MATHEMATICS
10 PLUS

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A TEACHING RESOURCE

NOVA SCOTIA
Education

MATHEMATICS 10 PLUS:

A TEACHING RESOURCE

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CONTENTS

INTRODUCTION

Mathematics 10 Plus follows the Mathematics 10 curriculum but is presented over 220 hours to allow additional time for revisiting concepts, procedures, and skills when required and to provide enrichment opportunities. Mathematics 10 Plus is a two-credit course, providing successful students with one mathematics credit (Mathematics 10—Course Code 008008) and one elective credit (Mathematics 10 Plus—Course Code 008157).

The *Atlantic Canada Mathematics Curriculum: Mathematics 10* is the principle resource for teachers of mathematics at this level. This document, *Mathematics 10 Plus: A Teaching Resource* gives the additional outcomes and provides yearly, unit, and lesson plans that show the intended philosophy of this course. This resource provides strategies to help teachers get to better know the needs of their students. These strategies include the use of pretests, writing (journal) exercises, take home activities and readings, and written reports. The Mathematics 10 Plus outcomes are listed in Section 1: Instructional Planning of this document. As well, the yearly plan given in Section 1: Instructional Planning lists both the Mathematics 10 outcomes that are required to be addressed (column 2) and Mathematics 10 Plus outcomes (column 3) that may be addressed as required. It is the intention of this course that the two sets of outcomes are addressed concurrently rather than sequentially.

How does one determine which students would benefit by reviewing topics previously explored in earlier grades, and which students are ready for enrichment?

In preparation for teaching the Mathematics 10 Plus course teachers must make decisions about the order of units to teach and how those units are connected to the outcomes for student achievement. They have to consider ways to determine when the review of previously taught concepts is important, and when students need enrichment. They might consider:

- pretests: Teachers might begin a section of unit with a pretest (see example of pretest in the Day 2 plan for the beginning of the developed unit) to determine the strengths and weaknesses of their students. The test should allow students to show their use of appropriate mathematical skills and provide evidence of conceptual and procedural understanding.
- verbal explanations: Teachers should provide students with many opportunities to show reasoning and understanding by asking students to write and or speak about what they are learning. (See examples of this in the Day 1 homework, Day 2 pretest, and in the Day 5 Reflection activity for the developed unit).
- take home assignments: Teachers can ask students to do activities at home

that are based on previous knowledge to determine their level of confidence and comfort with particular concepts. (See an example of this in the Day 8 homework assignment for the developed unit).

- reading activities: Teachers can make use of reading activities, like articles or parts of articles from the Mathematics Teacher, NCTM periodical, or from other magazines or papers, where students can prepare either written or oral reports on previously learned topics, concepts, and procedures. Reading articles are also a good way of enriching the curriculum. Ask some students to read an article and prepare a classroom demonstration or presentation. (See examples of these in the Appendix).

Mathematics 10 Plus: A Teaching Resource is intended to provide support for teachers as they design their Mathematics 10 Plus program.

- Section 1: Instructional Planning, sample yearly plan shows possible ways to cluster and sequence outcomes. A unit plan and lesson plans for unit 5, How Far? How Tall? How Steep?, is given to demonstrate how the two sets of outcomes can be addressed in concert with one another.
- Section 2: Language and Mathematics gives teachers strategies to challenge students to think and reason about mathematics and communicate their ideas to others orally or in writing in a clear and convincing manner. Strategies were chosen to promote vocabulary development, problem solving, and reflection in the mathematics classroom.
- Section 3: Assessment emphasizes that the way we assess has changed. It is no longer sufficient to test only a student's recall of a topic. It is also important to have students show that they understand and can apply the ideas in mathematics and that they are capable of using manipulatives and calculators to aid in this understanding.
- Section 4: Technology identifies the underpinnings of technological competence in the school program and how it applies to the mathematics classroom.

SECTION 1: INTRUCTIONAL PLANNING

Rationale for Yearly Plan

Starting with chapter 5 allows the student to explore numbers and number patterns, while connecting them to visual representations. This is an excellent opportunity to explore patterns with squares, represent numbers in multiple ways, order and operate on numbers and solve interesting problems, both routine and non-routine. In chapter 5, students will understand the importance of recognizing patterns, creating visual representations, reading word problems carefully and repeatedly, using variables to represent unknown quantities, using appropriate notation, solving simple equations, using the calculator correctly and efficiently, using units appropriately, and making concluding statements. Students have a "way in" as there is very little prior knowledge required for success. The symbolism is used only after students have come to the understanding of the patterns and their importance. Work in this chapter and the relevant 10 Plus outcomes should span the first or second week in September to the end of the first week in October.

Chapter 1 represents a change of pace for the student. The approach is very visual and numerical as it develops the concepts associated with observing patterns, extracting meaning from data and making predictions with some degree of confidence. This is an opportunity to work with whole numbers as well as decimals resulting from measurement and discuss reasonableness of the statistics that arise from the calculations, continuing to develop students' number sense. The normal distribution provides a context for further work on percent. The more students can see themselves in the data, the more they are likely to relate to the concepts that are being developed. The data displays developed here should be continued throughout the year. For example, when tests are returned to students, the teacher could illustrate results with a stem-and-leaf plot, box-and-whisker plot, frequency polygon and/or giving either the five-point summary or information about measures of central tendency and standard deviation with discussion to reinforce understanding. The chapter concludes with scatter plots and a brief exploration of lines of best fit which serve as in introduction to two-variable statistics and the concepts, language and visual representations associated with dependence. The time line for this chapter runs from the second week in October to the third week in November.

