Multiple Variations (John Van deWalle) or Parallel Tasks (Marian Small)

Marian Small says “the teacher is faced with groups of students who differ in their cognitive readiness for the mathematics that the teacher is mandated to teach. The teacher has to make difficult program decisions. She or he is expected to expose all students who are not on an individualized program to content that is predetermined by provincial authorities while recognizing that many students are either not ready for or have already learned the content to be delivered. Respecting individual needs is difficult for even the most experienced teacher but is, nonetheless, an important aspect of teaching.” (Small, 2005 p. 2)

> Marian Small uses the term “parallel tasks” to refer to learning opportunities that offer choices to students. When offering parallel tasks, the teacher poses two or more different questions at different levels. The questions are parallel in terms of the big idea to which they relate and their context. This differentiation strategy is similar to a common task with multiple variations.

Tomlinson emphasizes that “in differentiated classrooms, teachers begin where the students are, not the front of the curriculum guide.” (Tomlinson,1999 p.12). Teachers adjust content, process, or product in response to student readiness, interest, or learning profile. This differentiation will not be needed all the time; however, teachers select differentiation in response to student need. One suggestion Van de Walle makes about differentiation is to take a common problem-solving task, and adjust it for different levels (Van de Walle, 2006 p. 29). He finds that students tend to select the numbers that are challenging enough for them while giving them the chance to be successful in finding a solution.

For example, several sets of numbers can be inserted in a computation problem or task. The sets of numbers should reflect a range of challenge for students from less difficult to more difficult. Students are taught to choose the numbers that give them the most challenge without being too complicated. The benefits are that this is manageable for the teacher and all students feel that they have worked on the same task. The students have practiced learning the same concept, but they have applied their level of problem-solving skills to different numbers. Students are also encouraged to use more than one set of numbers to solve the problem, thereby increasing their understanding and comfort with the concepts.

Grade 7 Examples :

Example A:
Grade 7 / SCO A10: illustrate, explain and express ratios, fractions, decimals, and percents in alternate forms.

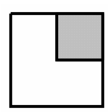
Use only the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 exactly one time each to create numbers that fit this ordering: fraction < decimal < ratio < percent < whole number

Variations could include:

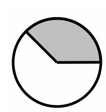
a) not having to use every number
b) allowing repetition of number
c) using a smaller set of numbers and making the ordering smaller, such as fraction < decimal< percent

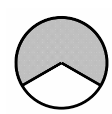
Example B:
Grade 7 / SCO A11 demonstrate number sense for percent

Estimate the percent that the shaded portion of each of the following diagrams represents. Explain your reasoning.



This question can start with a simple visual representation (the square shown above) and move to more difficult representations (the 2 circles below).





Example C:
Grade 7 / SCO B8 – estimate and determine percent when given the part and the whole

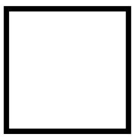
Investigate percent in surface area using Power Polygons or Pattern Blocks/Fraction Blocks. Join two or more Power Polygons or Pattern Blocks/Fraction Blocks to form a new convex polygon and determine or estimate the percent of the total area represented by each piece. Trace on regular or dot paper (variation). Underneath the drawings, write the percent that would describe how the area of each piece compares to the area of the new polygon (part to whole). Do this for at least four pairs of polygons.
When using the Power Polygons ask for an estimation of the percent that would describe how the area of each piece compares to the area of the new polygon (part to whole).
When using the Pattern Blocks/Fraction Blocks ask for a calculation of the percent that would describe how the area of each piece compares to the area of the new polygon (part to whole).

Grade 8 Examples:

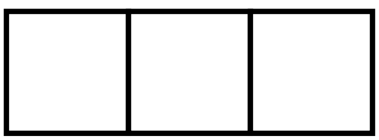
Example A:

Grade 8/SCO A8: represent and apply fractional percents, and percents greater than 100, in fraction or decimal forms, and vice versa

Task A: The shape below represents 100% of John`s garden. Draw diagrams to show 50%, 125%, and 200% of John`s garden.



Task B: The shape below represents 100% of John`s garden. Draw diagrams to show 50%, 150%, and 200% of John`s garden.



Variations could include:

d) changing the shape of the garden.
e) changing the percents in the question.
f) providing grid paper.

Example B:

Grade 8/SCO B3: create and solve problems which involve finding a, b, or c in the relationship a% of b = c, using estimation and calculation

Task A: 75% of a number is between 40 and 50. What could the number be?
Task B: 150% of a number is between 40 and 50. What could the number be?

Example C:

Grade 8/SCO B4: apply percentage increase and decrease in problem situations

Task A: The original package of M & M's contains 124 grams of candy. The newest package reads "***Now 25% More".*** How many grams are in the new package of M & M's?
Task B: The original package of M & M's contains 124 grams of candy. The newest package reads "***Now 50% More".*** How many grams are in the new package of M & M's?

When creating questions, it is important to consider prior knowledge and experiences of the students. Use of student names in problems as well as activities that are meaningful to them will generate interest in working towards a solution.

