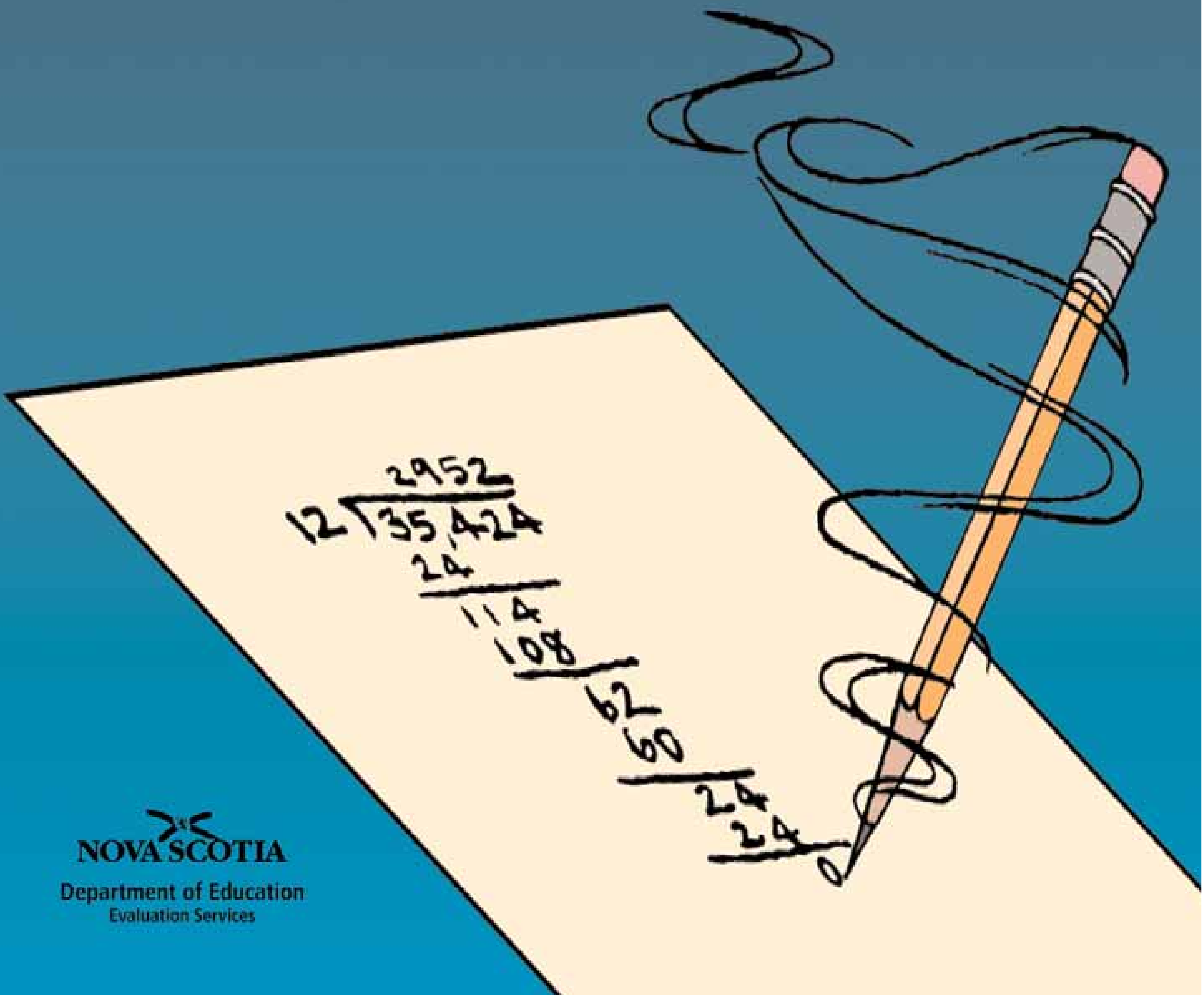
A Study Guide for Students

GRADE 12

PREPARING FOR

Nova Scotia
Examinations in Mathematics



NOVA SCOTIA
Department of Education
Evaluation Services

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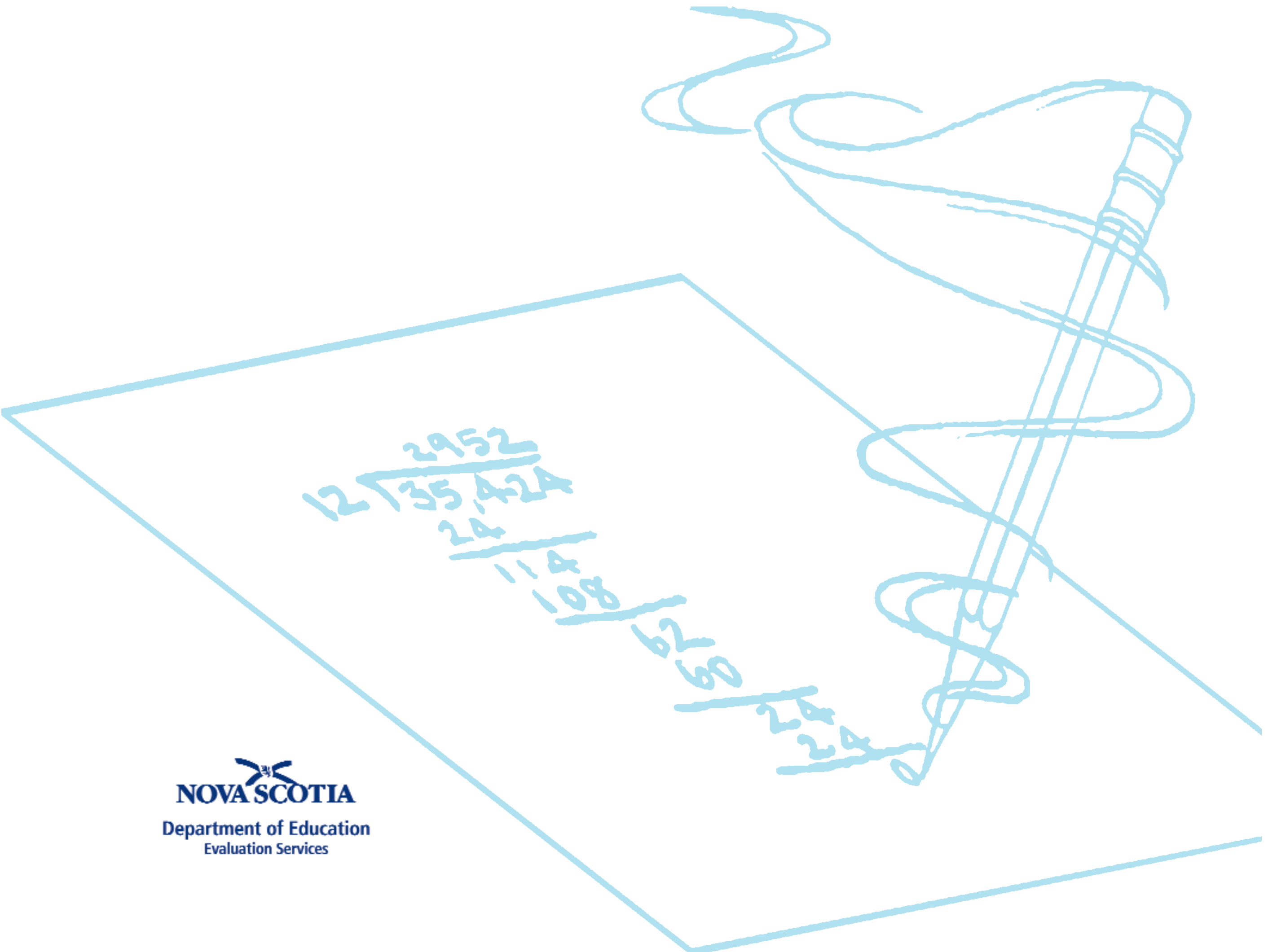

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A Study Guide for Grade 12 Students – Preparing for Nova Scotia Examinations in Mathematics (2005)

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Introduction

You may not realize it, but math is a big part of our everyday lives. Spending money, making movies and music, building houses, getting medical treatments – these things all involve math. In fact, much of what we do and many of the things we use are only possible because of someone's understanding of math. That's why it's so important for everyone to have basic math skills. The math skills you learn in elementary, junior high, and high school can open many doors in your future. They will help you in further studies, in the workplace, and at home.

As a grade 12 math student, you've spent many years learning math skills and concepts. In the upcoming provincial math exams, you will have an opportunity to show what you've learned. The provincial exam will test you on all of the math principles and concepts you learned in the Mathematics 12 or Advanced Mathematics 12 course.

This Study Guide is meant to help you prepare for the provincial exam. It's a summary of the information you learned in class this year, and it outlines what you are expected to know by the end of the course. It's divided into the four units you've been studying: Quadratics, Exponential Growth, Circle Geometry, and Probability.

In this guide you will find...

- curriculum outcomes for each unit
- math concepts you should know
- examples of questions along with possible solutions
- questions for you to try on your own

As well as possible solutions to sample questions, we have also included examples of how your thinking process might work to help you choose suitable methods to solve a given problem.

As you prepare for this exam, your motto should be "practice, practice, and more practice." You will have to work hard — like studying for *any* math exam, this means reviewing and studying course material and doing plenty of practice questions. We also recommend that you talk about math. Talk to your teachers and your classmates about math concepts and solutions to math questions or problems.

While this Study Guide is a good start, you should also use your math textbook, class notes, and any tests or assignments to get ready for the exam.

Being prepared is the most important step toward success. **Good luck with the exam!**

Study Tips

FOR LEARNING MATHEMATICS

Nova Scotia students are expected to learn mathematics "with understanding." Learning with understanding means being able to apply concepts, procedures, and processes in the right places.

Often, developing understanding of a subject requires effort. What can you do to help yourself learn mathematics? Here are some ideas:

1) Be an active participant in class.

2) Do your homework and assignments. Don't get behind in your work.

3) Manage your time wisely.

4) Prioritize your activities—your education comes FIRST.

5) Keep a complete and organized set of notes. Remember that your textbook is also a resource for you to read.

6) Reflect on your learning by reviewing material that you have previously learned.

7) Prepare for tests and exams many days in advance.

And how about when you're actually writing the exam? Here are a few ideas that can help:

1) Scan the entire exam.

2) Do the questions you consider to be routine or easy first. Manage your time wisely, making sure you don't spend too much time on any one question.

3) If the method to solve a problem is not specified, use the most efficient one.

4) Show all required work clearly. (Refer to the Study Guide for examples of complete solutions.)

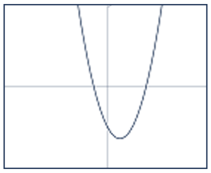
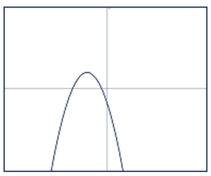
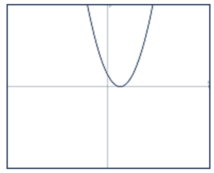
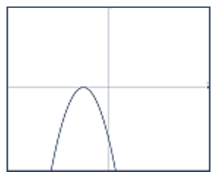
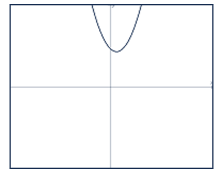
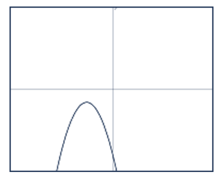
5) When finished, check your work.

Quadratics

THINGS TO REMEMBER

If $y = ax^2 + bx + c$

where a, b and c are real numbers and $a \neq 0$

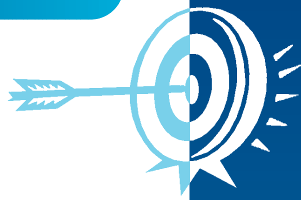
If $b^2 - 4ac > 0$ then either	If $b^2 - 4ac = 0$ then either	If $b^2 - 4ac < 0$ then either
 **OR** 	 **OR** 	 **OR** 
The function has 2 real zeros therefore 2 distinct x-intercepts. The equation $ax^2 + bx + c = 0$ has two real roots.	The function has a double real zero and therefore one x-intercept. The equation $ax^2 + bx + c = 0$ has a double real root.	The function has two non-real zeros and therefore no x-intercepts. The equation $ax^2 + bx + c = 0$ has two non-real roots.
Zeros of the function and the roots of the equation are exactly: $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$ If $b^2 - 4ac$ is a perfect square an exact rational answer is required. If $b^2 - 4ac$ is not a perfect square a decimal approximation is possible.	Zeros of the function and the roots of the equation are exactly: $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$ An exact rational answer is obtained.	Zeros of the function and the roots of the equation are exactly: $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$ Answers will be complex or imaginary and should be expressed as $x = a \pm bi$ (where $i = \sqrt{-1}$)
Vertex at $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$	Vertex at $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$	Vertex at $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$
Transformational form is: $\frac{1}{a}\left(y - f\left(-\frac{b}{2a}\right)\right) = \left(x + \frac{b}{2a}\right)^2$	Transformational form is: $\frac{1}{a}\left(y - f\left(-\frac{b}{2a}\right)\right) = \left(x + \frac{b}{2a}\right)^2$	Transformational form is: $\frac{1}{a}\left(y - f\left(-\frac{b}{2a}\right)\right) = \left(x + \frac{b}{2a}\right)^2$
Axis of symmetry is $x = -\frac{b}{2a}$	Axis of symmetry is $x = -\frac{b}{2a}$	Axis of symmetry is $x = -\frac{b}{2a}$

Quadratics

SEQUENCES

PAGES **2** TO **11** IN THE TEXT

Outcomes



I am expected to...

- **A7** describe and interpret domains and ranges using set notation
- **C4** demonstrate an understanding of patterns that are arithmetic, power, and geometric and relate them to corresponding functions
- **C29** analyse tables and graphs to distinguish between linear, quadratic, and exponential relationships

What do I HAVE to know?

- What is a sequence? **[C4]**
- How could I model a sequence? **[C4]**
- How do I represent a sequence in a table? **[C4]**
- When is a graph discrete? How do I express the domain and range? **[A7, C4]**
- What is a power sequence? **[C4]**
- What is an arithmetic sequence? **[C4]**
- How do I identify if a power sequence is linear, quadratic, or cubic? **[C4, C29]**
- How could I model linear and quadratic functions? **[C4, C29]**
- Do I know when to use the formula $t_n = t_1 + (n - 1)d$? **[C4, C29]**
- Can I explain why this formula generates a sequence that is arithmetic? **[C4]**
- What conclusion can I reach from finding the differences called D_1, D_2, and D_3? **[C4, C29]**

What MIGHT it look like...
on the provincial exam?

EXAMPLE 1

Which of the following sequences could be generated by a quadratic function?

A. {1, 2, 3, 4, ...}   B. {-5, -3, 3, 13, ...}
C. {2, 4, 8, 12, ...}   D. {2, 6, 18, 54, ...}

Ways of *Thinking* about Solutions

How do I use lists or tables to tell if a sequence can be generated by a quadratic function?

I should think about common differences.

If the second-level difference is constant, we know we have a quadratic.

A. 1 2 3 4
1 1 1

B. -5 -3 3 13
2 6 10
4 4
Yes

So, the answer is B.

Can I DO these on my own?

1. Which of the following sequences could be generated by a quadratic function?

A. {2, 4, 6, 8, ...}   B. {1, 4, 16, 64, ...}
C. {2, 5, 8, 12 ...}   D. {2, 8, 18, 32, ...}

2. From the text, try the following questions:
Pages 5 and 6 (questions 12 and 13)
Page 72 (questions 1–3)

Quadratics

USING GRAPHING TECHNOLOGY

PAGES **15** TO **22** IN THE TEXT

Outcomes

I am expected to...

- **A7** describe and interpret domains and ranges using set notation
- **C1** model real-world phenomena using quadratic functions
- **C3** sketch tables and graphs from descriptions and collected data
- **C8** describe and translate between graphical, tabular, written, and symbolic representations of quadratic relationships
- **C29** analyse tables and graphs to distinguish between linear, quadratic, and exponential relationships
- **C23** solve problems involving quadratic equations
- **F1** analyse, determine, and apply scatter plots and determine the equations for curves of best fit, using appropriate technology

What do I HAVE to know?

- How do I enter data into my graphing calculator? **[C1, F1]**
- How do I determine the equation of the curve of best fit for my data? **[C1, F1]**
- What work should I show when I am using a graphing calculator? **[C3]**
- Can I state the domain and range of a quadratic function? **[A7]**
- Can I determine values from a graph or an equation of a function? **[C8, C23, F1]**
- By examining a graph, can I determine whether it is linear, quadratic, or exponential? **[C8, C23]**
- From a graph can I determine the maximum or minimum values? **[C8, C23]**
- How do I know when to use the vertex of a parabola to solve a problem? **[C1, C23, F1]**

Quadratics

USING GRAPHING TECHNOLOGY

- What information do the x–coordinate and y–coordinate of the vertex provide? **[F1]**
- How do I know, given the graph or the equation of a quadratic function, if it has a maximum or minimum value? **[C8, C29, C23]**
- How do I use the "maximum" or "minimum" feature on the graphing calculator to determine the coordinates for the vertex? **[C8, C23]**

What MIGHT it look like... on the provincial exam?