Chapters 3 and 4 are heavily symbolic. It is important to connect the symbolism, language and algebraic skills with visual and contextual understanding. It is good practice to make use of opportunities for students to work in groups and explain their thinking and reasoning. Work in chapter 3 spans the time from the last week in November to the third week of January. This allows one week of review if students are to prepare for an exam. Some exploration of the subsets of the real numbers is needed so that students can distinguish as needed in discussions of domain and range, as well as in describing solutions of equations. It is important to take the time to develop the processes associated with solving equations using algebra tiles and then, through discussion and student observation, synthesize the understanding so that students see that they are “undoing” equations by performing inverse operations and further, when an operation is performed, both sides of the equation must be treated the same to preserve the balance. Any “shortcuts” must be careful to preserve these two critical concepts. (If students can identify what needs to be undone, and if they “do it” to both sides, they will be able to use the same basic understanding when solving equations requiring more sophisticated methods.) Suggested resources: Pilmer, Algebra with Pizzazz, Shell material.

In section 3.5, return to using algebra tiles. Discuss area – Demonstrate the use of the area model for 23 times 67. Investigation 9 is important for connecting the graph, the intercepts and the factors. More practice must be provided, however. In assessing this unit, opportunities must be provided for students to demonstrate their understanding with regard to all of the outcomes listed and not simply the factoring skill.

February begins chapter 4 which builds on the skills and concepts of chapter 3. Students sketch graphs from stories and data and practice the skill of interpreting a graphical representation. They will also have the experience of creating a story which corresponds to a given graph and recognizing what information can be determined as well as which information is not determined by the graph. The use of a CBR or CBL provides students with validation of their observations and provides a useful demonstration activity. Students can begin to study the dynamics of change as indicated by graphs. The concept of function is introduced in unit 4.2. A comprehensive definition of function can be developed as students work through the exercises in the text. Supplemental activities can be found in Pilmer and the appendix; students should be comfortable with function notation applied to graphs as well as to equations and tables. It may be necessary to spend some time reviewing the visual cues and geometric properties associated with transformations. To begin section 4.3, it is useful to discuss the graph of $y = x^2$ at length and in detail. It may be useful to introduce it by playing “What’s My Rule”. Then, all possible representations of the relation are given. More ordered pairs may have to be computed to answer the following questions. Is the relation a function? How do you know? How can you tell from the graph? from the table? Is it linear? How do you know from the table? From the graph? From the rule (equation)? What is the domain? The range? (Use interval notation and set notation). Will the graph continue? Is it discrete or continuous? Construct a story to fit the graph. Does it fit the whole graph or only a part of it? Where does the shape of the graph come from? The equation is called a quadratic equation and the shape of the curve is called a parabola. Students may need support recognizing what the various transformations look like before they can look at a graph and decide what type has taken place. Section 4.4 and 4.5 continue the study of scatter plots, begun in grade 6, and continued each year through to this course where they will learn to find the equation for the line of best fit first using the median-median approach, both by hand and by using the graphing calculators regression processes, then by the least squares method (calculator only). Near the end of this section students will be given opportunities to explore tables, and situations that require a curve of best fit. Both QuadReg and ExpReg will be used. Students should be able to determine the best fit model by inspecting the curve drawn on the scatter plot and the correlation coefficient value, or by understanding the situation described by the scatter plot and what shape might best describe it, or by examining the patterns or lack of in the residuals.

Chapter 2 presents a change of pace. It is important to verbalize and discuss the definitions associated with the various networks. As this is an area where the homework assignments are not onerous, it presents a possible opportunity to work a parallel unit on understanding and operating on fractions.

How to build skills with numbers:

Although students are encouraged to have and use a calculator, suggest that for basic one digit multiplications and two digit additions and subtractions, they try to answer the question first in their head and use the calculator only as a check. To help with multiplication and developing number sense, create a “times table” sheet.

Examine a Story – work with a set of numbers – what operations keep you in the set (closure property)? Story – what operation can’t you do and stay in the set? Write a story to show understanding.

Supplemental work on matrices – equality; addition; subtraction; multiplication by a scalar; multiplying matrices; applications.

Chapter 7:

Use either Modeling mathematics or Baker’s Choice

Explain the process: Post a sheet of chart paper with the headings – refer to the sheet to show students where they are in the process:

- Understand the problem – discussion of constraints (in words only) and profit and the relationship between these
- students experiment to determine possible and impossible solutions by relating them to the constraints – use the table, plot the points in different colours – observe pattern

- need to take a closer look – investigate one constraint at a time (make a simpler problem) – groups on chart paper – seems to be a boundary – what does boundary look like – what happens on the boundary – write equation – what happens on the good side of the boundary – inequality – overlap graphs – need to refine procedures
- investigate inequalities – one variable to understand notation – two variable gives region on graph – indicate region with shading – practice
- return to Heather's problem – graph all constraints to determine the feasible region
- complete the sentence "If you choose any point in the feasible region . . ."
- investigation of profit line or optimal solution line – Pick a point; determine the profit (or optimal solution). Find another point that gives exactly the same profit (use what we know about solving equations to do this). Find a third point with the same profit. What do you notice. Why? Shortcut – use Autograph once you have generated the equation. Why should the lines be parallel? Work through a couple of different profit lines – show how the equation in the y=mx+b form evolves from the Ax+By=C form and how the slope with be the same in every case. How could we make the profit greater? Where will the line Ax+By=50 be in relation to the others? What about Ax+By=300? How will we know where the greatest profit is?
- How can we determine the coordinates of the point on the optimal solution line that represents the best solution? It is the point last touched by the optimal solution line as it passes through the feasible region.
- Summarize the steps for solving a linear programming problem
  - Read the problem
  - Identify the variables. Represent them using letters.
  - Identify the constraints. Represent them as inequalities.
  - Graph the inequalities to determine the feasible region.
  - Write the optimal solution line equation. Graph a possible line to represent a particular optimal solution.
  - Use the optimal solution line to determine the point(s) that cause the optimal solution. Determine the coordinates for that point or points. OR
  - Determine the coordinates of the vertices of the feasible region.
  - Substitute the coordinates of the vertices in the optimal solution equation to determine the values of the variables that give an optimal solution.
  - Write a concluding statement.