Differentiation Using Open-ended Questions

Open-ended questions are questions that require a student to think more deeply and to give a response that involves more than recalling a fact or reproducing a skill which has been previously taught. Open-ended questions have more than one acceptable answer and can be approached by more than one way of thinking.

Van de Walle (2006) suggests that a problem-based approach to teaching will reach most students in a classroom. Well designed open-ended questions provide most students with an obtainable yet challenging task. Open-ended questions allow for differentiation of product. Products vary in quantity and complexity depending on the student's understanding. By generating possible solutions, all students are able to demonstrate their mathematical thinking.

Some mathematical problems are called closed questions because they call for a single answer with limited reflection. For example, the following question is an example of a closed question.

What is 10% of 62?

However, teachers want students to do more than recall facts. They want them to reflect, engage in challenging dialogue, grapple with mathematical concepts, and learn from one another. Consider re-phrasing the previous question to this:

Explain a situation where 10% is a little and another situation where 10% is a lot.

Open-Ended Questions:

- should elicit a range of responses
- requires the student not just to give an answer, but to explain why the answer makes sense
- may allow students to communicate their understanding of connections across mathematical topics
- should be accessible to most students and offer students an opportunity to engage in the problem-solving process
- should draw students to think deeply about a concept and to select strategies or procedures that make sense to them
- can create an open invitation for interest-based student work

When we think about the questions we ask our students, we need to consider the following:

- the mathematical goals of the lesson
- the misconceptions that students might have about a concept
- the connections to be made to prior knowledge
- how the understanding will be assessed

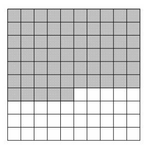
When planning the questions we will ask our students, teachers need to

- understand the mathematics embedded in the question
- present the question clearly using accessible language
- set clear and reasonable expectations for students and their work
- allow for individual approaches, methods, and/or answers
- add variety or more data to a question to make it more accessible to all students
- understand and demonstrate the correct usage of manipulatives

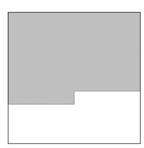
Grade 7 Examples :

A) Grade 7 / SCO A11 demonstrate number sense for percent

Closed Question: What percent of this grid is shaded in?



Open Question: Estimate the percent of this grid that is shaded in. Explain your reasoning.



B) Grade 7 / SCO B8 – estimate and determine percent when given the part and the whole

Closed question**:**

Raymond broke his chocolate bar into 8 pieces, and gave 2 to each of his twin sisters. What percent of the bar does he still have?

Open question:

Create a situation in which a part is about 50% of the whole.

Grade 8 Examples:

A) Grade 8/SCO A8: represent and apply fractional percents, and percents greater than 100, in fraction or decimal forms, and vice versa

Closed Question: Tell which is greater and why? (assume the whole is the same)

125% or $\frac{6}{5}$

Open Question: Choose a fraction greater than 1 and a percent greater than 100%. Tell which is greater and how you know. (assume the whole is the same)

B) Grade 8/SCO B3: create and solve problems which involve finding a, b, or c in the relationship a% of b = c, using estimation and calculation

Closed question:

40 is ____% of 800

Open question:

Fill in values for the blanks to make this statement true:

40 is ______% of ______

Variations can include:

- the number 40 can be changed
- ask a different question such as: ____ is ___% of 200, or ____ is 20% of _____

C/ Grade 8/SCO B4: apply percentage increase and decrease in problem situations

Closed Question:
Last year an I-Pod was selling for $125 at Walmart. This year the price was lowered to $100. What percent decrease does the new price represent?

Open Question:
Last year an I-Pod was selling for $______ at Walmart. This year the price was lowered to $______. What percent decrease does the new price represent?
Variations could include:

- having only one blank and giving one of the numbers
- having a blank for either decrease or increase

Closed questions can be adjusted to become an open-ended question. Teachers can also generate open-ended questions based on the interests of the students or connected to the current class topic. It is important to plan these questions ahead of class time. In *Good Questions for Math Teaching: Why Ask Them and What to Ask (K-6)*, Sullivan and Lilburn suggest two methods for this process:

Method 1: Working Backward
1. Identify a topic.
2. Think of a closed question and write down the answer.
3. Make up an open question that includes (or addresses) the answer.

Example:
1. Percent
2. 40 % of 20 =
3. Complete this sentence to make a true statement. ___% of ____ = 8.

Method 2: Adjusting an Existing Question
1. Identify a topic.
2. Think of a typical question.
3. Adjust it to make an open question.

Example:
1. Percent
2. Jeri spent 70% of her savings of $250. How much did she spend??
3. Show three different ways you can find 705 of $250. Which way do you prefer and why?

Open-ended questions engage students on many levels, have multiple possible solution paths, lend themselves to a variety of representations and connect several mathematical ideas. Open-ended questions stimulate higher level thinking skills and problem-solving. They are particularly appropriate for mixed-ability classes.