EXAMPLE 1

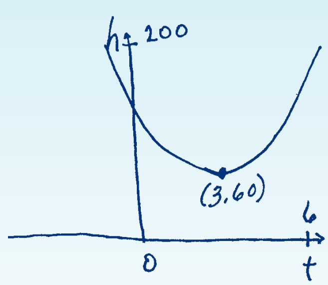
At the Halifax Airshow, a plane performs a power dive. The equation $h = 10t^2 - 60t + 150$ expresses the relationship between height, h, in metres, and time, t, in seconds during the dive.

(a) What is the minimum height that the plane reaches during the dive?

(b) When will the plane be at a height of 100 m during the dive?

Ways of Thinking about Solutions

For part **(a)**, what is a minimum value, and where do I find it on the graph?
How do I use my calculator to graph the given function and find the minimum value?
For part **(b)**, how do I use my table feature to evaluate the function for $h = 100$?"



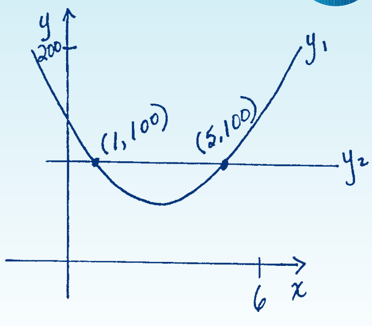
a) The plane reaches a minimum height of 60 m.

b) Using the table feature

x	y_1
⋮	
1	100
⋮	
5	100

∴ the plane will be at a height of 100 m after 1 second, and again after 5 seconds.

or



Graph $y_1 = 10x^2 - 60x + 150$
and $y_2 = 100$
Read the x-values of the intersection points.

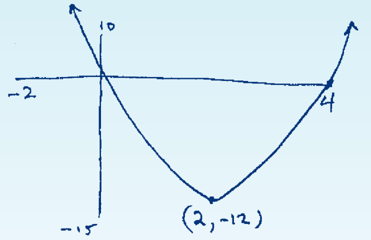
EXAMPLE 2

The function $y = 3x^2 - 12x$ has

A. a minimum value of –12
B. a minimum value of 2
C. a maximum value of –12
D. a maximum value of –2

Ways of Thinking about Solutions

How do I know from the equation if I have a maximum or minimum value?
Looking at the coefficient of x^2 indicates whether I have a maximum or minimum value.
The graph shows that it is a minimum because it opens upward.



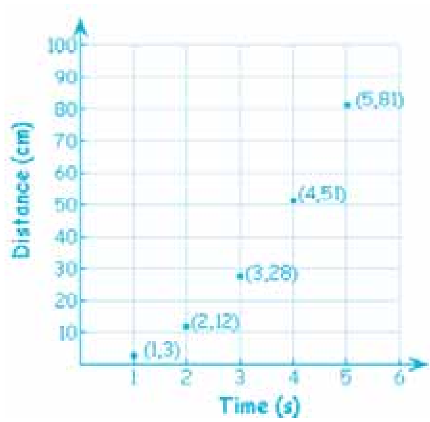
Using the minimum feature on the calculator (2nd, CALC, 3) the minimum value is -12, so A is correct.

or

I see on the graph that the minimum value must be negative, so A is the right answer.

EXAMPLE 3

A ball is released and rolls down an inclined plane. The distance it travels with respect to time since release is recorded in the following scatter plot:



It was determined that a quadratic function would best represent this data set.

(a) Using your graphing calculator, do a quadratic regression and fill in the values for *a*, *b*, *c*, and R^2 that you obtained on your graphing calculator.

QuadReg
$y=ax^2+bx+c$
a=
b=
c=
R^2=

The quadratic function is ______________________________.

(b) What is the significance of the R^2 value obtained?

(c) Given the ordered pair (6, ?), determine the missing coordinate. What does this ordered pair represent in the context of the given problem?

Ways of Thinking about Solutions

How do I enter data into my graphing calculator?

How do I use the calculator to graph the curve of best fit?

a) Enter the data into List1 and List2, then use quadReg L_1, L_2

$a = 3.5$
$b = -1.5$
$c = 1$
$R^2 = 1$

so, the equation is $y = 3.5x^2 - 1.5x + 1$

b) A value of 1 for R^2 means that the equation fits the data perfectly.

To answer part (c), how do I use either the equation obtained, the graph of the function, or its table to determine the *y–value* when $x = 6$?

c) $y(6) = 3.5(6)^2 - 1.5(6) + 1 = 118$, so $(6, 118)$.

This coordinate tells us that the ball will have travelled 118 cm in 6 sec.

Can I DO these on my own?

1. A batter hits a ball, and its height, h, in metres, with respect to time, t, in seconds, is expressed by the function $h = -5t^2 + 10t + 1$. What is the maximum height of the ball and what is the time required for the ball to reach its maximum height?

2. From the text, try questions 5 and 6 on page 16 and question 5 on page 72.

Quadratics

TRANSFORMATIONS

PAGES **24** TO **31** IN THE TEXT

Outcomes

I am expected to...

A7 describe and interpret domains and ranges using set notation

B1 demonstrate an understanding of the relationships that exist between arithmetic operations and the operations used when solving equations

C8 describe and translate between graphical, tabular, written, and symbolic representations of quadratic relationships

C9 translate between different forms of quadratic equations

C31 analyse and describe the characteristics of quadratic functions

C32 demonstrate an understanding of how the parameter changes affect the graphs of quadratic functions

What do I HAVE to know?

- How can I change a quadratic function between transformational form, standard form, and general form? **[B1, C9]**
- Can I identify the transformations on $y = x^2$ when the function is in either transformational form or in standard form? **[C31, C32]**
- Can I determine the vertex from an equation of a quadratic function? **[C31, C32]**
- Can I determine the domain and range from a graph? **[A7]**
- What is an axis of symmetry and how is it related to the vertex? **[C31]**
- Can I express the transformations as a mapping rule? Can I write an equation or sketch a graph from a mapping rule? **[C8, C31, C32]**
- How do I use a graph to determine the transformations? **[C31, C32]**

Quadratics

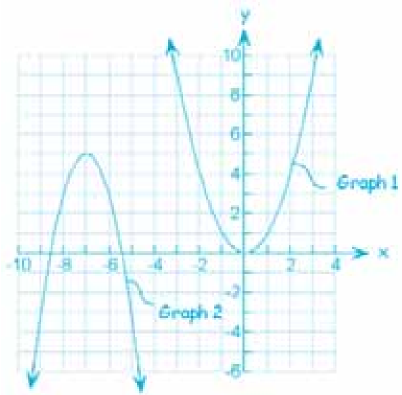
TRANSFORMATIONS

- How do I determine an equation of a quadratic function given its graph? **[C8, C32]**
- Do I know the significance of the *y*-intercept? **[C31, C32]**
- What effect will changing the value of *a* or *c* in the function $y = ax^2 + bx + c$ have on the graph? **[C32]**

What MIGHT it look like... on the provincial exam?

EXAMPLE 1

What mapping rule describes the transformation of graph 1 onto graph 2?



A. $(x, y) \to (x + 5, -\frac{1}{2}y - 7)$

B. $(x, y) \to (x - 7, -\frac{1}{2}y - 5)$

C. $(x, y) \to (x + 5, -2y + 7)$

D. $(x, y) \to (x - 7, -2y + 5)$

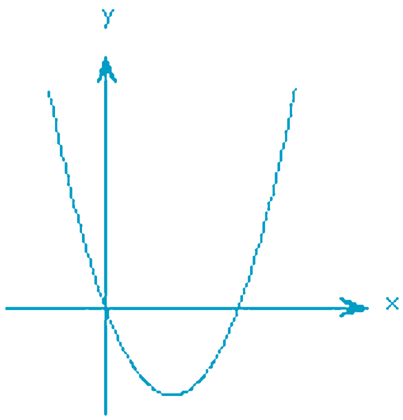
Ways of Thinking about Solutions

How do I use the graph to describe the transformations in a mapping rule?

The vertex has moved from (0,0) to (-7,5), therefore D is the only possible answer.
I need to check if D includes a reflection and a stretch of 2. I see -2 as the coefficent of the "y" in the mapping rule, ∴ D is correct

EXAMPLE 2

The function $y = ax^2 + bx + c$ is represented by the following graph:



Which one of the following statements is true?

A. $c > 0$

B. $c = 0$

C. $c < 0$

D. c is an imaginary number

Ways of Thinking about Solutions

What does the value of c in the function $y = ax^2 + bx + c$ represent, and how do I determine the *y–intercept* of the function?

'C' is the y-intercept value, which on this graph is zero, so B. c = 0, must be the correct answer.

EXAMPLE 3

A parabola has a minimum value at its vertex (1, 3). Which one of the following statements describes the domain and range of the function represented by this parabola?

A. $\{x \in \mathbb{R}\}$ *and* $\{y \in \mathbb{R}\}$

B. $\{x \in \mathbb{R} \mid x \leqslant 1\}$ *and* $\{y \in \mathbb{R}\}$

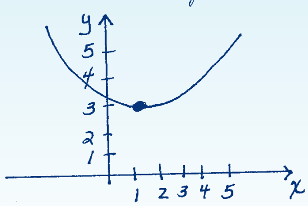
C. $\{x \in \mathbb{R} \mid x \geqslant 1\}$ *and* $\{y \in \mathbb{R} \mid y \geqslant 3\}$

D. $\{x \in \mathbb{R}\}$ *and* $\{y \in \mathbb{R} \mid y \geqslant 3\}$

Ways of Thinking about Solutions

Is this graph opening upward or downward? How can I determine the domain and range from a graph?

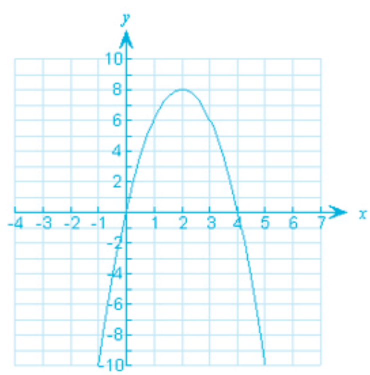
If there is a vertex at (1, 3) and it represents a minimum, then the graph looks something like this



So the range is:
$y \geq 3 \quad y \in R$
The domain is $x \in R$
$\therefore$ D must be the right answer

EXAMPLE 4

A quadratic function is represented by this graph:



(a) Write the equation of the quadratic function (in general form).

(b) State the domain and range.

Ways of *Thinking* about Solutions

How do I determine an equation of a function, given a graph, and how do I write it in general form?

a) The pattern from the vertex for $y = x^2$ is over 1, up 1, over 2, up 4.
From the graph, I can see that the pattern is over 1, down 2; over 2, down 8, therefore a vertical stretch of 2 ($a = 2$) and a reflection in the x-axis (parabola opening downwards).

$$-\tfrac{1}{2}(y-8) = (x-2)^2$$

So in general form:
$$y - 8 = -2(x^2 - 4x + 4)$$
$$y = -2x^2 + 8x - 8 + 8$$
$$y = -2x^2 + 8x.$$

or

The vertex is at $(2, 8)$
so $\frac{1}{a}(y-8) = (x-2)^2$

To determine the value of "a", substitute a coordinate point from the graph into the equation .... $(1, 6)$ is on the graph....

$$\frac{1}{a}(6-8) = (1-2)^2$$
$$-\frac{2}{a} = 1$$
$$-2 = a$$
$$\therefore \frac{-1}{2}(y-8) = (x-2)^2$$

So in general form:
$$y - 8 = -2(x^2 - 4x + 4)$$
$$y = -2x^2 + 8x - 8 + 8$$
$$y = -2x^2 + 8x$$

or

I could have used the graphing calculator:
- determine at least 3 points from graph
- enter into lists
- use quadReg.

For part (b), how can I express the domain and range?

Domain : $\{x \in R\}$
Range : $\{y \in R \mid y \leq 8\}$

or

Domain : $x \in (-\infty, \infty)$
Range : $y \in (-\infty, 8]$

EXAMPLE 5

A batter hits a ball, and its height, *h*, in metres, with respect to time, *t*, in seconds, is expressed by the function $h = -5t^2 + 10t + 1$. What was the initial height of the ball when it was hit?

Ways of Thinking about Solutions

Do I know that the *y–intercept* is the initial position of a projectile, and how do I determine the *y*–intercept from the function?

The y-intercept is (0,1), so the initial height was 1 m.

or

$h(0) = -5(0)^2 + 10(0) + 1$
$= 1$
$\therefore$ the initial height was 1 m.

6

EXAMPLE 6

Which one of the following is correct? The graph of the function $y = (x + 7)^2 + 4$ is the image of $y = x^2$ after

A. a horizontal translation of 7 and a vertical translation of 4
B. a horizontal translation of -7 and a vertical translation of 4
C. a horizontal translation of 7 and a vertical translation of -4
D. a horizontal translation of -7 and a vertical translation of -4

Ways of *Thinking* about Solutions

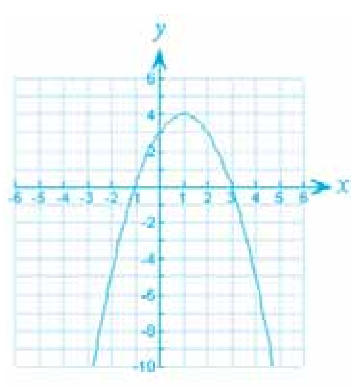
How can I determine the transformations on $y = x^2$ when the equation of the function is in standard form?

$y = (x+7)^2 + 4$
horizontal translation of -7
vertical translation of 4
$\therefore$ B is correct

7

EXAMPLE 7

Which quadratic function best represents this graph? (a, h, and k are positive real numbers)



A. $y = -a\,(x - h)^2 + k$
B. $y = -a\,(x + h)^2 - k$
C. $y = a\,(x + h)^2 - k$
D. $y = a\,(x - h)^2 + k$

Quadratics

TRANSFORMATIONS

Ways of Thinking about Solutions

Given a graph, how do I determine the equation of the function, and how can I see the transformations of $y = x^2$ when the equation is in standard form?

The graph opens downward so the coefficient of x^2, "a" must be negative. So either A or B.
The vertex is (+, +), so A must be correct.

$y = -a(x-h)^2 + k$

opens downward

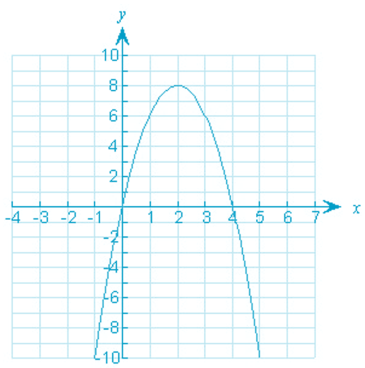
ht is "h"

vt is "k"

Can I DO these on my own?

1. Given that the quadratic function $y = 3(x - 2)^2 + 5$ is the image of $y = x^2$ after some transformation, determine the transformations and determine its mapping rule.

2. The function $y = ax^2 + bx + c$ is represented by this graph:



Which of the following statements is true?

A. $a > 0$ *and* $c > 0$

B. $a > 0$ *and* $c < 0$

C. $a < 0$ *and* $c < 0$

D. $a < 0$ *and* $c = 0$

3. From the text, try questions 6, 8, 9, and 10 on pages 30 and 31.

Quadratics

QUADRATIC FUNCTIONS

PAGES **31** TO **35** IN THE TEXT

Outcomes

I am expected to...

- **B1** demonstrate an understanding of the relationships that exist between arithmetic operations and the operations used when solving equations
- **C8** describe and translate between graphical, tabular, written, and symbolic representations of quadratic relationships
- **C9** translate between different forms of quadratic equations
- **C23** solve problems involving quadratic equations

What do I HAVE to know?

- How can I rewrite a quadratic function from general form to transformational form? **[B1, C9]**
- What does "completing the square" mean? **[B1, C9]**
- When do I need to complete the square? **[B9, C9, C23]**

What MIGHT it look like...

on the provincial exam?

EXAMPLE 1

Given the function $y = -3x^2 + 6x + 3$, show how to change the equation of the function into transformational form.

Ways of Thinking about Solutions

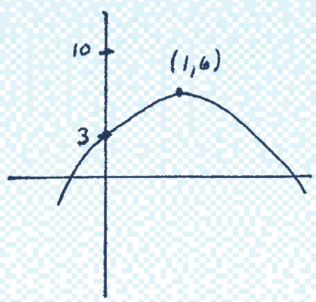
Can I rewrite the equation of a function from general form to transformational form by completing the square? Do I know what procedure to follow to complete the square?