Chapter 6

Use Polydron or GO Frame pieces to help students get a feel for surface area and volume. Allow students to create shapes and apply the definition of pyramids and prisms. Review formulas for area of rectangle, triangle, circle. Use these and trigonometry to determine a simple formula to find the area of any regular n-gon (see question 17, p.369 in the text). Use area formulae and the concept of volume = area of base times height to determine the volume of prisms (rectangular, triangular, pentagonal, hexagonal). Use this to get area of pyramids (one third area of prism with same base and height – done in grade 9). Explore the Economy of Design to determine relationships between minimum surface areas and maximum volumes. Examine similarity of 3D objects.

A suggested laboratory activity: bring in shapes used for packaging; students identify shape; take the required measurements; determine surface area; determine volume. Write a report. It is at this point when a discussion about accuracy and precision is meaningful. All students should not use the same shapes – provide a dozen different ones and each student must do 2 or 3 and write a report describing their process and indicating why that particular package is suitable for the contents. Examples: Toblerone bar, cereal boxes, Quality Street, chips, tuna can, etc.

Get into the chapter activities – explore – conjecture – determine relationships – have fun.

Yearly Plan

Chapter 5: How Far? How Tall? How Steep? (5 weeks)

Math 10 Outcomes	Math 10 Plus Outcomes
5.1 Ratios Based on Right Triangles • apply the properties of similar triangles D2 • solve problems involving similar triangles and right triangles D8 • determine the accuracy and precision of a measurement D7 • solve problems involving measurement using bearings and vectors D6	• choose appropriate strategies for calculating (mental math, estimation, appropriate technology, paper and pencil), and for problem solving 10^{+}B3 • develop and use ratio, rate and proportions as tools for solving problems 10^{+}D3 • demonstrate an understanding that using strategies is useful in solving routine and non-routine problems 10^{+}C7 • represent numbers in multiple ways (including exponents, ratios, percents, proportions, and scientific notation) and apply appropriate representations to solve problems 10^{+}A3

Math 10 Outcomes	Math 10 Plus Outcomes
5.2 The Pythagorean Theorem • apply the Pythagorean theorem D14 • demonstrate an understanding of and write a proof for the Pythagorean theorem E7 • solve problems involving similar triangles and right triangles D8 • use inductive and deductive reasoning when observing patterns, developing properties, and making conjectures E8 • use deductive reasoning, construct logical arguments, and be a able to determine, when given a logical argument, if it is valid E9	• choose appropriate strategies for calculating (mental math, estimation, appropriate technology, paper and pencil) , and for problem solving 10^{+}B3 • demonstrate an understanding of rational and irrational numbers, compare and order them, and apply them in meaningful situations 10^{+}A1 • represent numbers in multiple ways (including exponents, ratios, percents, proportions, and scientific notation) and apply appropriate representations to solve problems 10^{+}A3
5.3 Square Roots and Their Properties • approximate square roots A4 • demonstrate an understanding of and apply properties to operations involving square roots A8 • develop algorithms and perform operations on irrational numbers B2 • apply the Pythagorean theorem D14 • use inductive and deductive reasoning when observing patterns, developing properties, and making conjectures E8	• demonstrate an understanding of rational and irrational numbers, compare and order them, and apply them in meaningful situations 10^{+}A1 • represent numbers in multiple ways (including exponents, ratios, percents, proportions, and scientific notation) and apply appropriate representations to solve problems 10^{+}A3 • choose appropriate strategies for calculating (mental math, estimation, appropriate technology, paper and pencil) , and for problem solving 10^{+}B3 • represent patterns and relationships in multiple ways (context, concrete, pictorial, verbal, and symbol) and use these representations to predict and solve problems 10^{+}C1
5.4 Defining Trigonometric Ratios • explore and apply functional relationships and notation, both formally and informally C21 • relate the trigonometric functions to the ratios in similar right triangles D3 • apply trigonometric functions to solve problems involving right triangles including the use of angle of elevation D5 • use calculators to find trigonometric values of angles and angles when trigonometric values are known D4 • solve problems using trigonometric ratios D12	• develop and use ratio, rate and proportions as tools for solving problems 10^{+}D3 • represent numbers in multiple ways (including exponents, ratios, percents, proportions, and scientific notation) and apply appropriate representations to solve problems 10^{+}A3

Math 10 Outcomes	Math 10 Plus Outcomes
5.5 Applications of Trigonometry • solve problems involving similar triangles and right triangles D8 • solve problems using trigonometric ratios D12 • solve problems involving measurement using bearings and vectors D6 • apply trigonometric functions to solve problems involving right triangles including the use of angle of elevation D5	• demonstrate an understanding that using strategies is useful in solving routine and non-routine problems 10^{+}C7 • represent numbers in multiple ways (including exponents, ratios, percents, proportions, and scientific notation) and apply appropriate representations to solve problems 10^{+}A3

Chapter 1: Data Management (6 weeks)

Math 10 Outcomes	Math 10 Plus Outcomes
1.1 Variables and Relationships • analyze graphs or charts of given situations to identify specific information A2 • gather data, plot the data using appropriate scales, and demonstrate an understanding of independent and dependent variables, domain, and range C3 • design and conduct experiments using statistical methods and scientific inquiry F1 • (optional) demonstrate an understanding of concerns and issues that pertain to the collection of data F2 • solve problems by modeling real-world phenomena F6	• collect data, display it accordingly as a histogram, a stem-and-leaf plot, a box plot, a scatter plot and interpret the displays, both with technology and by hand when appropriate 10^{+}F1
1.2 Measuring • (optional) determine accuracy and precision of a measurement D7 • demonstrate an understanding of the concerns and issues that pertain to the collection of data F2	• model, solve and create problems that utilize addition, subtraction, multiplication and divisions of fractions and decimals 10^{+}B2
1.3 Describing Data • analyze graphs or charts of given situations to identify specific information A2 • design and conduct experiments using statistical methods and scientific inquiry F1demonstrate an understanding of the concerns and issues that pertain to the collection of data F2	• choose appropriate strategies for calculating (mental math, estimation, appropriate technology, paper and pencil) , and for problem solving 10^{+}B3 • collect data, display it accordingly as a histogram, a stem-and-leaf plot, a box plot, a scatter plot and interpret the displays, both with technology and by hand when appropriate 10^{+}F1