Appendix A Tracking Templates

A-1 Whole Class Sorter
A-2 Unit Planning: Identifying Student Needs
A-3 Unit Planner
A-4 Class Jot Notes
A-5 General Checklists
A-6 Focused Template for Individual Student
A-7 Sample Tracking for Independent Work

Appendix A-1

Whole Class Sorter

Activity/assignment Date:	Meeting expectations (strong performance)	Meeting expectations	Not yet meeting expectations

Appendix A-2

Unit Planning: Identifying Student Needs

<table>
<tr><td>Unit:</td><td>Total Weeks:</td><td>From:</td><td>To:</td></tr>
<tr><td colspan="4">Outcomes:</td></tr>
<tr><td colspan="4">Key Concepts:</td></tr>
<tr><td colspan="2">Students who may require support:</td><td colspan="2">Specific details:</td></tr>
<tr><td colspan="2"></td><td colspan="2"></td></tr>
<tr><td colspan="2"></td><td colspan="2"></td></tr>
<tr><td colspan="2">Students who may require extensions:</td><td colspan="2">Specific details:</td></tr>
<tr><td colspan="2"></td><td colspan="2"></td></tr>
</table>

Appendix A-3

Unit Planner*

Unit:

Assessment Tool/Evidence	Outcome(s)

Support Codes: 1. individual instruction 2. guided group 3. choice task 4. open-ended question 5. Reduced quantity

Name	Support	Outcome Achievement	Comments
		consistently / usually sometimes or with some support with full support	
		consistently / usually sometimes or with some support with full support	
		consistently / usually sometimes or with some support with full support	
		consistently / usually sometimes or with some support with full support	
		consistently / usually sometimes or with some support with full support	
		consistently / usually sometimes or with some support with full support	
		consistently / usually sometimes or with some support with full support	
		consistently / usually sometimes or with some support with full support	
		consistently / usually sometimes or with some support with full support	
		consistently / usually sometimes or with some support with full support	
		consistently / usually sometimes or with some support with full support	
		consistently / usually sometimes or with some support with full support	
		consistently / usually sometimes or with some support with full support	

*Based on a model developed by Amy Doucette, École St. Catherine's School

Appendix A-4

Class Jot Notes

(Record student names at the top of each box before copying.)

Appendix A-5

General Checklists

Outcomes to be addressed:

Name:										

Appendix A-6

Focused Template for Individual Student:

Name: Differentiation planned to support student:				
Outcomes:	Not yet	Meeting	Meeting expectations (Strong Performance)	Comments
•				
•				
•				
•				

Appendix A-7

Sample Tracking Form for Student's Independent Work:

Date	**Activity**	**Date Completed**	**Problem(s) Encountered**	**Independent strategy used to Solve Problem(s)**

Bibliography

Dodick, Paula, Karam, Deborah, and Maas, Daniel. *Roads to Reasoning: Developing Thinking Skills through Problem Solving, Grade Six.* Whitby, Ont: McGraw-Hill Ryerson, 2001.

Erickson, Sheldon. *Proportional Reasoning*. Fresno, Ca. AIMS Education Foundation. 2000.

Nova Scotia Department of Education. *Atlantic Canada Mathematics Curriculum: Grades 4-6*. Halifax, NS, 1999.

Nova Scotia Department of Education. *Atlantic Canada Mathematics Curriculum: Grade 7*. Halifax, NS, 1998.

Nova Scotia Department of Education. *Mathematics Grade 7: A Teaching Resource*. Halifax, NS, revised 2010.

Nova Scotia Department of Education. *Continuing a Coherent Mathematics Program: a Study Document for Educators at the Junior High Level.* Halifax, 2006.

Principles and Standards for School Mathematics. Reston, VA: The National Council of Teachers of Mathematics, Inc, 2000.

Small, Dr. Marian. *Big Ideas from Dr. Small Creating a Comfort Zone for Teaching Mathematics Grades 4-8*. Toronto: Nelson Education Ltd., 2009.

Small, Dr. Marian. *Good Questions: Great Ways to Differentiate Mathematics Instruction.* Toronto, Ontario: Nelson, Thomson Canada Ltd, 2009.

Small, Dr. Marian. *Making Math Meaningful to Canadian Students, K-8*. Upper Saddle River: Nelson Education Limited, 2008.

Schuster, Lainie, and Anderson, Nancy. *Good Questions for Math Teaching : Why Ask Them and What to Ask, Grades 5-8*. Minneapolis: Math Solutions Publications, 2002.

Tomlinson, Carol A. *How to Differentiate Instruction in Mixed Ability Classrooms*. Virginia, USA: Association for Supervision and Curriculum Development, 2001.

Van de Walle, John A. *Elementary and Middle School Mathematics : Teaching Developmentally.* Boston: Pearson Education, 2004.

Van De Walle, John A., and Louann H. Lovin. *Teaching Student-Centered Mathematics: Grades 5-8*. Boston, USA: Pearson Education Inc, 2005.