Algebraically from general to transformational form.

$$y = -3x^2 + 6x + 3$$
$$-\tfrac{1}{3}y = x^2 - 2x - 1$$
$$-\tfrac{1}{3}y + 1 = x^2 - 2x$$
$$-\tfrac{1}{3}y + 1 + 1 = x^2 - 2x + 1$$
$$-\tfrac{1}{3}y + 2 = (x-1)^2$$
$$-\tfrac{1}{3}(y-6) = (x-1)^2$$

or

Graphically



From the graph, I can see the vertex is $(1, 6)$, therefore $-\frac{1}{a}(y-6) = (x-1)$

The coefficient of x^2 is -3 therefore the parabola is reflected in the x-axis with a vertical stretch of 3.

$\therefore -\frac{1}{3}(y-6) = (x-1)^2$

EXAMPLE 2

At the Halifax Airshow, a plane performs a power dive. The equation $h = 10t^2 - 60t + 150$ expresses the relationship between height, h, in metres, and time, t, in seconds during the dive. (Solve this question algebraically.)

(a) What is the minimum height that the plane reaches during the dive?

(b) When will the plane be at a height of 100 metres during the dive?

Ways of Thinking about Solutions

For part (a), Should I rewrite the equation in transformational form, and how can I do this algebraically? How do I know when I will need to use the "completing the square" procedure? What do I have to do to complete the square?

I'm asked for 'minimum height' therefore I must change the equation in transformational form.

$$h = 10t^2 - 60t + 150$$
$$h - 150 = 10(t^2 - 6t)$$
$$h - 150 + 90 = 10(t^2 - 6t + 9)$$
$$h - 60 = 10(t-3)^2$$
$$\tfrac{1}{10}(h-60) = (t-3)^2$$

Vertex is at (3, 60)
∴ minimum height is at 60m.

[See also the graphical approach taken to solve this problem when it first appears on pages 4 and 5]
For part (b), how do I solve the equation for *t* when given an *h*-value?

If $h = 100$ then $\frac{1}{10}(100-60) = (t-3)^2$

$$\tfrac{1}{10}(40) = (t-3)^2$$
$$4 = (t-3)^2$$
$$\sqrt{4} = \sqrt{(t-3)^2}$$
$$\pm 2 = t - 3$$
$$t = 5 \text{ or } t = 1$$

Can I DO these on my own?

1. Given the function $y = -2x^2 + 4x - 5$,
(a) Write the function in standard form.
(b) Write the vertex coordinates.
(c) What is the equation of the axis of symmetry?

2. From the text, try the following questions:
Pages 32 to 35 (questions 15, 17–23, and 26–30)
Pages 72 and 73 (questions 6–9 and 11)

Quadratics

DETERMINING QUADRATIC FUNCTIONS

PAGES **36** TO **39** IN THE TEXT

Outcomes

I am expected to...

B1 demonstrate an understanding of the relationships that exist between arithmetic operations and the operations used when solving equations

C8 describe and translate between graphical, tabular, written, and symbolic representations of quadratic relationships

C23 solve problems involving quadratic equations

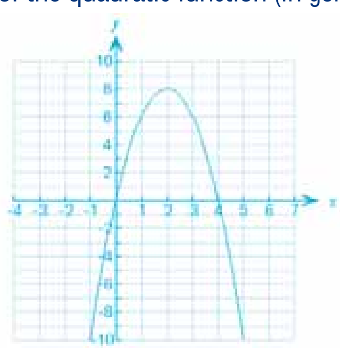
What do I HAVE to know?

- When I read a problem, do I know how to model it using a quadratic function? **[C23]**
- To determine a quadratic equation, what information do I need? **[C8]**
- If I'm given the coordinates of the vertex and one other point on the parabola, how can I determine the equation of the quadratic function? **[B1, C8]**

What MIGHT it look like...

on the provincial exam?

1. Write the equation of the quadratic function (in general form) shown in this graph:



Quadratics

DETERMINING QUADRATIC FUNCTIONS

Ways of Thinking about Solutions

What information do I need to determine the equation of a function from a graph?

How do I write the function in general form?

The vertex is (2, 8) so $\frac{1}{a}(y-8) = (x-2)^2$

Another point on the graph is (3, 6) so

$$\frac{1}{a}(6-8) = (3-2)^2$$

$$\frac{-2}{a} = 1$$

$$a = -2$$

$$\therefore -\frac{1}{2}(y-8) = (x-2)^2$$

so $y-8 = -2(x-2)^2$

$$y = -2(x^2-4x+4)+8$$

$$y = -2x^2+8x-8+8$$

$$y = -2x^2+8x$$

Can I DO these on my own?

1. What is the equation of a quadratic function that has a vertex at (–3, 7) and passes through (4, –1)?

2. The arch of a tunnel has the shape of a parabola. Its highest point is 9 m above the centre of the road, which is 5 m from the edge of the tunnel. Can a truck that is 3 m wide and 5 m high pass through the tunnel?

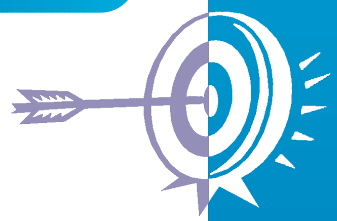
3. From the text, try the following questions:
Pages 38 and 39 (questions 39–45)
Page 73 (questions 10(a) and 10(b))

Quadratics

ROOTS OF QUADRATIC EQUATIONS

PAGES **41** TO **54** IN THE TEXT

Outcomes



I am expected to...

- **A3** demonstrate an understanding of the role of irrational numbers in applications
- **A9** represent non-real roots of quadratic equations as complex numbers
- **B1** demonstrate an understanding of the relationships that exist between arithmetic operations and the operations used when solving equations
- **B10** derive and apply the quadratic formula
- **C8** describe and translate between graphical, tabular, written, and symbolic representations of quadratic relationships
- **C22** solve quadratic equations
- **C23** solve problems involving quadratic equations

What do I HAVE to know?

- Can I use the four methods for solving quadratic equations: graphing, factoring, completing the square, and using the quadratic formula? **[C8, C22]**
- Can I recognize which method is most efficient to use? **[C22]**
- Do I know how to use the CALC menu to solve quadratic equations? **[C22]**
- When I solve a quadratic equation, what does the solution represent? **[C22]**
- What is the relationship between roots, *x–intercepts*, zeros, and solutions of quadratic equations? **[C22]**
- Can I solve an equation in the form $ax^2 + bx + c = 0$ to show where the quadratic formula comes from? **[B10]**

ROOTS OF QUADRATIC EQUATIONS

- Do I know that $x = -\frac{b}{2a}$ is the axis of symmetry?
- Can I determine the *y–value* of the vertex knowing the *x*-coordinate? The vertex is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$
- Do I understand that I have non-real roots (imaginary roots) when the discriminant is negative? **[A9]**
- Do I know how to write non-real roots as complex numbers? **[A9]**
- Do I know when it is better to write irrational roots as “exact” roots, or as decimal approximations? **[A3, C23]**
- Do I realize that when I read the words “maximum” or “minimum” I need to determine the vertex of the quadratic? **[C22, C23]**

What MIGHT it look like…
on the provincial exam?

EXAMPLE 1

The function $h = –5h^2 + 20t + 2$ describes the height of a baseball, *h*, in metres, as a function of time, *t*, in seconds, from the instant the ball is hit. Mark solved the equation $–5t^2 + 20t + 2 = 0$, and its positive root represents

A. the initial height of the ball
B. the maximum height of the ball
C. the time it takes for the ball to reach a maximum height of 2 m
D. the time it takes for the ball to hit the ground

Ways of Thinking about Solutions

When I solve a quadratic equation, what does the solution represent?

When solving $-5t^2 + 20t + 2 = 0$ I know that the height is 0. Therefore my positive "t" value represents the time it takes the ball to hit the ground.
$\therefore$ D is the correct answer.

ROOTS OF QUADRATIC EQUATIONS

2

EXAMPLE 2

Solve the following equation. If the root(s) are non-real, express in terms of *i*.

$2x^2 + 6x = -17$

Ways of Thinking about Solutions

Do I know what solving the equation means?

First step: Rewrite the equation in general form. $(ax^2+bx+c=0)$

$$2x^2+6x+17=0$$

Consider factoring... will not factor.
Use the quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-6\pm\sqrt{(6)^2-4(2)(17)}}{2(2)}$$

$$x=\frac{-6\pm\sqrt{-100}}{4}$$

If the discriminant is negative, do I know how to write these roots as complex numbers?

$x=\frac{-6\pm10i}{4}$ or $x=-\frac{3}{2}+\frac{5i}{2}$ and $x=-\frac{3}{2}-\frac{5i}{2}$

3

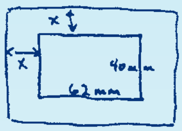
EXAMPLE 3

A photograph measures 40 mm by 62 mm. A frame of uniform width is placed around the photograph, doubling the area. What is the width of the frame?

Ways of *Thinking* about Solutions

Begin by sketching a diagram that incorporates all the information, and then form an equation.

Sketch a diagram, if possible:



Area of picture $= 62 \times 40$

$= 2480$

Area of picture + frame $= 2(2480)$

$(62 + 2x)(40 + 2x) = 4960$

$2480 + 80x + 124x + 4x^2 = 4960$

$4x^2 + 204x - 2480 = 0$

$x^2 + 51x - 620 = 0$

Do I know how to solve a quadratic equation and when it is better to write irrational roots as "exact" roots or as decimal approximations?

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-51 \pm \sqrt{2601 + 2480}}{2}$$

(Since I am looking for a measurement, I must find the decimal approximation)

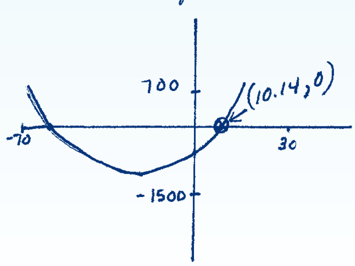
$$= \frac{-51 \pm 71.28}{2}$$

$$= -61.14 \text{ or } 10.14$$

The width can't be negative, so the width of the frame is 10.14 mm.

or

Use the graphing calculator to graph $y = x^2 + 51x - 620$



When $y = 0$, $x^2 + 51x - 620 = 0$

$\therefore$ the positive zero of the function represents the solution.

The width of the frame is 10.14 mm.

Can I DO these on my own?

From the text, try the following questions:

- Pages 44 to 47 (questions 8–12, 15, 16, 21, and 22)
- Page 49 (questions 28–32)
- Pages 52 and 53 (questions 37, 39-41)
- Page 54 (questions 44, 45, 48, and 49)
- Page 73 (questions 14–16)

Quadratics

THE NATURE OF QUADRATIC ROOTS

PAGES **55** TO **57** IN THE TEXT

Outcomes

I am expected to...

A4 demonstrate an understanding of the nature of the roots of quadratic equations

A9 represent non-real roots of quadratic equations as complex numbers

B10 derive and apply the quadratic formula

C15 relate the nature of the roots of quadratic equations and the *x*–intercepts of the graphs of the corresponding functions

What do I HAVE to know?

- Do I know how to calculate the "discriminant"? What can I conclude about the roots of the equation when I know the value of the discriminant? **[A4]**
- Do I understand that $\sqrt{-1} = i$? **[A9]**
- Do I know that if I'm given two roots of a quadratic equation, I can derive the equation? **[A4, C15]**

What MIGHT it look like... on the provincial exam?

EXAMPLE 1

If the roots of a quadratic equation are –2 and 4, the discriminant is

A. an imaginary number
B. 0
C. a positive number
D. a negative number

Ways of Thinking about Solutions

Do I know that if the discriminant has a positive value there will be two distinct *x*–intercepts for the graph of the corresponding function?

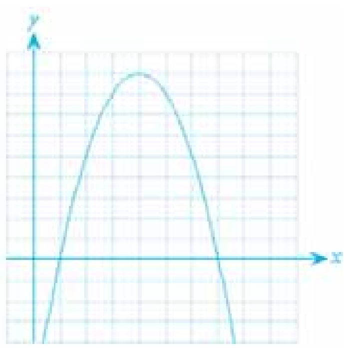
If there are 2 real different roots $\left(\frac{-b \pm \sqrt{b^2-4ac}}{2a}\right)$

The discriminant (b^2-4ac) must be positive
so, <u>C</u> is the answer.

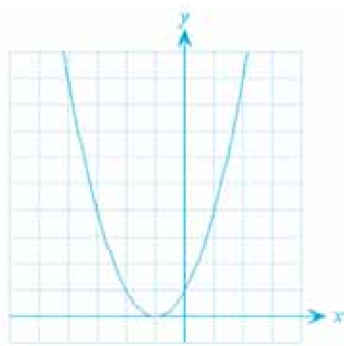
EXAMPLE 2

Select the correct graph for $y = -(x - m)(x - n)$, if $m>0$ *and* $n>0$.

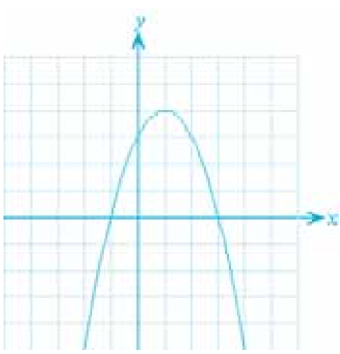
A.



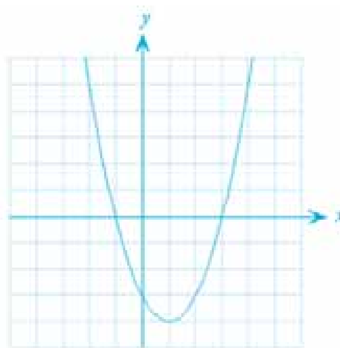
B.



C.



D.



Ways of *Thinking* about Solutions

Looking at the coefficient of the x^2 term, I can determine if there is a reflection in the x–axis. Then, because the function is in factored form, I can determine whether the zeros are positive or negative.

The coefficient of x^2 is negative, therefore there is a reflection in the x-axis so either A or C are possible solutions. If $m>0$ and $n>0$, $\therefore$ both roots are positive so A is correct

Can I DO these on my own?

From the text, try the following questions:

- Pages 55 to 57 (questions 51–60)
- Page 74 (question 19)

Quadratics

SEQUENCES

PAGES **11** TO **14** IN THE TEXT

Outcomes

I am expected to...

C10 **[ADV]** determine the equation of a quadratic function using finite differences

What do I HAVE to know?

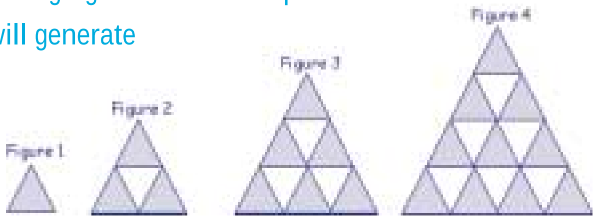
- Do I know when to use the finite difference to determine a quadratic equation in general form?
- Do I know that D_2 is equal to twice the *a*–value in the general form of the quadratic $y = ax^2 + bx + c$?
- If I know the *a*–value, how do I calculate the *b* and *c–values*?
- Knowing the *a*, *b*, and *c–values*, can I write the general form of the equation?

Can I DO these on my own?

1. Determine algebraically, using finite differences, the function that represents the relationship between the *x*–values and the *y*-values in the given table.

x	1	2	3	4	5	6
y	5	19	43	77	121	175

2. The number of shaded triangles in the following figures forms a sequence. Algebraically determine the function that will generate the sequence.



3. From the text, try the following questions
Pages 12 to 14 (questions 38–42)
Page 73 (questions 10(c), 10(d), and 12)

ADVANCED

Quadratics

THE NATURE OF QUADRATIC ROOTS

PAGES **58** AND **59** IN THE TEXT

Outcomes

I am expected to...

A4 demonstrate an understanding of the nature of the roots of quadratic equations

Note: Although this is not an advanced outcome, students in the advanced course should also understand that the sum and product of the roots r_1 and r_2 are related to the coefficients of the general quadratic equation $ax^2 + bx + c = 0$ in this way $r_1 + r_2 = -\frac{b}{a}$ and $r_1 \times r_2 = \frac{c}{a}$.

What do I HAVE to know?

- Do I know how to use the sum and product of roots of a quadratic equation to determine an equation that has those roots?
- Do I know that there is an infinite number of quadratic equations with those roots?

Can I DO these on my own?

1. A parabola crosses the x-axis at $-\frac{1}{2}$ and 5.

(a) Write a function in general form representing such a parabola.

(b) Write a function representing all the parabolas that have *x–intercepts* at $-\frac{1}{2}$ and 5.