Math 10 Outcomes	Math 10 Plus Outcomes
1.3 Describing Data (continued) • construct various displays of data F3calculate various statistics using appropriate technology, analyse and interpret the displays, and describe the relationships F4 • analyze statistical summaries, draw conclusions, and communicate results about distributions of data F5 • create and analyse plots using appropriate technology C4 • solve problems using graphing technology C17	
1.4 Defining Data Spread and 1.5 Large Distributions and the Normal Curve • analyse statistical summaries, draw conclusions, and communicate results about distributions of data F5 • calculate various statistics using appropriate technology, analyse and interpret the displays, and describe the relationships F4 • calculate and apply the mean and standard deviation using technology to determine whether a variation makes a difference F13 • make and interpret frequency bar graphs while conducting experiments and exploring measurement issues F14 • solve problems using graphing technology C17 • explore measurement issues using the normal curve F12 • calculate and apply mean and standard deviation using technology, to determine if a variation makes a difference F13 • determine whether differences in repeated measurements are significant or accidental D9	• demonstrate an understanding that using strategies is useful in solving routine and non-routine problems 10^{+}C7 • represent numbers in multiple ways (including exponents, ratios, percents, proportions, and scientific notation) and apply appropriate representations to solve problems 10^{+}A3 • collect data, display it accordingly as a histogram, a stem-and-leaf plot, a box plot, a scatter plot and interpret the displays, both with technology and by hand when appropriate 10^{+}F1
1.6 Using Data to Predict • create and analyse plots using appropriate technology C4 • gather data, plot the data using appropriate scales, and demonstrate an understanding of independent and dependent variables, and domain and range C3 • solve problems using graphing technology C17	

Math 10 Outcomes	Math 10 Plus Outcomes
1.6 Using Data to Predict (continued) • explore non-linear data, using power and exponential regression, to find a curve of best fit F7 • determine and apply a line of best fit, using the least squares method F8 • demonstrate an intuitive understanding of correlation F9 • use interpolation and extrapolation and the equation to predict and solve problems F10 • sketch lines and curves of best fit, and determine the equation for the line of best fit by hand and with technology F2	

Chapter 3: Patterns, Relations, Equations, and Predictions (7 weeks)

Math 10 Outcomes	Math 10 Plus Outcomes
3.1 Describing Patterns • express problems in terms of equations and vice versa C1 • model real-world phenomena with linear, quadratic exponential, and power equations C2 • gather data, plot the data using appropriate scales, and demonstrate an understanding of independent and dependent variables and domain and range C3 • construct and analyse tables relating two variables C9 • develop and apply strategies for solving problems C15 • describe real-world relationships depicted by graphs and tables of values F11 • identify, generalize, and apply patterns C8 • solve problems using graphing technology C17 • determine if a graph is linear by plotting points in a given situation C32	• demonstrate an understanding of the interrelationships of subsets of real numbers 10^{+}A2 • represent patterns and relationships in multiple ways (context, concrete, pictorial, verbal, and symbol) and use these representations to predict and solve problems 10^{+}C1 • construct and analyse tables and graphs to describe how changes in one quantity affect a related quantity 10^{+}C2 • solve and create problems involving linear equations and inequalities 10^{+}C3 • graph, and write in symbols and in words, the solution set for equations and inequalities involving all real numbers 10^{+}A4 • apply algebraic operations on polynomial expressions and equations to simplify, expand, factor, and to solve relevant problems 10^{+}B5 • choose appropriate strategies for calculating (mental math, estimation, appropriate technology, paper and pencil) , and for problem solving 10^{+}B3

Math 10 Outcomes	Math 10 Plus Outcomes
3.2 Solving Problems by Solving Equations • apply properties of numbers when operating upon expressions and equations A6 • model (with concrete materials and pictorial representations) and express the relationships between arithmetic operations and operations on algebraic expressions and equations B1 • interpret solutions to equations based on context C16	• solve and create problems involving linear equations and inequalities 10+C3 • apply algebraic methods to solve linear equations and inequalities 10+C6 • explore and explain, using physical models, the connections between arithmetic and algebraic operations 10+B4
3.3 Decision Making and Patterns • apply properties of numbers when operating upon expressions and equations A6 • model (with concrete materials and pictorial representations) and express the relationships between arithmetic operations and operations on algebraic expressions and equations B1 • sketch graphs from words, tables, and collected data C5 • identify, generalize and apply patterns C8 • describe real-world relationships depicted by graphs, tables of values, and written descriptions C10 • interpret solutions to equations based on context C16 • investigate and find the solution to a problem by graphing two linear equations with and without technology C18 • solve equations using graphs C25 • solve linear and simple radical, exponential, and absolute value equations and linear inequalities C27 • explore and describe the dynamics of change depicted in tables and graphs C28	• graph, and write in symbols and in words, the solution set for equations and inequalities involving all real numbers 10+A4 • model, solve and create problems that utilize addition, subtraction, multiplication, and division of fractions and decimals 10+B3 • explore and explain, using physical models, the connections between arithmetic and algebraic operations 10+B4 • apply algebraic operations on polynomial expressions and equations to simplify, expand, factor, and to solve relevant problems 10+B5 • represent patterns and relationships in multiple ways (context, concrete, pictorial, verbal, and symbol) and use these representations to predict and solve problems 10+C1 • construct and analyse tables and graphs to describe how changes in one quantity affect a related quantity 10+C2 • solve and create problems involving linear equations and inequalities 10+C3 • apply algebraic methods to solve linear equations and inequalities 10+C6 • demonstrate an understanding that using strategies is useful in solving routine and non-routine problems 10+C7