2. From the text, try the following questions:

Pages 57 to 59 (questions 61–72)

Page 74 (question 18)

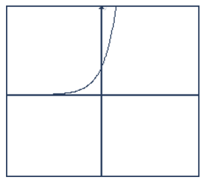
Notes

Exponential Growth

THINGS TO REMEMBER

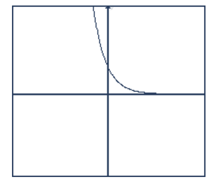
For $y = ab^x$

If $a > 0$ and $b > 1$



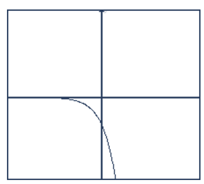
Exponential growth
Increasing function
Asymptote is y=0

If $a > 0$ and $0 < b < 1$



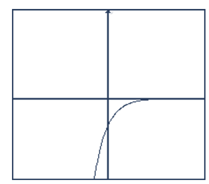
Exponential decay
Decreasing function
Asymptote is y=0

If $a < 0$ and $b > 1$



Decreasing function
Asymptote is y=0

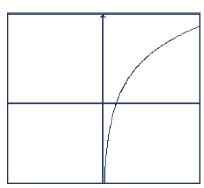
If $a < 0$ and $0 < b < 1$



Increasing function
Asymptote is y=0

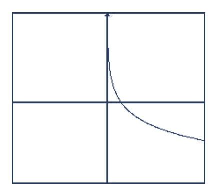
For $y = a \log_b x$

$a > 0$



Increasing function
Asymptote at x=0

$a < 0$



Decreasing function
Asymptote at x=0

Notice that the log curve does not cut the y-axis and therefore the log of zero or the log of a negative number is not defined in real numbers

Rules for using Exponents and Logs

Exponents	Logs
$a^m \cdot a^n = a^{m+n}$ $a^m \div a^n = \dfrac{a^m}{a^n} = a^{m-n}$ $(a^m)^n = a^{m \cdot n}$ $\left(\dfrac{a^n}{b^m}\right)^p = \dfrac{a^{n \cdot p}}{b^{m \cdot p}}$ $a^{-n} = \dfrac{1}{a^n}$ and $a^n = \dfrac{1}{a^{-n}}$ $a^0 = 1$	$\log_n(a \cdot b) = \log_n a + \log_n b$ $\log_n(a \div b) = \log_n\left(\dfrac{a}{b}\right) = \log_n a - \log_n b$ $\log_n a^m = m \log_n a$ $\log_n a = \dfrac{\log a}{\log n}$ $x = n^y \quad \Leftrightarrow \quad y = \log_n x$ $\therefore y = \log_n x$ means "y is the exponent to which you raise the base n to get the answer x.
If $a^p = a^q$ then $p = q$	If $\log_n x = \log_n y$ then $x = y$

Exponential Growth

GEOMETRIC SEQUENCES

PAGES **110** TO **115** IN THE TEXT

Outcomes

I am expected to...

A7 describe and interpret domains and ranges using set notation

C3 sketch tables and graphs from descriptions and collected data

C4 demonstrate an understanding of patterns that are arithmetic, power, and geometric and relate them to corresponding functions

C29 analyse tables and graphs to distinguish between linear, quadratic, and exponential relationships

C33 analyse and describe the characteristics of exponential and logarithmic functions

What do I HAVE to know?

- What is a geometric sequence? **[C4, C33]**
- If I graph a geometric sequence from a table, is the graph discrete or continuous? What are the domain and range, and how do I express them? **[A7, C3, C4]**
- What is a common ratio, and how do I determine it? **[C4, C33]**
- How can I determine if a function is linear, quadratic, cubic, or exponential? **[C4, C29, C33]**
- How do I distinguish arithmetic and other power sequences, from geometric sequences? **[C4, C29, C33]**
- Can I explain why the formula $t_n = t_1(r)^n$ generates a sequence that is geometric? **[C4, C33]**

Exponential Growth

GEOMETRIC SEQUENCES

- Do I know that graphs of geometric sequences can form growth and decay curves? **[C3, C33]**
- How do I know when an exponential graph is a growth curve or a decay curve? **[B2, C3, C33]**

What MIGHT it look like... on the provincial exam?

EXAMPLE 1

What type of function would best model the data in the table below?

x	y
1	21.4
2	45.6
3	72.6
4	102.4
5	135.0

A. linear
B. quadratic
C. logarithmic
D. exponential

Ways of Thinking about Solutions

How do I determine from a table the type of function that best models the sequence of *y*-values ?

Does the value of "x" change by the same increment? If yes, verify first and second level differences for the y-values.

y	D_1	D_2
21.4		
	24.2	
45.6		2.8
	27	
72.6		2.8
	29.8	
102.4		2.8
	32.6	
135		

D_2 has constant terms of 2.8.
∴ the sequence can be modelled with a quadratic function.
The answer is B

2

EXAMPLE 2

Which of the following table of values is an exponential function?

A.

x	2	3	4	5
y	9	16	25	36

B.

x	2	6	8	15
y	6	12	24	48

C.

x	2	4	6	8
y	1	7	13	19

D.

x	2	5	8	11
y	24	12	6	3

Ways of Thinking about Solutions

The *x*-values have to change by the same increment... and from the table I have to check for a common ratio between *y*-values.

The tables A, C, and D have x-values that change by the same increment, but the y-values for A and C do not produce a common ratio, while D does. So D is the correct answer. (In table B, the y-values form a common ratio, but the x-values do not change by the same increment.

3

EXAMPLE 3

Is 2^x, 2^{x+2}, 2^{x+4} a geometric sequence? Explain your reasoning.

Exponential Growth

GEOMETRIC SEQUENCES

Ways of Thinking about Solutions

To be geometric there has to be a common ratio.

Find a common ratio

$$\frac{t_2}{t_1} \stackrel{?}{=} \frac{t_3}{t_2}$$

$$\frac{2^{x+2}}{2^x} = 2^2 = 4$$

$$\frac{2^{x+4}}{2^{x+2}} = 2^2 = 4$$

Since the successive terms have a common ratio of 4, it is geometric. (Substituting a specific value for x does not determine that $2^x, 2^{x+2}, 2^{x+4}$ is a geometric sequence for all values of x.)

Can I DO these on my own?

1. Which is a geometric sequence?

A. 1, 3, 5, 7, ...

B. 2, 4, 6, 8, ...

C. 4, 7, 12, 19, ...

D. 1.5, 3.0, 6.0, 12.0, ...

2. Show how to determine the function that generates the following sequence:

$\{3^{-2}, 3^{-1}, 1, 3, \ldots\}$

3. Try these questions from the text:

Pages 112 and 113 (questions 7, 8 10, 11)

Pages 199 and 200 (questions 2 and 3)

Exponential Growth

EXPONENTIAL FUNCTIONS

PAGES **115** TO **141** IN THE TEXT

Outcomes

I am expected to...

- **A5** demonstrate an understanding of the role of real numbers in exponential and logarithmic expressions and equations
- **A7** describe and interpret domains and ranges using set notation
- **B12** apply real number exponents in expressions and equations
- **C2** model real-world phenomena using exponential functions
- **C3** sketch tables and graphs from descriptions and collected data
- **C25** solve problems involving exponential and logarithmic equations
- **C33** analyse and describe the characteristics of exponential and logarithmic functions
- **C34** demonstrate an understanding of how the parameter changes affect the graphs of exponential functions
- **F1** analyse determine, and apply scatter plots and determine the equations for curves of best fit, using appropriate technology

HAVE

- What type of real-life situations can be modelled by exponential functions? **[C2]**
- Do I know that compound interest grows exponentially, and do I know how to calculate it? **[C2, C33]**
- Do I understand that the focal point on an exponential graph is the point (0, 1) and that all exponential functions of the form $y = b^x$, where $b \neq 0$, pass through this point? **[C33]**
- What is an asymptote? **[C3, C33]**

Exponential Growth

EXPONENTIAL FUNCTIONS

- Do I understand that data on an exponential curve approaches an asymptote? **[C3, C33]**
- Can I determine and express the domain and range of an exponential function? **[A7]**
- Can I evaluate expressions with positive, zero, and negative exponents ? **[A3, B12]**
- Do I know that the graph of $y = 2^{-x}$ is a reflection in the y-axis of the graph of $y = 2^{x}$? **[C3, C33, C34]**
- How does changing the values of *a* and *b* for $y = ab^{x}$ transform its graph? **[C34]**
- Can I distinguish between a growth curve and a decay curve? **[A5, C33, C34]**
- Do I know how to determine the equation of an exponential function from a table? From a graph? **[C2, C11, C33, C34]**
- Do I know how to use ExpReg on the graphing calculator to determine the equation of an exponential function? **[F1]**
- From a table of values, do I know that the initial value *a*, in $y = ab^{x}$ is found where the *x*-value is zero? **[C33, C34]**
- Do I know how to determine the horizontal asymptote of an exponential graph from the corresponding function? **[C33, C34]**
- Do I know how to solve word problems using exponential equations and functions? **[A5, B12, C2, C25]**

What MIGHT it look like...
on the provincial exam?

EXAMPLE 1

27^{-2} has the same value as

A. -27^{2}

B. 3^{-6}

C. $\left(-\frac{1}{9}\right)^{4}$

D. $\left(-\frac{1}{3}\right)^{4}$

Ways of Thinking about Solutions

Can I rewrite the given expression in a different form and/or using a different base?

$$27^{-2} = \left(\frac{1}{27}\right)^{2} = \left(\frac{1}{3^{3}}\right)^{2} = \left(\frac{1}{3}\right)^{6} = 3^{-6}$$

or

$$\left(3^{3}\right)^{-2} = 3^{-6}$$

So B is the answer

EXAMPLE 2

What is one half of 2^{20}?

A. 2^{10}

B. 1^{20}

C. 2^{19}

D. 1^{10}

Ways of Thinking about Solutions

What does “one-half of” mean?

$\frac{1}{2} \cdot 2^{20}$ **or** $\frac{2^{20}}{2} = 2^{20-1} = 2^{19}$

$2^{-1} \cdot 2^{20}$

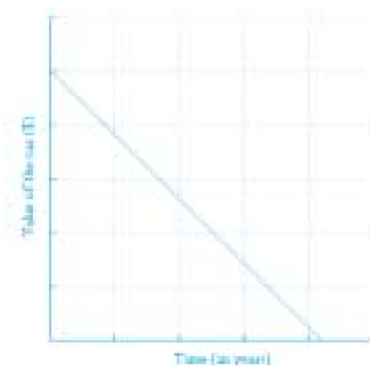
$2^{20-1} = 2^{19}$

So C is correct

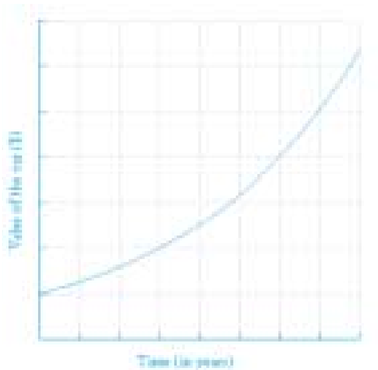
EXAMPLE 3

Select the graph that best represents this situation: “A car depreciates at a rate of 30% per year.”

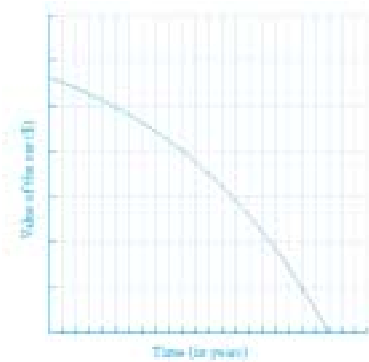
A.



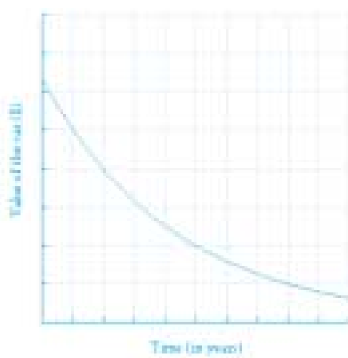
B.



C.



D.



Ways of Thinking about Solutions

How can I tell from the graph that the value is depreciating?

This situation must be represented by a decay curve, therefore only D can be the correct answer.

EXAMPLE 4

Which function is the same as $y = 4(2)^x$?

A. $4y = 2^x$ B. $y = 2^{x+2}$
C. $y = 2^{2x}$ D. $y = 8^x$

Ways of Thinking about Solutions

How do I rewrite the given function?

$y = 4(2^x)$
$= 2^2(2^x)$
$= 2^{x+2}$, so B is correct.

EXAMPLE 5

Which of the following functions forms a decay curve?

A. $y = 0.2^{-x}$ B. $y = 0.2^x$
C. $y = 2^{2x}$ D. $-y = 0.2^{-x}$

Ways of Thinking about Solutions

A decay curve has to have a base less than one but greater than zero.

Look at the value of "b" (common ratio) and for any reflection (-x and -y).
Choices A, B, and D have a value of "b" between 0 and 1 and therefore possibly decay curves.
A has a reflection in the y-axis (-x) ∴ a growth curve.
D has a reflection in the y-axis (-x) ∴ a growth curve AND a reflection in the x-axis, ∴ a decreasing function but not a decay curve.
∴ B is the correct answer.

6

EXAMPLE 6

Show how to evaluate the expression $(4^{-1} + 3^{-2}) \div (8^0 + 4^{-½})$ without the use of a calculator.

Ways of Thinking about Solutions

Any number to the exponent zero equals one.

Taking the reciprocal of a number changes the sign of its exponent.

A number with an exponent of $\frac{1}{2}$ is equivalent to the square root of that number.

$$(4^{-1} + 3^{-2}) \div (8^0 + 4^{-1/2})$$

$$\left(\frac{1}{4} + \frac{1}{9}\right) \div \left(1 + \sqrt{1/4}\right)$$

$$\frac{13}{36} \div \left(1 + \frac{1}{2}\right)$$

$$\frac{13}{36} \div \frac{3}{2} \rightarrow \frac{13}{\cancel{36}_{18}} \times \frac{\cancel{2}^{1}}{3} = \frac{13}{54}$$

7

EXAMPLE 7

Find the equation of the function represented in the following table. Do not use a graphing calculator.

x	0	2	4	6
y	4.7	14.1	42.3	126.9

Ways of Thinking about Solutions

How do I determine an equation from a table? What does it mean when the consecutive *x*-values increase by 2 instead of 1?

Does it have a common ratio?

$\frac{t_2}{t_1} = \frac{t_3}{t_2}$

$\frac{14.1}{4.7} \stackrel{?}{=} \frac{42.3}{14.1}$

$3 = 3$

Since the x-values increase by 2 the horizontal stretch is 2.

The common ratio is 3 and "a" = 4.7

$\therefore\ y = 4.7(3)^{x/2}$

HS = 2

8

EXAMPLE 8

Joey has been collecting antique toy cars as an investment. The value, *V*, in dollars, of a toy car with respect to its age, *t*, in years, can be modelled using the function

$$V = 2(3)^{\frac{t}{4}}$$

Which of the following statements is true?

A. The car's initial value was \$2 and it tripled in value every 4 years.
B. The car's initial value was \$3 and it doubled in value every 4 years.
C. The car's initial value was \$4 and it tripled in value every 2 years.
D. The car's initial value was \$2 and it quadrupled in value every 3 years.

Ways of *Thinking* about Solutions

Do I understand what the values 2, 3, and 4 in the equation represent?

The initial value is #2, it triples every 4 years. So A is correct.

EXAMPLE 9

$\frac{3^{120}}{3}$ is equal to

A. 3^{-120}

B. 3^{119}

C. 1^{120}

D. 3^{117}

Ways of *Thinking* about Solutions

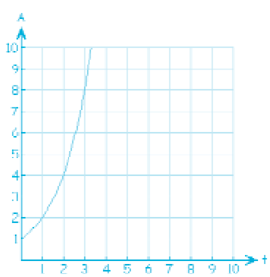
Dividing exponential terms when bases are the same tells me to subtract their exponents.

$3^{120-1} = 3^{119}$; so B. is correct

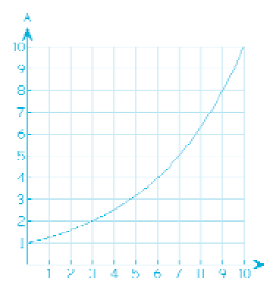
EXAMPLE 10

Which curve best describes the growth of bacteria that doubles every three hours, where *t* is time in hours and *A* is amount of bacteria per square millimetre?

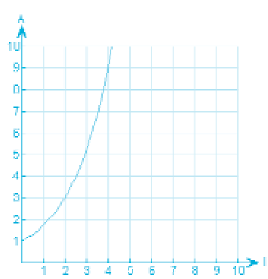
A.



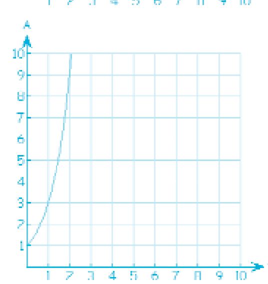
B.



C.



D.



Ways of *Thinking* about Solutions

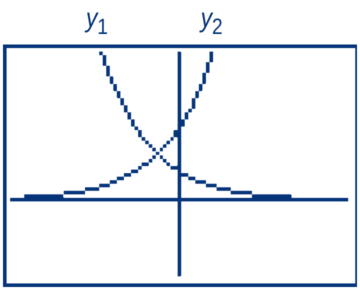
How can I tell from the graph that the growth of bacteria doubles every three hours?

The initial amount on all four graphs is 1 at $t=0$
$\therefore$ at $t=3$, the amount must be 2
B is the correct answer

11

EXAMPLE 11

Two exponential functions, y_1 and y_2, of the form $y = ab^x$, are graphed:



How do the values of a compare in the two functions?

A. The value of a for y_1 is greater than the value of a for y_2.
B. The value of a for y_1 is less than the value of a for y_2.
C. The value of a for y_1 is equal to the value of a for y_2.
D. For the function y_1, $0 < a < 1$, and for the function y_2, $a > 1$.

Ways of *Thinking* about Solutions

How can I tell from a graph what the *a–value*s are?

The 'a' values are the y-intercepts. I can see that y_2 has a higher a-value. So B is the answer.

EXAMPLE 12

Solve this equation for x:

$$\left(\frac{1}{8}\right)^{x+2} = (4)^{-(x+1)}$$

Ways of Thinking about Solutions

What is the first step when solving an exponential equation?

Since this equation has only one term on each side, I have to try and make the bases the same in order to equate their exponents.

$$\left(\frac{1}{8}\right)^{x+2} = 4^{-(x+1)}$$

$$(2^{-3})^{x+2} = (2^2)^{-(x+1)}$$

$$2^{-3x-6} = 2^{-2x-2}$$

So $-3x-6 = -2x-2$

$-x = 4$

$x = -4$

Can I DO these on my own?

1. Evaluate the following without using a calculator. Show at least one intermediate step needed to obtain your final answer.

(a) $\left(-\frac{1}{2}\right)^{-3}$

(b) $\left(-64\right)^{\frac{2}{3}}$

(c) $\left(\frac{9}{16}\right)^{-\frac{1}{2}}$

(d) $5^0 + \left(\frac{1}{3}\right)^{-1}$

2. Without using the regression feature of the graphing calculator, determine the exponential function represented by the data in the following table.

x	-3	0	3	6
y	15	12	9.6	7.68

3. Try these questions from the text:

Page 121 (questions 38 and 39)

Page 123 (questions 50, 51, 52(a), (b), and (c))

Pages 129 and 130 (questions 9–12)

Pages 200 and 201 (questions 5–11)

Exponential Growth

EXPONENTIAL EQUATIONS AND EXPRESSIONS

PAGES **156** TO **170** IN THE TEXT

Outcomes

I am expected to...

A5 demonstrate an understanding of the role of real numbers in exponential and logarithmic expressions and equations

B1 demonstrate an understanding of the relationships that exist between arithmetic operations and the operations used when solving equations

B12 apply real number exponents in expressions and equations

C2 model real-world phenomena using exponential functions

C11 describe and translate between graphical, tabular, written, and symbolic representations of exponential and logarithmic relationships

C24 solve exponential and logarithmic equations

C25 solve problems involving exponential and logarithmic equations

What do I HAVE to know?

- Do I know how to use my graphing calculator to solve an exponential equation? **[C24]**
- Do I know how to solve an exponential equation algebraically? **[C24, B1]**
- When I read a problem that talks about "half-life" or "doubling period," what value should I use for the common ratio, and what is a horizontal stretch? **[C2, C11, C25]**
- Do I know when to use the formula: $A = P(1+\frac{r}{n})^{nt}$, and do I know what it represents? **[C2, C11, C25]**
- Do I understand, when given a function $f(x)$, that $f(4)$ means "find y when $x = 4$"? **[C24]**
- Do I understand, when given a function $f(x)$ and asked to find x when $f(x) = 4$, that this means "find the x-value when $y = 4$"? **[C24]**

Exponential Growth

EXPONENTIAL EQUATIONS AND EXPRESSIONS

- Can I apply my laws of exponents (found on page 167 of the text)?
- Do I know how to evaluate expressions with fractional exponents? **[A5, B12]**

What MIGHT it look like… on the provincial exam?

EXAMPLE 1

The price of a particular product doubles every 35 years. If the price of the product was $16.40 on January 1, 1996, the price of the product will be $36.50 in the year

A. 2028 B. 2031
C. 2036 D. 2040

Ways of Thinking about Solutions

The answer can't be B because 2031 is 35 years after 1996 and therefore the amount should have doubled, to $32.80. Since $36.50 is slightly more than $32.80, the correct answer must be C or D. Maybe I should check with the formula.

$$y = 16.40(2)^{t/35}$$
$$36.50 = 16.40(2)^{t/35}$$
$$\frac{36.50}{16.40} = 2^{t/35}$$
$$2.23 = 2^{t/35}$$
$$\frac{t}{35} = \log_2 2.23$$
$$t = 35\,\frac{\log 2.23}{\log 2}$$
$$= 40.5$$

∴ 40 years later.
The best answer is C.

EXAMPLE 2

$\sqrt[3]{x^2}$ can be expressed as

A. $x^{\frac{3}{2}}$

B. $x^{\frac{2}{3}}$

C. $\frac{x^2}{3}$

D. $\left(x^2\right)^3$

Ways of *Thinking* about Solutions

How can I rewrite the given expression?

$(x^2)^{1/3} = x^{2/3}$

EXAMPLE 3

A general rule used by car dealerships is that the trade-in value of a car decreases by 30 per cent each year.

(a) Suppose you own a car whose trade-in value V is presently \$3750. Determine how much it will be worth one year from now, two years from now, three years from now.
Fill in the table of values:

t time in years	V value of the car
1	
2	
3	

Ways of *Thinking* about Solutions

What does it mean when a car depreciates by 30 per cent each year?

- 1 year from now, it will be worth 70% of \$3150, which is \$2625
- 2 years from now, it will be worth 70% of \$2625, which is \$1837.50.
- 3 years from now, it will be worth 70% of \$1837.50, which is \$1286.25.

(b) Without using the equation of the function, explain why an exponential function can be used to represent the data in part (a).

Ways of *Thinking* about Solutions

How can I describe why an exponential function is the best model?

Each of the values would be obtained by multiplying the previous value by 0.7, which would then be a common ratio. Exponential functions are those that represent numbers that have a common ratio.

(C) Write the particular equation expressing the trade-in value *V* of your car as a function of the number of years *t* from the present.

Ways of *Thinking* about Solutions

How do I determine the values of *a* and *b* for this particular equation?

an exponential function looks like $y = ab^x$
'a' is the initial → \$3750
'b' is the common ratio → 70%
'x' is the time in years
'y' is the new value after x-years.

So, $y = 3750(.70)^x$

EXAMPLE 4

Find the roots of the following equations:

A. $\sqrt[3]{9} = 81^{3x-5}$

B. $\left(\frac{1}{4}\right)^{x-5} - 5 = 3$

Ways of *Thinking* about Solutions

To solve an exponential equation, simplify the equation in order to obtain one term on each side. Then make the bases the same, if possible.

A.

$$\sqrt[3]{9} = 81^{3x-5}$$
$$3^{2/3} = (3^4)^{3x-5}$$
$$3^{2/3} = 3^{12x-20}$$
$$\frac{2}{3} = 12x - 20$$
$$2 = 36x - 60$$
$$-36x = -62$$
$$x = \frac{62}{36} = \frac{31}{18}$$

or

A.

$$9^{1/3} = 9^{6x-10}$$
$$\frac{1}{3} = 6x - 10$$
$$1 = 18x - 30$$
$$31 = 18x$$
$$\frac{31}{18} = x$$

B.

$$\left(\frac{1}{4}\right)^{x-5} - 5 = 3$$
$$\left(\frac{1}{4}\right)^{x-5} = 8$$
$$(2^{-2})^{x-5} = 2^3$$
$$-2x + 10 = 3$$
$$-2x = -7$$
$$x = 7/2$$

EXAMPLE 5

At the start of 2004, Jonathan invested \$500 in a fund that doubles every seven years. In what year will he have \$1,200 in his account?

Ways of Thinking about Solutions

How do I model this situation using an exponential equation?

initial amount: \$500
common ratio → 2
new amount: \$1200
double every 7 years ...

$1200 = 500(2)^{x/7}$

$\frac{1200}{500} = 2$

$2.4 = 2^{x/7}$

... can't make the bases the same ... change to logs

$\log 2.4 = \log 2^{x/7}$

$\log 2.4 = \frac{x}{7}\log 2$

$\frac{7\log 2.4}{\log 2} = x$

OR

$\frac{x}{7} = \log_2 2.4$

$x = \frac{7\log 2.4}{\log 2}$

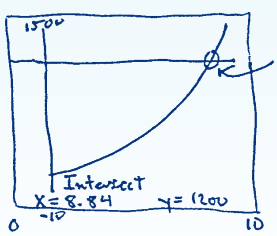
$x = 8.84$

so, 8.8 years after 2004, or, in 2012.

OR

using a graph...

$y_1 = 500(2)^{x/7}$
$y_2 = 1200$
find intersection point...



x = 8.84 years

so 2004
+ 8.84
in 2012

Can I DO these on my own?

1. Which of the following is equal to $a^{\frac{4}{5}}$?

A. $\sqrt[5]{a^4}$

B. $\left(a^{\frac{1}{4}}\right)^5$

C. $\left(a^4\right)^5$

D. $\dfrac{1}{a^{\frac{5}{4}}}$

2. Given $8(2)^x = 32$ Which of the following statements is false?

A. $16^x = 32$

B. $2^{x+3} = 32$

C. $2^x = 4$

D. $x = \log_2 4$

3. A certain bacterial culture initially has 200 bacteria/cm^2 and the number doubles every 20 minutes.

(a) Find the equation representing this situation.

(b)How many bacteria/cm^2 would there be after four hours?

4. Try these questions from the text:

Pages 158 to 161 (questions 8–19)

Pages 164 to 167 (questions 1–19, 21(a) and (b))

Pages 202 and 203 (questions 18–21 and 25–29)

Exponential Growth

LOGARITHMS

PAGES **172** TO **182** IN THE TEXT

Outcomes

I am expected to...

- **A5** demonstrate an understanding of the role of real numbers in exponential and logarithmic expressions and equations
- **B1** demonstrate an understanding of the relationships that exist between arithmetic operations and the operations used when solving equations
- **B12** apply real number exponents in expressions and equations
- **B13** demonstrate an understanding of the properties of logarithms and apply them
- **C11** describe and translate between graphical, tabular, written, and symbolic representations of exponential and logarithmic relationships
- **C19** demonstrate an understanding, algebraically and graphically, that the inverse of an exponential function is a logarithmic function
- **C24** solve exponential and logarithmic equations
- **C25** solve problems involving exponential and logarithmic equations

What do I HAVE to know?

- How are the graphs of $y = b^x$ and $y = \log_b x$ related? **[C19]**
- When I look at a logarithmic curve in the form of $y = \log_b x$, do I know that $(1, 0)$ is the x-intercept? Do I know that $x = 0$ is the vertical asymptote? **[C11]**
- Do I know how to change an equation from exponential form to logarithmic (often called “log”) form? **[C19]**
- Can I convert an equation from log form to exponential form and vice versa? **[A5, B13]**

Exponential Growth

LOGARITHMS

- Do I know how to solve exponential equations using logarithms? **[A5, B12]**
- Do I know how to use the laws of logarithms to express several log terms as a single log term? **[B13]**
- Do I know how to solve logarithmic equations? **[B13, C24]**
- Do I understand that logs are used when I want to solve for a variable that is in the exponent when I can't make the bases the same? **[C24, C25]**

What MIGHT it look like...
on the provincial exam?

EXAMPLE 1

Marla is trying to determine the value for x in the equation $7^x = 22$.
Which one of the following will she use to obtain the value of x?

A. $x = \log\left(\frac{22}{7}\right)$

B. $\log_7 x = \log 22$

C. $x = \frac{\log 22}{\log 7}$

D. $x = \frac{\log 7}{\log 22}$

Ways of Thinking about Solutions

When the bases are different, how do I change the exponential equation to a log equation?

$7^x = 22$

$x = \log_7 22$ – how can I do this on my calculator?

$x = \frac{\log 22}{\log 7}$

2

EXAMPLE 2

$2\log_3 9 + \log_3 7 - \log_3 3$ expressed as a single logarithm is

A. $2\log_3 13$

B. $2\log_3 21$

C. $\log_3 42$

D. $\log_3 189$

Ways of *Thinking* about Solutions

To express as a single log, I have to use the laws of logarithms. Do I know how to do this?

$2\log_3 9 = \log_3 9^2 \ldots$

multiply the 9^2 and the 7, then divide by 3

now I have $\log_3\left(\frac{9^2 \cdot 7}{3}\right) \rightarrow \log_3(189)$

so, D. is correct.

3

EXAMPLE 3

Given $\log_3 x = -1$, the value of x is

A. $x = \frac{1}{2}$

B. $x = \frac{1}{3}$

C. $x = -1$

D. $x = -3$

Ways of *Thinking* about Solutions

To solve this log equation, change it to an exponential equation.

$3^{-1} = x$

$\therefore x = 1/3$

B is the correct answer

EXAMPLE 4

A certain bacterial culture initially has 200 bacteria/cm^2 and the number doubles every 20 minutes. How long would it take until there are 1000 bacteria/cm^2? Express your answer accurate to two decimal places.

Ways of Thinking about Solutions

How do I model this situation using an exponential equation, and how do I solve this equation?

initial value ↗ doubles

$1000 = 200(2)^{\frac{t}{20}}$ → doubling period

$\frac{1000}{200} = 2^{t/20}$

$5 = 2^{t/20}$

$\frac{t}{20} = \log_2 5$

$t = \frac{20 \log 5}{\log 2}$

$= 46.44$

The exponent has a variable, therefore change to a log equation

It will take 46.44 minutes

EXAMPLE 5

Solve the following equation for *x*. Express your answer accurate to two decimal places.

$$\log_5 2x - \log_5 3 = \log_5 4$$

Ways of Thinking about Solutions

The first step in solving a log equation is to simplify both sides of the equation in order to obtain one term on each side of the equation.

$$\log_5 2x - \log_5 3 = \log_5 4$$

$$\log_5 \frac{2x}{3} = \log_5 4$$

$$\frac{2x}{3} = 4$$

$$2x = 12$$

$$x = 6$$

Can I DO these on my own?

1. The expression $3\log_2(3) + \log_2(5) - \log_2(9)$ is equivalent to

A. $3\log_2\left(\frac{15}{9}\right)$

B. $3\log_2(-1)$

C. $3\log_2(5)$

D. $\log_2(15)$

2. If $\log_x 9 = \frac{1}{2}$, then x is equal to

A. 3

B. 4.5

C. 18

D. 81

3. Solve for x:

(a) $x = 3\log_3 12$

(b) $\log_x 34 = 0.5$

(c) $\log(x) + \log(x - 1) = 2\log(x)$

4. Try these questions from the text:

Pages 174 and 175 (questions 11–15)

Pages 177 and 178 (questions 1–6)

Pages 180 to 182 (questions 9–14 and 17)

Pages 203 and 204 (questions 33–39)

Exponential Growth

TRANSFORMATIONS

PAGES **143** TO **155** IN THE TEXT

In addition to Mathematics 12 outcomes, you are also responsible for:

Outcomes

I am expected to...

- **A7** describe and interpret domains and ranges using set notation
- **C2** model real-world phenomena using exponential functions
- **C3** sketch tables and graphs from descriptions and collected data
- **C11** describe and translate between graphical, tabular, written, and symbolic representations of exponential and logarithmic relationships
- **C24** solve exponential and logarithmic equations
- **C33** analyse and describe the characteristics of exponential and logarithmic functions
- **C34** demonstrate an understanding of how the parameter changes affect the graphs of exponential functions
- **C35 [ADV]** write exponential functions in transformational form and as mapping rules to visualize and sketch graphs

What do I HAVE to know?

- Can I describe the transformations on the function $y = ab^x$ in words and in a mapping rule. **[C33, C34, C35ADV]**
- Can I explain how each of the transformations is identifiable in the function $A(y - C) = \text{base}^{B(x - D)}$? **[C33, C34, C35ADV]**
- Can I write an exponential function in transformational form, given the transformations in words, in a mapping rule, or from a graph? **[C34, C35ADV]**
- Can I rewrite an exponential function into transformational form? **[C11, C35ADV]**

Exponential Growth

TRANSFORMATIONS

- Can I sketch graphs of exponential functions using transformations? **[C3, C11, C35ADV]**
- Can I complete a table of values using a mapping rule? **[C11]**
- Can I sketch a graph by determining how the transformations affect the focal point, (0, 1), and then using the patterns to the right and left of the focal point? **[C3, C33, C34, C35ADV]**
- Do I understand how the range of the function is related to the asymptote? **[A7, C33]**
- Can I find the equation of the horizontal asymptote and do I understand what it represents in the context of a word problem? **[C3, C33]**
- Do I understand when a function is either increasing and/or decreasing? **[C33]**
- Can I use a graph to solve exponential equations? **[C11, C33, C24]**

Can I DO these on my own?

1. The function $f(x) = 12(2)^{2(x+1)} + 3$ has a horizontal asymptote at

A. $y = 3$ B. $y = -3$

C. $y = 12$ D. $y = -1$

2. For the exponential function $y = -4(3)^{x+2} - 1$, do the following:

(a) Write the equation in transformational form.