Math 10 Outcomes	Math 10 Plus Outcomes
3.4 Predictions and Lines: y = mx + b • determine the slope and y-intercept of a line from a table of variables C13 • determine the equation of a line using the slope and y-intercept C14 • rearrange equations C24 • investigate and make and test conjectures concerning the steepness and direction of a line C29 • graph by constructing a table of values, by using graphing technology, and when appropriate by intercept-slope method C33	• model, solve and create problems that utilize addition, subtraction, multiplication, and division of fractions and decimals 10^{+}B3 • explore and explain, using physical models, the connections between arithmetic and algebraic operations 10^{+}B4 • apply algebraic operations on polynomial expressions and equations to simplify, expand, factor, and to solve relevant problems 10^{+}B5 • represent patterns and relationships in multiple ways (context, concrete, pictorial, verbal, and symbol) and use these representations to predict and solve problems 10^{+}C1 • construct and analyse tables and graphs to describe how changes in one quantity affect a related quantity 10^{+}C2 • solve and create problems involving linear equations and inequalities 10^{+}C3 • determine the equations of lines by obtaining their slopes and yintercepts from graphs 10^{+}C4 • determine the equations of lines, and solve equations and inequalities by using technology 10^{+}C5 • apply algebraic methods to solve linear equations and inequalities 10^{+}C6 • demonstrate an understanding that using strategies is useful in solving routine and non-routine problems 10^{+}C7
3.5 More Patterns • demonstrate an understanding of the zero product property and its relationship to solving equations by factoring A5 • use concrete materials, pictorial representations, and algebraic symbolism to perform operations on polynomials B3 • solve quadratic equations by factoring C26 • expand and factor polynomial expressions using perimeter and area models C35	• choose appropriate strategies for calculating (mental math, estimation, appropriate technology, paper and pencil) , and for problem solving 10^{+}B3 • explore and explain, using physical models, the connections between arithmetic and algebraic operations 10^{+}B4 • apply algebraic operations on polynomial expressions and equations to simplify, expand, factor, and to solve relevant problems 10^{+}B5

Math 10 Outcomes	Math 10 Plus Outcomes
3.5 More Patterns (continued)	• represent patterns and relationships in multiple ways (context, concrete, pictorial, verbal, and symbol) and use these representations to predict and solve problems 10^{+}C1 • construct and analyse tables and graphs to describe how changes in one quantity affect a related quantity 10^{+}C2 • solve and create problems involving linear equations and inequalities 10^{+}C3 • demonstrate an understanding that using strategies is useful in solving routine and non-routine problems 10^{+}C7
3.6 Other Patterns • express problems in terms of equations and vice versa C1 • solve linear and simple radical, exponential, and absolute value equations and linear inequalities C27	• explore and explain, using physical models, the connections between arithmetic and algebraic operations 10^{+}B4 • apply algebraic operations on polynomial expressions and equations to simplify, expand, factor, and to solve relevant problems 10^{+}B5 • represent patterns and relationships in multiple ways (context, concrete, pictorial, verbal, and symbol) and use these representations to predict and solve problems 10^{+}C1 • construct and analyse tables and graphs to describe how changes in one quantity affect a related quantity 10^{+}C2

Chapter 4: Modeling Functional Relationships (5 weeks)

Math 10 Outcomes	Math 10 Plus Outcomes
4.1 Tables, Graphs, and Connections • analyse graphs or charts of situations to derive specific information A2 • sketch graphs from words, tables, and collect data C5 • identify, generalize, and apply patterns C8 • describe real-world relationships depicted by graphs and tables of values F11	• represent patterns and relationships in multiple ways (context, concrete, pictorial, verbal, and symbol) and use these representations to predict and solve problems 10^{+}C1 • construct and analyse tables and graphs to describe how changes in one quantity affect a related quantity 10^{+}C2

Math 10 Outcomes	Math 10 Plus Outcomes
4.2 Relations and Functions • explore and apply functional relationships and notation, both formally and informally C21 • graph by constructing a table of values, by using graphing technology, and when appropriate, by intercept-slope method C33	• represent patterns and relationships in multiple ways (context, concrete, pictorial, verbal, and symbol) and use these representations to predict and solve problems 10$^+$C1 • construct and analyse tables and graphs to describe how changes in one quantity affect a related quantity 10$^+$C2
4.3 Equipping Your Function Toolkit • model real-world phenomena with linear, quadratic, exponential and power equations, and linear inequalities C2 • analyse and describe transformations of quadratic functions and apply them to absolute value functions C22 • express transformations algebraically and with mapping rules C23 • graph equations and inequalities and analyse graphs both with and without graphing technology C31 • apply transformations when solving problems E4 • use transformations to draw graphs E5	• demonstrate an understanding of the properties of transformations and their mapping notation 10$^+$E1 • demonstrate an understanding that using strategies is useful in solving routine and non-routine problems 10$^+$C7
4.4 Algebraic Models: Part 1 • model real-world phenomena with linear, quadratic, exponential and power equations, and linear inequalities C2 • create and analyse scatter plots using appropriate technology C4 • determine and apply the line of best fit using the least squares method and the median-median method with and without technology, and describe the differences between the two methods F8 • use interpolation, extrapolation and equations to predict and solve problems F10 • calculate various statistics using appropriate technology, analyse and interpret displays and describe the relationships F4 • demonstrate an intuitive understanding of correlation F9	• represent patterns and relationships in multiple ways (context, concrete, pictorial, verbal, and symbol) and use these representations to predict and solve problems 10$^+$C1 • construct and analyse tables and graphs to describe how changes in one quantity affect a related quantity 10$^+$C2 • solve and create problems involving linear equations and inequalities 10$^+$C3 • determine the equations of lines, and solve equations and inequalities by using technology 10$^+$C5 • collect data, display it accordingly as a scatter plot and interpret the display both with and without technology and by hand when appropriate 10$^+$F1 • sketch lines and curves of best fit, and determine the equation for the line of best fit by hand and with technology 10$^+$F2 • use technology to determine the curve of best fit 10$^+$F3

Math 10 Outcomes	Math 10 Plus Outcomes
4.5 Algebraic Models: Part 2 • create and analyse scatter plots using appropriate technology C4 • solve problems using graphing technology C17 • evaluate and interpret non-linear equations using graphing technology C20 • construct various displays of data F3 • explore non-linear data using power and exponential regressions to find a curve of best fit F7 • use interpolation, extrapolation and equations to predict and solve problems F10 • compare regression models of linear and non-linear functions C30	• collect data, display it accordingly as a scatter plot and interpret the display both with and without technology and by hand when appropriate 10^{+}F1 • sketch lines and curves of best fit, and determine the equation for the line of best fit by hand and with technology 10^{+}F2 • use technology to determine the curve of best fit 10^{+}F3