(b) Find the coordinates of the focal point.

(c) State the equation of the horizontal asymptote.

(d) State the domain and range.

3. Try these questions from the text:

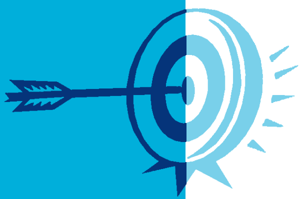
Pages 143 to 162

ADVANCED

Exponential Growth

SOLVING EXPONENTIAL EQUATIONS AND SIMPLIFYING EXPONENTIAL EXPRESSIONS

PAGES **161** TO **171** IN THE TEXT



Note: Although there are no new outcomes to be achieved, there is a difference in the types of questions you may be asked, and in the sophistication of your responses.

Outcomes

I am expected to...

A5 demonstrate an understanding of the role of real numbers in exponential and logarithmic expressions and equations

B1 demonstrate an understanding of the relationships that exist between arithmetic operations and the operations used when solving equations

B12 apply real number exponents in expressions and equations

C2 model real-world phenomena using exponential functions

C11 describe and translate between graphical, tabular, written, and symbolic representations of exponential and logarithmic relationships

C24 solve exponential and logarithmic equations

C25 solve problems involving exponential and logarithmic equations

HAVE

- Do I know how to solve a system of equations when the x- and y-variables are in the exponent? **[C24]**
- Can I recognize and solve exponential equations that are in the form of a quadratic equation (e.g., $(4^x)^2 - 17(4^x) + 16 = 0$)? **[C24]**

Exponential Growth

SOLVING EXPONENTIAL EQUATIONS AND SIMPLIFYING EXPONENTIAL EXPRESSIONS

- Can I factor a term in the form of b^{x+y} to get two factors $b^y(b^x)$? **[A5, B12]**
- Can I simplify, factor, prove, and evaluate exponential expressions and equations (as required for the questions on page 171 in the text)? **[A5, B12]**
- Can I solve more complex logarithmic equations (like those on page 175 in the text)? **[C24]**

Can I DO these on my own?

Try these questions from the text:

Page 167 (question 21 parts c to g)

Page 171 (questions 27–36)

Page 202 (questions 22 and 23)

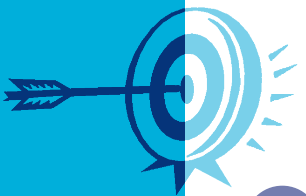
ADVANCED

Exponential Growth

LOGARITHMS

PAGES **175** TO **188** IN THE TEXT

Note: Although there are no new outcomes to be achieved, there is a difference in the types of questions you may be asked, and in the sophistication of your responses.



Outcomes

I am expected to...

A5 demonstrate an understanding of the role of real numbers in exponential and logarithmic expressions and equations

B1 demonstrate an understanding of the relationships that exist between arithmetic operations and the operations used when solving equations

B12 apply real number exponents in expressions and equations

B13 demonstrate an understanding of the properties of logarithms and apply them

C11 describe and translate between graphical, tabular, written, and symbolic representations of exponential and logarithmic relationships

C19 demonstrate an understanding, algebraically and graphically, that the inverse of an exponential function is a logarithmic function

C24 solve exponential and logarithmic equations

C25 solve problems involving exponential and logarithmic equations

Exponential Growth

LOGARITHMS

What do I HAVE to know?

- Can I apply the logarithm laws when simplifying expressions and solving equations? **[A5, B12, B13, C24]**
- Do I know that, when solving an exponential equation and the bases cannot be made the same, logs are required to find the value for the variable in the exponent? **[C24]**
- Can I solve problems that involve equations that are logarithmic, or equations that require logarithms to solve? **[C24, C25]**

Can I DO these on my own?

Try these questions from the text:

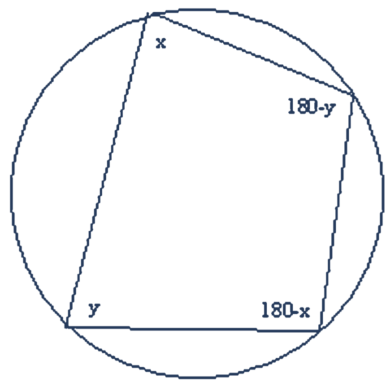
Page 204 (questions 40–45)

Page 184 to 188 (questions 29–43)

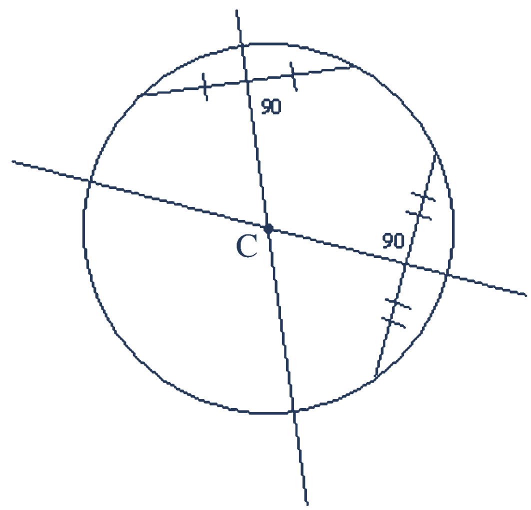
Circle Geometry

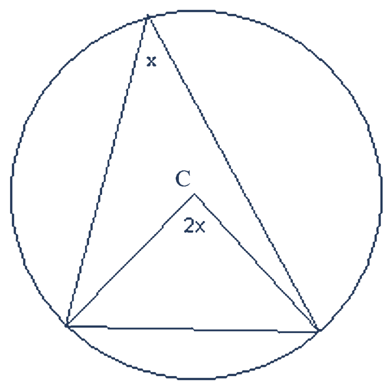
THINGS TO REMEMBER

Circle Properties

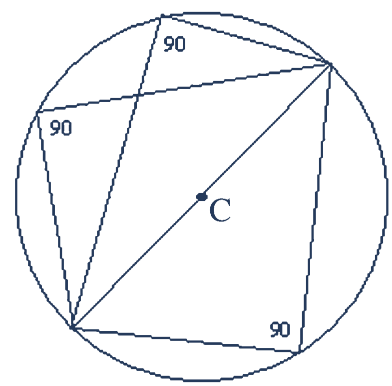


Advanced Math 12

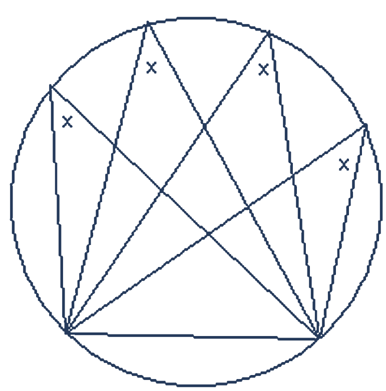




Advanced Math 12



Advanced Math 12



Advanced Math 12

"C" is the centre of the circle.

Circle Geometry

CHORD PROPERTIES

PAGES **206** TO **212** IN THE TEXT

Outcomes

I am expected to...

- **D1** develop and apply formulas for distance and midpoint
- **E4** apply properties of circles
- **E5** apply inductive reasoning to make conjectures in geometric situations
- **E7** investigate, make, and prove conjectures associated with chord properties of circles
- **E12** demonstrate an understanding of the concept of converse

What do I HAVE to know?

- Do I know that when I explore properties and make conclusions, I am making a conjecture (using inductive reasoning)? **[E5, E7]**
- Do I know that the diameter of the circle is its longest chord? **[E7]**
- Do I know how to locate the diameter of a circle and the center of a circle? **[E4, E7]**
- Can I show that when I draw the perpendicular bisectors of two chords in a circle, they will intersect at the centre of the circle? **[E7]**
- How do I know that equal chords are the same distance from the centre of a circle? **[E7]**
- Do I know how to write the converse of "if P, then Q" ? **[E12]**
- When a statement and its converse are true, can I write them using "iff"(if and only if)? **[E12]**
- Do I know that if a statement is true, its converse is not necessarily true? **[E12]**
- Can I solve problems using properties of chords? **[E4]**
- Can I use distance and midpoint formulas in proofs? **[D1]**

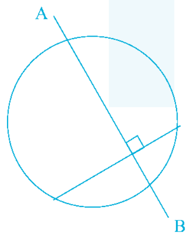
What MIGHT it look like...

on the provincial exam?

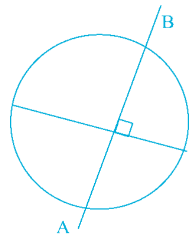
EXAMPLE 1

In which diagram is there enough evidence to conclude that the line segment AB passes through the centre of the circle?

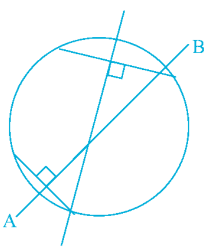
A.



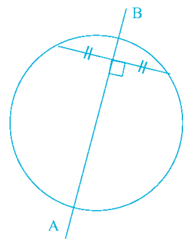
B.



C.



D.



Ways of Thinking about Solutions

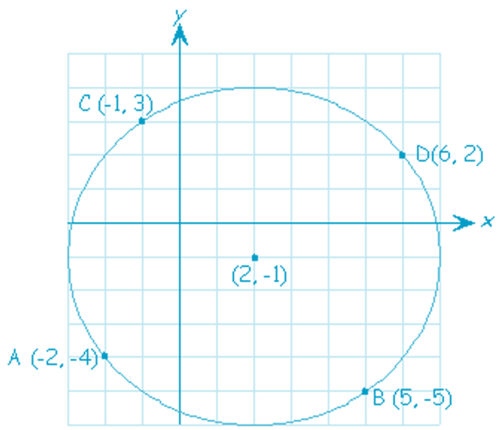
What chord property must I use to solve the problem?

When a line is the perpendicular bisector of a chord, then it must go through the centre of a circle
∴ D is the correct answer

E X A M P L E 2

Given two chords, $\overline{AB}$ and $\overline{CD}$, with points $A(-2, -4)$, $B(5, -5)$, $C(-1, 3)$, and $D(6, 2)$ on a circle with centre $(2, -1)$:

(a) Show algebraically that the distance from the midpoint of chord $\overline{AB}$ to the centre of the circle is the same as the distance from the midpoint of chord $\overline{CD}$ to the centre of the circle.



Ways of Thinking about Solutions

Do I know the midpoint and distance formulas?

For midpoint: midpt of $\overline{AB} = \left(\frac{-2+5}{2}, \frac{-4-5}{2}\right) = (1.5, -4.5)$

midpt of $\overline{CD} = \left(\frac{-1+6}{2}, \frac{3+2}{2}\right) = (2.5, 2.5)$

Distance from midpoint of $\overline{AB}$ to the centre:

$d = \sqrt{(2-1.5)^2 + (-1+4.5)^2} = \sqrt{12.5}$

Distance from midpoint of $\overline{CD}$ to the centre:

$d = \sqrt{(2-2.5)^2 + (-1-2.5)^2} = \sqrt{12.5}$

equal distance

(b) (i) Complete this theorem: If two chords on a circle are equidistant from the centre of the circle, then ...

Ways of *Thinking* about Solutions

Can I remember the properties of chords?

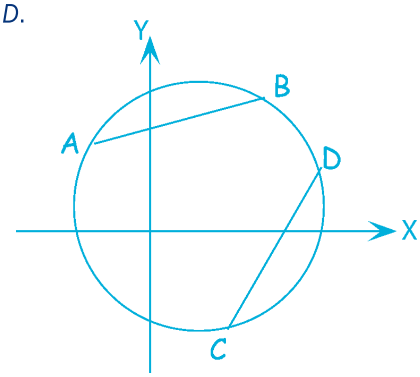
the two chords are congruent

(ii) State the converse of the theorem in part (i).

If two chords on a circle are congruent then they are equidistant from the centre.

EXAMPLE 3

For the following diagram, explain how you would find the centre of the circle, given the coordinates of *A*, *B*, *C*, and *D*.



Ways of *Thinking* about Solutions

What properties of chords should I use to find the centre of a circle?

I would determine the midpoints of chord $\overline{AB}$ and $\overline{CD}$. Find the $\perp$ bisector of each chord. The point of intersection of the 2 $\perp$ bisectors is the centre of the circle.

or

Fold A onto B, unfold and draw a line on the crease. Fold C onto D, unfold and draw a line on the crease. The intersection point of these 2 lines is the centre of the circle.

Can I DO these on my own?

From the text, try these questions:

- Page 209 (question 10)
- Page 295 (question 4)

Circle Geometry

COORDINATE GEOMETRY

PAGES **222** TO **231** IN THE TEXT

Outcomes

I am expected to...

- **D1** develop and apply formulas for distance and midpoint
- **E4** apply properties of circles
- **E5** apply inductive reasoning to make conjectures in geometric situations
- **E7** investigate, make, and prove conjectures associated with chord properties of circles

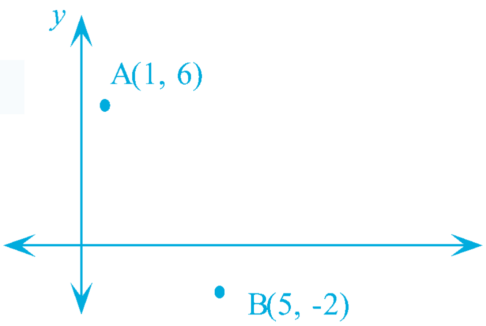
What do I HAVE to know?

- Can I write proofs involving the manipulation of coordinates? **[E4, E7]**
- Can I solve problems involving slope, distance, and midpoint by using coordinates? **[E7, D1]**
- Can I use the property of equal slopes to prove that two lines (two sides of a figure) are parallel? **[E7, E11]**
- Can I use the property of negative reciprocal to prove that two lines (two sides of a figure) are perpendicular? **[E7]**
- Do I know that the equation of a circle with radius r and its centre at the origin is $x^2 + y^2 = r^2$? **[E4, E7]**

What MIGHT it look like...
on the provincial exam?

EXAMPLE 1

Given the points *A* and *B* as shown, which expression will calculate length *AB*?



A. $\sqrt{(1-6)^2 + (5+2)^2}$

B. $\sqrt{(1+5)^2 + (6-2)^2}$

C. $\sqrt{(1-5)^2 + (6+2)^2}$

D. $\sqrt{(6+1)^2 + (-2+5)^2}$

Ways of *Thinking* about Solutions

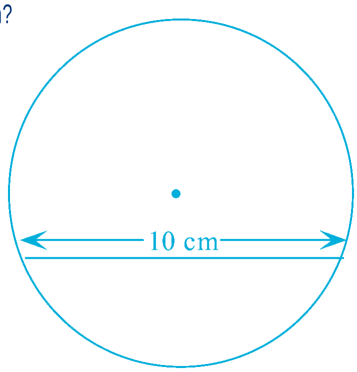
Should I use the distance formula?

$$D = \sqrt{(1-5)^2 + (6-(-2))^2}$$
$$= \sqrt{(1-5)^2 + (6+2)^2}$$

$\therefore$ C is correct

EXAMPLE 2

What is the distance in centimetres between the chord and the centre of the circle if the radius of the circle is 7 cm?



A. $4\sqrt{6}$

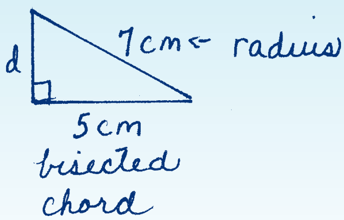
B. $2\sqrt{6}$

C. $\sqrt{51}$

D. $\sqrt{74}$

Ways of Thinking about Solutions

How can I use the given information (radius and chord length) to solve this?



$\therefore\ 7^2 = 5^2 + d^2$

$49 - 25 = d^2$

$\sqrt{24} = d$

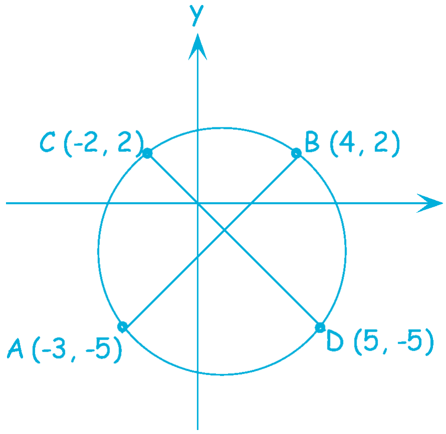
$2\sqrt{6} = d$

B is the correct answer

3

EXAMPLE 3

A, B, C, and D are four points on a circle:



(a) Prove that $\overline{AB}$ and $\overline{CD}$ are the same length.

Ways of Thinking about Solutions

Finding the length means using the distance formula.

$AB = \sqrt{(-3-4)^2 + (-5-2)^2} = \sqrt{49+49} = \sqrt{98}$

$CD = \sqrt{(-2-5)^2 + (2+5)^2} = \sqrt{49+49} = \sqrt{98}$

$AB = CD.$

(b) Do the chords bisect each other? Justify your answer.

Ways of Thinking about Solutions

What does "bisect" signify?

Midpoint $\overline{AB} = \left(\frac{-3+4}{2}, \frac{-5+2}{2}\right) = \left(\frac{1}{2}, -\frac{3}{2}\right)$

Midpoint $\overline{CD} = \left(\frac{-2+5}{2}, \frac{2-5}{2}\right) = \left(\frac{3}{2}, -\frac{3}{2}\right)$

not the same

$\therefore$ $\overline{AB}$ and $\overline{CD}$ do not bisect each other.

EXAMPLE 4

Line L_1, with the equation $y = -\frac{1}{3}x - \frac{8}{3}$, intersects the chord $\overline{AB}$, with points $A(3, -7)$ and $B(5, -1)$.

Prove that L_1 is perpendicular to $\overline{AB}$.

Ways of Thinking about Solutions

How can I use slopes to determine that lines are perpendicular?

Slope of $L_1 = -1/3$

Slope of $\overline{AB} = \frac{-7-(-1)}{3-5} = \frac{-6}{-2} = 3 \quad \therefore L_1 \perp \overline{AB}$

Can I DO these on my own?

From the text, try the following questions:

Page 231 (questions 31, 32, 34, and 35)

Page 295 (questions 6 to 9)

Circle Geometry

CHORD PROPERTIES AND PROOFS

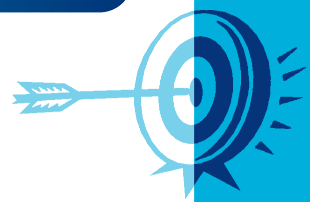
PAGES **206** TO **231** IN THE TEXT

ADVANCED

In addition to Mathematics 12 outcomes, you are also responsible for:

Outcomes

I am expected to...



E11 write proofs using various axiomatic systems and assess the validity of deductive arguments

What do I HAVE to know?

- Do I know that deductive proof involves drawing valid conclusion from establish facts. **[E11]**
- Do I know that SSS, SAS, ASA, SAA, and HL are sufficient conditions to prove that two triangles are congruent? **[E11]**
- Can I read a proof written by someone else and determine if each of the steps in the proof are correct? **[E11]**
- Do I know that CPCTC stands for "corresponding parts of congruent triangles are congruent"? **[E11]**

ADVANCED

Circle Geometry

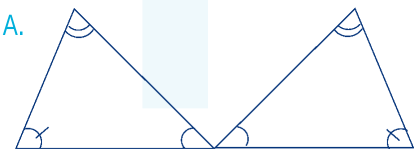
CHORD PROPERTIES AND PROOFS

What MIGHT it look like...
on the provincial exam?

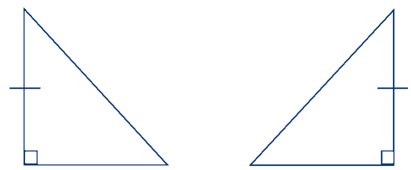
EXAMPLE 1

Which of the following pairs of triangles have sufficient information given to be proven congruent?

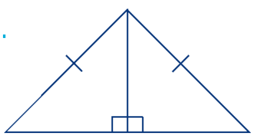
A.



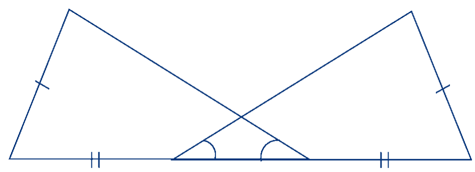
B.



C.



D.



Ways of Thinking about Solutions

Which congruence theorem can I use to prove the congruency of triangles?

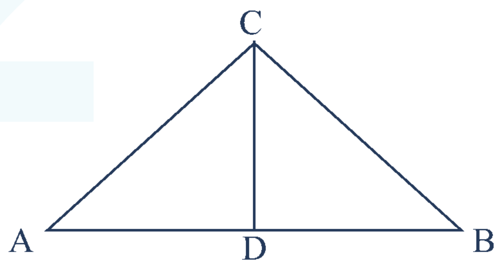
A) - AAA is not a congruence theorem
B) - right triangles and one side congruent : not enough information
C) - right triangle, hypothenuse and one leg congruent - HL theorem
D) - SSA - not a congruence theorem

∴ C is the correct answer.

2

EXAMPLE 2

Given that D is the midpoint of AB and $\angle A \cong \angle B$, you want to prove that triangles *ACD* and *BCD* are congruent.



Here are the first three statements of the proof:

$CD = CD$ (common side)
$AD = BD$ (definition of midpoint)
$\angle A \cong \angle B$ (given)

Which of the following would be the next statement to the proof?

A. $AD = CD$

B. $AC = BC$

C. $\angle A \cong \angle B$

D. $AD = \frac{1}{2}AB$

Ways of Thinking about Solutions

To prove the two triangles congruent, what am I missing?

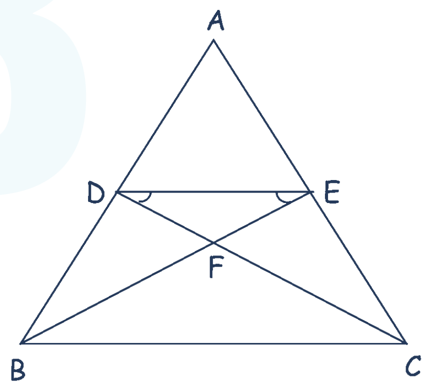
AC = BC - isosceles triangle
∴ B is the correct answer.

Circle Geometry

CHORD PROPERTIES AND PROOFS

EXAMPLE 3

Given: and $\angle EDF \cong \angle FED$ and $FB = FC$, prove that $DB = EC$.



Ways of **Thinking** *about Solutions*

$\overline{DB}$ and $\overline{EC}$ belong to triangles DBC and ECB, respectively.

Can I prove that these triangles are congruent?

$\angle EDF \cong \angle FED$	Given
$\triangle DEF$ is an isosceles $\triangle$	Def'n of isosceles $\triangle$
$DF = FE$	isosceles $\triangle$
$FC = FB$	Given
$DC = BE$	segment addition
$\angle DCB \cong \angle EBC$	Isosceles $\triangle$, $FC = FB$
$BC = BC.$	Common side
$\triangle DCB \cong \triangle EBC$	SAS
$\therefore$ $DB = EC$	CPCTC

Circle Geometry

CHORD PROPERTIES AND PROOFS

or

$\overline{DB}$ and $\overline{EC}$ belong to triangles *DFB* and *EFC*, respectively.
Can I prove that these triangles are congruent?

$\angle EDF \cong \angle FED$	(Given)
$\triangle DEF$ is an isosceles $\triangle$	(Def'n of isosceles $\triangle$)
$\therefore DF = EF$	(Isosceles $\triangle$)
$\angle DFB \cong \angle EFC$	(X theorem)
$FB = FC$.	(Given)
$\therefore \triangle DFB \cong \triangle EFC$.	(SAS.)
$\therefore DB = EC$	(CPCTC)

Can I DO these on my own?

Try these questions from the text:

- Page 217 (questions 25 and 26)
- Page 219 (question 29)
- Page 220 (question 32)

ADVANCED

Circle Geometry

ANGLES IN A CIRCLE

PAGES **232** TO **243** IN THE TEXT

Outcomes

I am expected to...

- **E4** apply properties of circles
- **E5** apply inductive reasoning to make conjectures in geometric situations
- **E8** **[ADV]** investigate, make, and prove conjectures associated with angle relationships in circles
- **E11** write proofs using various axiomatic systems and assess the validity of deductive arguments

What do I HAVE to know?

- Do I know what a central angle and an inscribed angle are and how they are related? **[E4, E8 ADV]**
- Do I know that an arc may be measured in degrees, like an angle? **[E4]**
- Do I know that the measure of an arc in a circle is equivalent to the measure of the central angle that it subtends? **[E4, E8 ADV]**
- Do I know that arcs and chords can also subtend inscribed angles? **[E4]**
- Do I know that all inscribed angles subtended by the same arc are congruent? **[E5, E8 ADV]**
- Do I know that if a chord is a diameter then it subtends an inscribed angle, whose measure is 90º? **[E5, E8 ADV]**
- Do I know that a cyclic quadrilateral is a quadrilateral inscribed in a circle, and that its opposite angles are supplementary? **[E5, E8 ADV]**

Circle Geometry

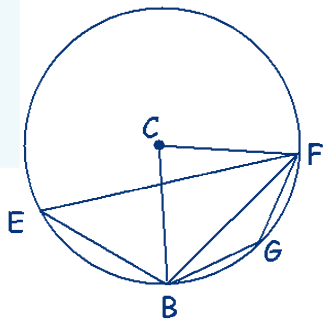
ANGLES IN A CIRCLE

What MIGHT it look like...

on the provincial exam?

EXAMPLE 1

Given $\angle FBC = 50°$, what is the measure of $\angle E$?



A. 30°

B. 40°

C. 50°

D. 60°

Ways of Thinking about Solutions

Since $\angle E$ and the central angle are both subtended by the same arc, can the information help me find the measure of the central angle?

Since $\angle$ FBC is equal to 50°, $\angle$ CFB is also equal to 50° (isosceles triangles) Therefore $\angle$C = 80°. $\angle$E is an inscribed angle subtented by the same arc as the central angle "C", therefore $\angle$E measures ½ of 80°.

B is the correct answer.

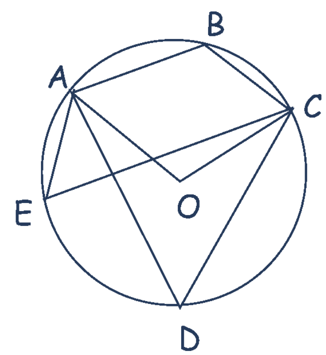
Circle Geometry

ANGLES IN A CIRCLE

2

EXAMPLE 2

For this diagram, which of the following statements is correct?



A. $\angle AOC \cong \angle ABC$

B. $m\angle AOC + m\angle ABC = 180^o$

C. $\angle AEC \cong \angle ADC$

D. $\angle OAB \cong \angle OCB$

Ways of Thinking about Solutions

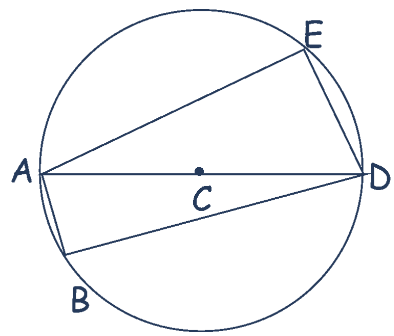
Examine the choices to see if I can apply any of the circle properties.

C must be correct because these two angles are both subtented by $\overset{\frown}{AC}$, and they are both inscribed angles.

3

EXAMPLE 3

Given that *C* is the centre of the circle, which one of the following statements is true?



A. $\angle CAE \cong \angle BDE$ B. $\angle ABD \cong \angle AED$

C. $\overline{AE} \cong \overline{BD}$ D. $\overline{AB} \cong \overline{DE}$

Ways of Thinking about Solutions

Examine the choices to see if I can apply any of the circle properties.

$\angle ABD$ and $\angle AED$ are both 90° because they are inscribed angles subtended by semi-circles, so they must be congruent. B is the correct answer.

Can I DO these on my own?

Try these questions from the text:

- Page 237 (questions 16 and 17)
- Pages 238 and 239 (question 18)
- Page 241 (question 33)
- Pages 242 and 243 (questions 35 – 46)

Circle Geometry

EQUATIONS OF CIRCLES AND ELLIPSES

PAGES **252** TO **267** IN THE TEXT

Outcomes

I am expected to...

E3 [ADV] write the equations of circles and ellipses in transformational form and as mapping rules to visualize and sketch graphs

E13 [ADV] analyse and translate between symbolic, graphical, and written representations of circles and ellipses

E14 [ADV] translate between different forms of equations of circles and ellipses

E15 [ADV] solve problems involving the equations and characteristics of circles and ellipses

E16 [ADV] demonstrate the transformational relationship between the circle and the ellipse

What do I HAVE to know?

- Can I explain why a circle is not a function? **[E3ADV]**
- Given the equation of a circle or an ellipse in general form, can I rewrite it in transformational or standard form? **[E14ADV]**
- Given the equation of an ellipse or circle in transformational or standard form, can I describe the transformations of $x^2 + y^2 = 1$ in words and as mapping rules? **[E3ADV]**
- Given the equation or mapping rule of a circle or an ellipse, can I sketch its graph? **[E3ADV]**

Circle Geometry

EQUATIONS OF CIRCLES AND ELLIPSES

ADVANCED

- Can I determine the equation of a circle or an ellipse given its graph or its mapping rule? **[E3ADV, E13ADV]**
- Can I rewrite the equation of a circle or an ellipse from transformational or standard form to general form? **[E14ADV]**
- Do I know how to identify when an equation represents a circle or an ellipse? **[E16ADV]**
- Can I determine the major and minor axes of an ellipse? **[E16ADV, E15ADV]**
- Can I solve problems involving circles and ellipses? **[E15ADV]**

What MIGHT it look like…

on the provincial exam?

1

EXAMPLE 1

Determine the centre and radius of the circle defined by this equation:

$x^2 + y^2 - 4x + 6y - 12 = 0$

Ways of Thinking about Solutions

To determine the centre and the radius, I need to rewrite the equation in standard or transformational form.

$$x^2 - 4x \quad + y^2 + 6y \quad = 12$$
$$x^2 - 4x + 4 + y^2 + 6y + 9 = 12 + 4 + 9$$
$$(x-2)^2 + (y+3)^2 = 25$$

So, the centre is $(\underset{h.t.}{2}, \underset{v.t.}{-3})$ and the radius is $\underset{\sqrt{25}}{5}$.

Circle Geometry

EQUATIONS OF CIRCLES AND ELLIPSES

2

EXAMPLE 2

In an ellipse, $A(-7, 2)$ and $B(3, 2)$ are the endpoints of the major axis and $C(-2, -1)$ and $D(-2, 5)$ are the endpoints of the minor axis. Determine the equation of the ellipse in general form.

Ways of Thinking about Solutions

Am I able to determine the lengths of the minor and major axes and the coordinates of the centre of the ellipse by examining the given coordinates?

I can tell from the co-ordinates that the major axis $\overline{AB}$ is horizontal and 10 units long. The vertical axis is 6 units long. The centre would be at the midpoint of $\overline{AB}$ or $\overline{CD}$, $(-2,2)$.

So, the equation of the ellipse: $\left[\frac{1}{5}(x+2)\right]^2 + \left[\frac{1}{3}(y-2)\right]^2 = 1$

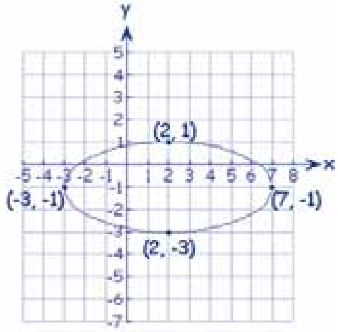
Now, translate to general form: $\frac{1}{25}(x^2+4x+4) + \frac{1}{9}(y^2-4y+4) = 1$

Multiply all terms by LCD 225: $\left(\frac{1}{25}(x^2+4x+4)\right)^{\times 225} + \left(\frac{1}{9}(y^2-4y+4)\right)^{\times 225} = (1)\times 225$

$9x^2+36x+36+25y^2-100y+100=225 \rightarrow 9x^2+25y^2+36x-100y-89=0$

EXAMPLE 3

Which of the following equations represents this graph?



A. $\left[\frac{(x+2)}{2}\right]^2 + \left[\frac{(y-1)}{5}\right]^2 = 1$

B. $\left[\frac{(x-2)}{5}\right]^2 + \left[\frac{(y+1)}{2}\right]^2 = 1$

C. $\left[\frac{(x+2)}{10}\right]^2 + \left[\frac{(y-1)}{4}\right]^2 = 1$

D. $\left[\frac{(x-2)}{25}\right]^2 + \left[\frac{(y+1)}{4}\right]^2 = 1$

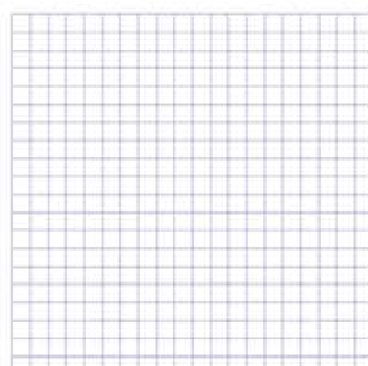
Ways of Thinking about Solutions

First I should determine the coordinates of the centre and the length of the minor and major axes.

The centre has to be (2, -1), so look for (x-2) and (y+1). Either B or D could be correct. The major axis is 10 units, therefore B is correct.

EXAMPLE 4

Write $25x^2 + 4y^2 + 150x - 16y + 141 = 0$ in transformational form and sketch its graph.