Chapter 2: Networks and Matrices (3 weeks)

Math 10 Outcomes	Math 10 Plus Outcomes
2.1 Creating and Travelling Network Graphs • model real-world situations with networks and matrices C7represent network problems using matrices and vice versa C37	• represent problem situations involving matrices 10^{+}A5 • model, solve, and create problems involving the matrix operations of addition, subtractions and scalar multiplication 10^{+}B6 • represent patterns and relationships in multiple ways (context, concrete, pictorial, verbal, and symbol) and use these representations to predict and solve problems 10^{+}C1
2.2 Digraphs and Adjacency Matrices • represent network problems as digraphs C37 • model real-world situations with networks C7 • solve network problems using matrices B6 • represent network problems as digraphs E6	• represent problem situations involving matrices 10^{+}A5 • model, solve, and create problems involving the matrix operations of addition, subtractions and scalar multiplication 10^{+}B6 • represent patterns and relationships in multiple ways (context, concrete, pictorial, verbal, and symbol) and use these representations to predict and solve problems 10^{+}C1

Math 10 Outcomes	Math 10 Plus Outcomes
2.3 Matrix Multiplication • develop, analyse and apply procedures for matrix multiplication B5 • solve network problems involving matrices B6 • develop and apply strategies for solving problems C15 •	represent problem situations involving matrices 10+A5 • model, solve, and create problems involving the matrix operations of addition, subtractions and scalar multiplication 10+B6 • represent patterns and relationships in multiple ways (context, concrete, pictorial, verbal, and symbol) and use these representations to predict and solve problems 10+C1

Chapter 7 Linear Programming (4 weeks)

Math 10 Outcomes	Math 10 Plus Outcomes
7.1 Exploring an Optimization Problem • analyse graphs or charts of situations to derive specific information A2 • identify and calculate the maximum and/or minimum values in a linear programming model B4 • apply linear programming to find optimal solutions to real world problems C6 • construct and analyse tables relating two variables C9	• represent patterns and relationships in multiple ways (context, concrete, pictorial, verbal, and symbol) and use these representations to predict and solve problems 10+C1 • construct and analyse tables and graphs to describe how changes in one quantity affect a related quantity 10+C2
7.2 Exploring Possible Solutions • relate sets of numbers to solutions of inequalities A1 • analyse graphs or charts of situations to derive specific information A2 • demonstrate and apply an understanding of discrete and continuous number systems A7 • identify and calculate the maximum and/or minimum values in a linear programming model B4 • model real-world phenomena with linear, quadratic, exponential and power equations, and linear inequalities C2 • apply linear programming to find optimal solutions to real world problems C6 • construct and analyse table relating two variables C9 • write an inequality to describe its graph C11 • express and interpret constraints using inequalities C12	• graph, and write in symbols and in words, the solution set for equations and inequalities involving all real numbers 10+A4 • represent patterns and relationships in multiple ways (context, concrete, pictorial, verbal, and symbol) and use these representations to predict and solve problems 10+C1 • construct and analyse tables and graphs to describe how changes in one quantity affect a related quantity 10+C2 • determine the equations of lines, and solve equations and inequalities by using technology 10+C5 • apply algebraic methods to solve linear equations and inequalities 10+C6 • solve and create problems involving linear equations and inequalities 10+C3

Math 10 Outcomes	Math 10 Plus Outcomes
7.2 Exploring Possible Solutions • interpret solutions to equations based on context C16 • rearrange equations C24 • solve linear ad simple radical, exponential, and absolute value equations and linear inequalities C27 • graph equations and inequalities and analyse graphs both with and without graphing technology C31 • graph by constructing a table of values, by using graphing technology, and when appropriate, by intercept-slope method C33 • investigate and make and test conjectures about the solution to equations and inequalities using graphing technology C34	
7.3 Connecting the Region and the Solution • relate sets of numbers to solutions of inequalities A1 • analyse graphs or charts of situations to derive specific information A2 • demonstrate and apply an understanding of discrete and continuous number systems A7 • identify and calculate the maximum and/or minimum values in a linear programming model B4 • apply linear programming to find optimal solutions to real world problems C6 • express and interpret constraints using inequalities C12 • solve problems using graphing technology C17 • solve systems of linear equations using substitution and graphing methods C19 • rearrange equations C24 • graph equations and inequalities and analyse graphs both with and without graphing technology C31 • investigate and make and test conjectures about the solution to equations and inequalities using graphing technology C34	• graph, and write in symbols and in words, the solution set for equations and inequalities involving all real numbers 10^{+}A4 • represent patterns and relationships in multiple ways (context, concrete, pictorial, verbal, and symbol) and use these representations to predict and solve problems 10^{+}C1 • construct and analyse tables and graphs to describe how changes in one quantity affect a related quantity 10^{+}C2 • determine the equations of lines, and solve equations and inequalities by using technology 10^{+}C5 • apply algebraic methods to solve linear equations and inequalities 10^{+}C6 • solve and create problems involving linear equations and inequalities 10^{+}C3 • demonstrate an understanding that using strategies is useful in solving routine and non-routine problems 10^{+}C7

Chapter 6: The Geometry of Packaging (6 weeks)