Ways of Thinking about Solutions

How do I rewrite this equation in transformational form?

$$25x^2 + 150x + 4y^2 - 16y = -141$$

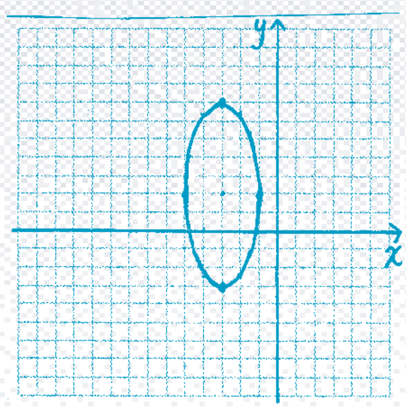
$$25(x^2 + 6x + \quad) + 4(y^2 - 4y + \quad) = -141$$

$$25(x^2 + 6x + 9) + 4(y^2 - 4y + 4) = -141 + 225 + 16$$

$$\frac{25(x+3)^2}{100} + \frac{4(y-2)^2}{100} = \frac{100}{100}$$

$$\frac{(x+3)^2}{4} + \frac{(y-2)^2}{25} = 1$$

$$\left[\frac{1}{2}(x+3)\right]^2 + \left[\frac{1}{5}(y-2)\right]^2 = 1 \quad \text{or} \quad \left(\frac{x+3}{2}\right)^2 + \left(\frac{y-2}{5}\right)^2 = 1$$



5

E X A M P L E 5

State the domain and range for $9(x - 1)^2 + 16(y + 2)^2 = 144$

Ways of Thinking about Solutions

To state the domain and range I should rewrite the equation in transformational form.

$$\frac{9}{144}(x-1)^2 + \frac{16}{144}(y+2)^2 = 1$$

$$\frac{1}{16}(x-1)^2 + \frac{1}{9}(y+2)^2 = 1$$

$$\left[\frac{1}{4}(x-1)\right]^2 + \left[\frac{1}{3}(y+2)\right]^2 = 1$$

H.S. of 4

centre at $(1,-2)$

Note: A quick sketch would be beneficial

Domain: $1-4 \le x \le 1+4$, $x \in R$ $\therefore$ Domain $-3 \le x \le 5$, $x \in R$

Range: $-2-3 \le y \le -2+3$, $y \in R$ Range. $-5 \le y \le 1$, $y \in R$

Can I DO these on my own?

Try these questions from the text:

- Page 255 to 257 (questions 7–20)
- Pages 259 and 260 (questions 23–30)
- Page 261 (question 35 and 36)
- Pages 263 and 264 (questions 37–41)
- Pages 265 and 267 (questions 42–51)
- Pages 297 and 298 (questions 22–28)

Notes

Probability

EXPERIMENTAL AND THEORETICAL PROBABILITIES

PAGES **300** TO **306** IN THE TEXT

Outcomes

I am expected to...

G2 demonstrate an understanding that determining probability requires the quantifying of outcomes

What do I HAVE to know?

- Do I understand that probability is the ratio of the number of ways of achieving success to the total number of possible outcomes? **[G2]**
- Do I understand that the complement of an event X is "event X not occurring," symbolized as $\overline{X}$? **[G2]**
- Do I understand the difference between experimental probability and theoretical probability? **[G2]**

What MIGHT it look like...
on the provincial exam?

EXAMPLE 1

Two students, Peter and Ken, performed a simulation that involved tossing three coins. They recorded the number of heads and tails from 10 trials. According to the results, shown in the table below, what is the experimental probability of getting three heads on a single trial?

Trial 1	HHT
Trial 2	HHT
Trial 3	HTT
Trial 4	HHH
Trial 5	HTT
Trial 6	THH
Trial 7	HTH
Trial 8	TTT
Trial 9	HHH
Trial 10	TTT

Probability

EXPERIMENTAL AND
THEORETICAL PROBABILITIES

Ways of Thinking about Solutions

Experimental probability is determined based on observed results.

There were 10 trials, 2 of which turned up heads – probability is the ratio $\frac{\text{number of successes}}{\text{total number of outcomes}}$

so $P(3H) = \frac{2}{10} = \frac{1}{5}$

Can I DO these on my own?

Try these questions from the text:

Pages 301 and 302 (questions 3, 4, and 11)

Probability

COUNTING AND PROBABILITY

PAGES **307** TO **319** IN THE TEXT

Outcomes

I am expected to...

G2 demonstrate an understanding that determining probability requires the quantifying of outcomes

G3 demonstrate an understanding of the fundamental counting principle and apply it to calculate probabilities of dependent and independent events

What do I HAVE to know?

- Do I know that the "sample space" is the total number of possible outcomes? **[G2]**
- Do I know that given $P(A)$ and $P(B)$, the Fundamental Counting Principle states that $P(A \text{ and } B) = P(A) \times P(B)$, and that this is often referred to as the Multiplication Principle? **[G3]**
- Do I understand the difference between $P(A \text{ and } B)$ and $P(A \text{ or } B)$? **[G3]**
- Do I understand when events are mutually exclusive? **[G3]**
- Do I understand that $P(A \text{ or } B) = P(A) + P(B)$, if A and B are mutually exclusive events? **[G3]**
- Can I explain the difference between dependent and independent events? **[G3]**
- When appropriate, can I organize given information within a Venn diagram? **[G3]**
- Do I understand that $P(A \text{ or } B)$ is determined by $P(A) + P(B) - P(A \text{ and } B)$ when events are not mutually exclusive? **[G3]**

What MIGHT it look like...

on the provincial exam?

EXAMPLE 1

If we shuffle a standard deck of 52 cards and randomly select one card, what is the probability of selecting a king or a queen?

A. $\frac{1}{52}$

B. $\frac{1}{26}$

C. $\frac{1}{13}$

D. $\frac{2}{13}$

Ways of Thinking about Solutions

This is simple probability using the word "or," and the two events are mutually exclusive.

$P(K) = \frac{4}{52}$ ← number of kings
52 → total number of cards

$P(Q) = \frac{4}{52}$

So, $P(K \text{ or } Q) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$

So D is the answer

EXAMPLE 2

Two coins and one die are simultaneously tossed on a table. What is the probability of obtaining two heads and a 6?

A. $\frac{5}{12}$

B. $\frac{1}{24}$

C. $\frac{1}{5}$

D. $\frac{1}{8}$

Ways of *Thinking* about Solutions

How many different ways can two coins land?

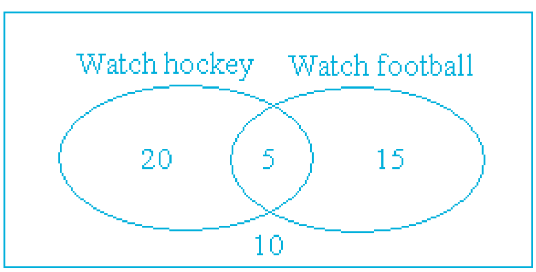
two coins can land HH, HT, TH, TT. Only one of these is HH, so $P(2H) = 1/4$

... the die has 6 faces, $P(6) = 1/6$

$\therefore$ $P(2H \text{ and } 6) = \frac{1}{4} \cdot \frac{1}{6} = \frac{1}{24}$, so B is correct

EXAMPLE 3

Some students were asked whether they watch hockey or football. The results are shown in the following Venn diagram:



What is the probability that a person chosen at random watches hockey?

A. $\frac{2}{5}$

B. $\frac{1}{2}$

C. $\frac{4}{5}$

D. $\frac{3}{10}$

Ways of Thinking about Solutions

Altogether, 50 students were asked, and 25 students watch hockey, so ...

$P(H) = \frac{25}{50} = \frac{1}{2}$, so B is correct

Can I DO these on my own?

1. Two boxes contain marbles. The first contains five red marbles and three white marbles. The second box contains four black marbles and seven green marbles. One marble is chosen at random from each box. What is the probability that a white and a black marble will be chosen?

A. $\frac{3}{22}$

B. $\frac{33}{32}$

C. $\frac{1}{12}$

D. $\frac{12}{25}$

2. Mr. Smith has three pairs of black pants and two pairs of grey pants in the first drawer of his dresser. In the second drawer, he has one white shirt and four multi-coloured shirts. He gets up late one morning and without looking quickly grabs a pair of pants from the first drawer and a shirt from the second drawer. What is the probability that he grabs a pair of black pants and a white shirt?

A. $\frac{3}{25}$

B. $\frac{4}{25}$

C. $\frac{4}{5}$

D. $\frac{2}{5}$

3. Two dice are rolled. What is the probability that both dice will land on the same number?

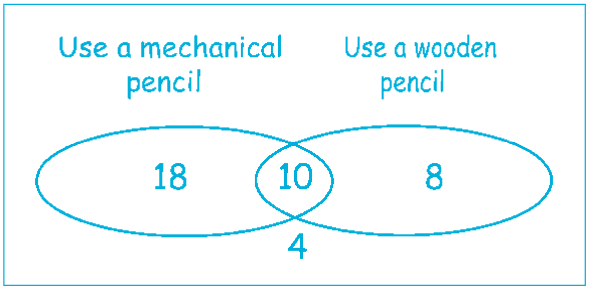
A. $\frac{1}{36}$

B. $\frac{1}{18}$

C. $\frac{1}{6}$

D. $\frac{1}{2}$

4. Forty people were surveyed and asked whether they use a mechanical pencil or a wooden pencil when writing a test. The results are shown in the Venn diagram below.



What is the probability that one person chosen at random does not use a wooden pencil when writing a test?

A. $\frac{9}{20}$

B. $\frac{4}{5}$

C. $\frac{7}{10}$

D. $\frac{11}{20}$

5. Try these questions from the text.
Page 310 (questions 10, 11, and 12)
Page 312 (questions 21 to 24)
Page 313 (questions 25 to 28)
Page 314 (questions 30 and 31)

Probability

COMBINATIONS AND PERMUTATIONS

PAGES **327** TO **339** IN THE TEXT

Outcomes

I am expected to...

A6 develop an understanding of factorial notation and apply it to calculating permutations and combinations

B8 determine probabilities using permutations and combinations

G7 distinguish between situations that involve combinations and permutations

G8 develop and apply formulas to evaluate permutations and combinations

What do I HAVE to know?

- When I read a problem can I identify if the situation requires combinations or permutations? **[G7]**
- Do I know when and how to use factorial notation? **[A6]**
- Do I know when and how to apply the formula ${}_nP_r = \dfrac{n!}{(n-r)!}$? **[A6, G8]**
- Do I know when and how to apply the formula for ${}_nC_r = \dfrac{n!}{r!(n-r)!}$? **[A6, G8]**

What MIGHT it look like...

on the provincial exam?

EXAMPLE 1

Tim and Rebecca are the first and second students in a line of seven students waiting to buy tickets for a concert. The number of different orders in which the remainder of the students can line up behind them is

A. 5!

B. 7!

C. (5!)(2!)

D. $\dfrac{7!}{2!}$

Ways of *Thinking* about Solutions

Since Tim and Rebecca are already in line to buy tickets, there are only five people to line up behind them.

There are 5 different people to be in the third position, 4 different in the fourth position and so on. So 5! is the answer.

EXAMPLE 2

Use a real-life example to explain why $_4C_4 = 1$.

Ways of *Thinking* about Solutions

$_4C_4$ means "how many different ways are there for choosing four items out of four items, if the order doesn't matter?" Now, in real life ...

A pizza store has a sale on 4-item pizzas. Let's see, how many ways are there to choose from the 4 toppings if I want all 4 toppings? There is only one way to choose ... "give me all 4 toppings."

EXAMPLE 3

A class is made up of 13 girls and 9 boys. If five students are chosen at random, what is the probability that five girls will be chosen?

Ways of *Thinking* about Solutions

There are 22 students altogether. Probability is the ratio of the number of students chosen to the total number of students. Is the number chosen a combination or a permutation?

number of successes => 5 girls chosen from 13 girls – the order of choice does not matter, so a combination

total number of outcomes => 5 will be chosen from 22 students

So. $\frac{{}_{13}C_5}{{}_{22}C_5} = \frac{1287}{26\,334} \doteq 0.05$

∴ The probability of choosing 5 girls is 0.05

or

$\frac{13}{22} \times \frac{12}{21} \times \frac{11}{20} \times \frac{10}{19} \times \frac{9}{18} = \frac{1287}{26\,334} \doteq 0.05$

EXAMPLE 4

The value of $\frac{1000!}{999!}$ is

A. 1

B. $1.\overline{001}$

C. 1000

D. undefined

Ways of Thinking about Solutions

My calculator gives me an error message, so the numbers must be too big.
I'll have to do this one by hand.

$$\frac{1000!}{999!} = \frac{1000 \times 999 \times 998 \times \ldots \times 1}{999 \times 998 \times 997 \times \ldots \times 1} = 1000$$

so $\underline{C}$ is the answer

Can I DO these on my own?

1. You want to put eight different books on a shelf, side by side. In how many ways can these books be arranged?

A. $8!$

B. $\frac{8!}{2!}$

C. ${}_8P_1$

D. ${}_8C_8$

2. Peter, Mary, and Susan are part of a group of 10 people. An executive consisting of Peter as the president, Mary as the treasurer, and Susan as the secretary could be formed from this group. What is the probability this executive will be formed?

A. $\frac{1}{{}_{10}P_3}$

B. $\frac{1}{{}_{10}C_3}$

C. $\frac{3}{{}_{10}P_3}$

D. $\frac{3}{{}_{10}C_3}$

3. A prom committee of seven will be chosen from 10 boys and 12 girls. Calculate the probability that the committee will be made up of all girls.

4. Use a *real-life* example to explain why ${}_5C_2 = {}_5C_3$.

5. Try these questions from the text:
Pages 330 and 331 (questions 8, 9, and 11)
Page 332 (questions 16 and 17)
Pages 333 and 334 (questions 21–26)
Page 363 (questions 12–16)

Probability

CONDITIONAL PROBABILITY

PAGES **319** TO **326** IN THE TEXT

Outcomes

I am expected to...

G5 [ADV] determine conditional probabilities

What do I HAVE to know?

- Do I understand what conditional probability means? **[G5ADV]**
- Do I know that $P(A|B)$ is used to denote conditional probability and that it means " the probability of event B occurring, given A"? **[G5ADV]**
- Can I apply the formula $P(A|B) = \dfrac{P(A \text{ and } B)}{P(B)}$? **[G5ADV]**

Can I DO these on my own?

1. A survey of 352 people resulted in this table:

	Developed a cold	Did not develop a cold	Total
Took vitamins regularly	46	139	185
Did not take vitamins regularly	115	52	167
Total	161	191	352

What is the probability that a person who developed a cold did not take vitamins regularly?

A. 115/352　　B. 115/167

C. 46/352　　D. 115/161

2. The following table shows data about students enrolled in grade 12 math at a high school: Assume that event A is "enrolled in Advanced Math 12" and event B is "male." Calculate

(a) $P(A)$

(b) $P(B \mid A)$

Sex	Advanced Math 12	Math 12	Total
Male	63	85	148
Female	72	112	184
Total	135	197	332

3. Use the following chart to calculate the probability of a male being blonde.

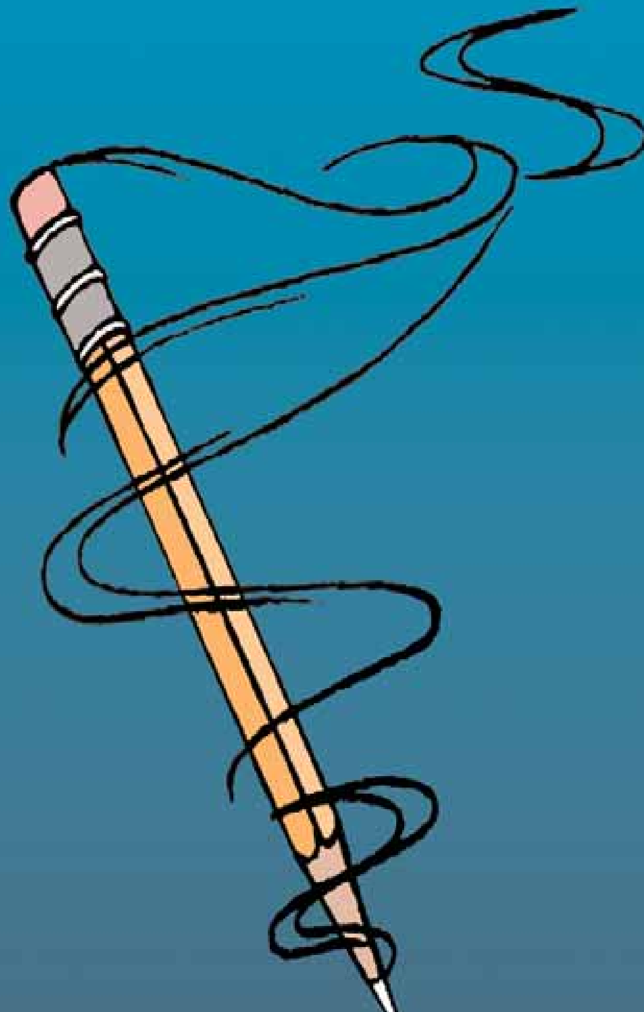
	Blonde	Not blonde
Male	15	18
Female	8	12

4. Try these questions from the text:

Pages 320 and 321 (questions 47–50)

Page 362 (question 10)

Notes



Good Luck on the Exam!