Math 10 Outcomes	Math 10 Plus Outcomes
6.1 Examining Factors in Container Design • explore, determine and apply formulas for perimeter, area, surface area and volume D1 • determine the precision and accuracy of a measurement D7 • demonstrate an understanding of the concepts of surface area and volume D13	• relate the volumes of pyramids, cones and spheres to the volumes of corresponding prisms and cylinders 10^{+}D1 • describe patterns and generalize the relationships between areas and perimeters of quadrilaterals, and areas and circumferences of circles 10^{+}D2 • examine and draw representations of 3-dimensional shapes, and from drawings, construct 3-dimensional shapes 10^{+}E2 • develop and apply properties of 2-dimensional figures, and apply them 10^{+}E3 • develop and apply relationships between parallel lines and congruent angles 10^{+}E4 • investigate, and demonstrate an understanding of the minimum sufficient conditions to guarantee congruent triangles 10^{+}E5 • make informal deductions using congruent triangle, polygon and angle properties 10^{+}E6
6.2 Regular Polygons • determine and apply formulas for perimeter, area, surface area and volume D1 • explore, discover, and apply properties of maximum area and volume D11 • explore, determine, and apply relationships between perimeter and area, surface area, and volume C36 • solve problems involving polygons and polyhedra E2	• relate the volumes of pyramids, cones and spheres to the volumes of corresponding prisms and cylinders 10^{+}D1 • describe patterns and generalize the relationships between areas and perimeters of quadrilaterals, and areas and circumferences of circles 10^{+}D2
6.3 Surface Area • demonstrate an understanding of the concepts of surface area and volume D13 • determine and apply formulas for perimeter, area, surface area, and volume D1	• relate the volumes of pyramids, cones and spheres to the volumes of corresponding prisms and cylinders 10^{+}D1 • describe patterns and generalize the relationships between areas and perimeters of quadrilaterals, and areas and circumferences of circles 10^{+}D2

Math 10 Outcomes	Math 10 Plus Outcomes
6.4 Economy of Design • demonstrate an understanding of the role of irrational numbers in applications A3 • approximate square roots A4 • solve problems involving polygons and polyhedra E2	• Demonstrate an understanding of rational and irrational numbers, compare and order them, and apply them in meaningful situations 10^{+}A1 • Develop and use ratio, rate and proportions as tools for solving problems 10^{+}D3
6.5 Similarity and Size • determine and apply relationships between the perimeters and areas of similar figures and between the surface and volumes of similar solids D10	• relate the volumes of pyramids, cones and spheres to the volumes of corresponding prisms and cylinders 10^{+}D1 • describe patterns and generalize the relationships between areas and perimeters of quadrilaterals, and areas and circumferences of circles 10^{+}D2 • develop and use ratio, rate and proportions as tools for solving problems 10^{+}D3
6.6 Variations in Packaging • explore properties of, and make and test conjectures about 2D and 3D figures E1 • use deductive reasoning and construct logical arguments and be able to determine, when given a logical argument, its validity E9 • use inductive reasoning when observing patterns developing properties, and making conjectures E8 • construct and apply altitudes, medians, angle bisectors and perpendicular bisectors to examine their intersection points E3	• develop and apply properties of 2-dimensional figures, and apply them 10^{+}E3 • develop and apply relationships between parallel lines and congruent angles 10^{+}E4 • investigate, and demonstrate an understanding of the minimum sufficient conditions to guarantee congruent triangles 10^{+}E5 • make informal deductions using congruent triangle, polygon and angle properties 10^{+}E6 • Develop and use ratio, rate and proportions as tools for solving problems 10^{+}D3

Section 5.1

See the Assessment Section for the **Instruction Process**

- teach through interaction
- a culture of questioning and deep thinking
- learn from shared discussions with one-another and with teacher
- formative assessment practices are essential ... provide evidence used to adapt the teaching to meet the students needs.
- continuous flow of information concerning student achievement that does not threaten the students ... they become involved in the process ... they are encouraged to take risks ... make mistakes ... and enhance their learning.
- observations, conversations and interviews, and/or interactive journals help students to
  - articulate their ideas, their understandings, and
  - where they feel they need more help ...
- this also helps teachers
  - keep in touch with each student's progress,
  - provide them with insight into their thinking processes, and their understandings.

Day 1:

Discussion with students about

- the year in general
- assessment for learning
- the learning process
- assessment of learning
  - of understanding concepts
  - of procedural skills

Homework:

- Read Focus A, and #1, 2, p.212 (text)
- write a report or summary of the above

Day 2:

Introduce the learning

- discuss the outcomes (section 5.1)
- show samples and discuss what the learned product looks like
- plan with students - set standards, and time lines
- form base groups (mixed ability)

Begin the pretest at Focus A:

- select students randomly to read their report/summary to class (teacher needs to listen carefully to note (anecdotal) particular student's needs)
- without any discussion edit the reports in groups - form a group report
- have each group now read an edited report to the class

Part 2 of pretest (independently):

- do #3, and 8, pp.100–102 (grade 9 Correspondence booklet)
- do #1 e f i j, and 7, p.152 (Nexus)
- hand in when complete (mark before next class - not to count)

Day 3:

Use information from students' results of pretest part 2

- selected (because of the different approaches) students will present to others, sitting independently, their solutions to each of the four questions on the pretest
- teacher-whole-group discussion - different methods (focus on different strategies used)

In base groups

- do activity (5.1 Lesson 1 of this document)
- listen in to the discussion in the groups about the activity
- lead large-group-discussion on the results of the activity when appropriate
- do Investigation 1, parts A to C, p.213, 214 in text, and complete for homework

Day 4:

Get into base groups ... compare results for Investigation 1

Assign each group to record ratios as decimals to the nearest tenth (on large chart paper) results from C.

Lead discussion about Investigation 1 A to C.

- why scale diagram in A?
- how was parallelism assured?
- measuring to the nearest tenth - why?
- meaning of "opposite", "adjacent", and "hypotenuse"
- what do we notice? [ans: the ratio decimals within each column are the same]

Complete discussion on 5.1 Lesson 1, and do 5.1 Lesson 2.

From results on pretest and from observations, form two groups - those that need more time (group 1), and those that do not (group 2). Depending on numbers in each of groups 1 and 2, break into smaller groups and assign:

- Groups 1 ... do Investigation 1, part D, p.214, and #1, p.152 (Nexus)
- Groups 2 ... #3, 4, and 5, p.214 (text).
- Both groups ... complete for homework.

Day 5:

Get into groups assigned from groups 1 and 2

Group 1's
- compare results from the homework, and
- prepare to present part D and #1(Nexus) to whole class
- present first

Group 2's
- compare results from homework, and
- prepare to present #3, 4, 5, p.214 to the whole class
- present second

Reflection activity
- write about “What makes two triangles similar”, and
- “If two triangles are similar, what do you know?”
- complete sentences, fully discuss
- hand in to be marked (formative)

Group Work

Groups 1
- do #5 to 9, p.197, 198 (Nexus)

Groups 2
- do #5 to 9, p.197, 198 (Nexus), and
- Read Focus C from text pp.216–217 and
- prepare a summary report

Day 6:

Get into groups assigned from Groups 1 and 2

Group 1s
- compare answers to homework (Nexus)
- prepare to present to whole class
- present first
- independently do #6, 7, pp.214–216 (text) - hand in (formative)
- in groups, do 9 and 10 pp.215–216

Group 2s

- compare and correct answers to homework (Nexus)
- prepare to present Focus C to whole class
- independently do #6, 7, and 9, pp.214–216 (text) - hand in (fromative)
- in groups do 10 and11, p.216

Discuss the answers to pp.214–216 (text)

- either by presentation, or
- by students writing answers on board (whole class)
- Groups 2 present Focus C to whole class
- Homework for all:
- #12 15, pp.217–218

Day 7:

Assessment of Learning day ...

Groups 1: do test 1

Groups 2: do test 2, then begin the “chpt. project”, p.219

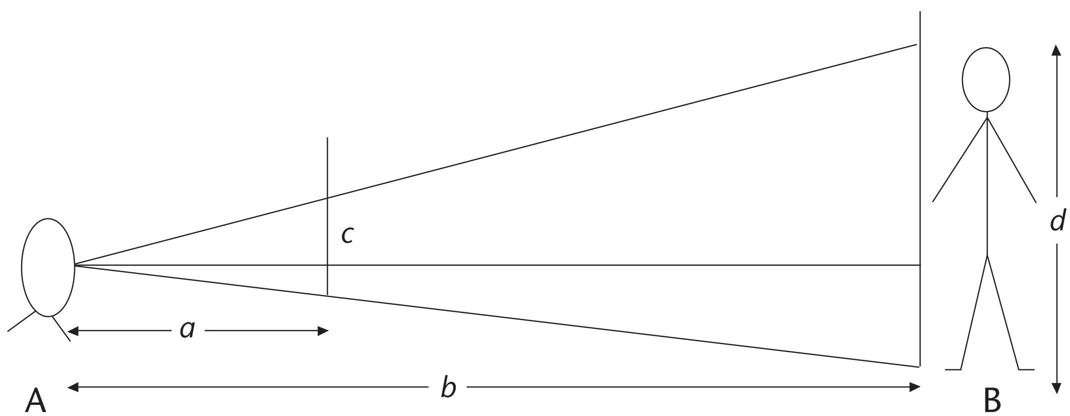
(Both tests can be found in the Appendix)

5.1 Lesson 1 Introduction to Indirect Measurement – Teacher Notes

- Activity based
- Builds conceptual understanding of similar triangles
- Provides an opportunity to practice measuring

The activity involves indirect measurement. It can be set up so that the whole class is engaged in the activity in groups of 3–4 or as a demonstration using two groups of 3–4 students each (student A, student B and a measurer/ recorder). Doing two rather than one demo provides the opportunity to conduct and then review the procedure and also to compare results and assess measuring strategies. Each group of students will need one 30 cm ruler and a metre stick or tape measure.

Students A and B are several metres apart. Student A has a 30 cm ruler at arm's length, a, and "eyes" student B, at a distance, b.

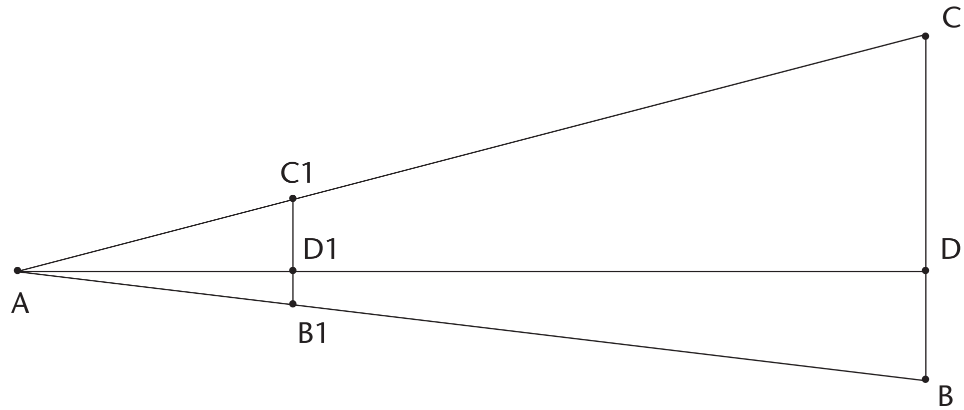


The following measurements are determined:

- b is the distance between students A and B (to the nearest cm)
- a is the distance from A's eye to the 30 cm ruler (to the nearest 0.5 cm)
- c is the measurement on the ruler that corresponds to B's height as determined by A (to the nearest 0.1 cm)

Students are instructed to determine B's height from these measurements. It is expected that they will use some variation of $\frac{d}{b}=\frac{c}{a}$ or $\frac{d}{c}=\frac{b}{a}$. Record the determination of B's height. It is useful to engage in discussion about the triangles that can be found in the diagram. Focus the discussion around the following questions:

- How many triangles?
- Identify the right angled triangles.
- Which triangles are similar? How do you know?
- How do similar triangles help us to determine B's height?



Now determine B's height directly and compare to the result obtained by indirect measurement. Ask students to suggest why the results are not exactly the same. What could be done to improve the results? If time allows, this presents an opportunity to engage a little error analysis.

What if? Student A read the ruler 0.1 cm higher or lower? What difference would it make in the final measurement?

5.1 Lesson 2 Patty Paper Angle Investigation – Teacher Notes

- Hands-on Geometry with no sharp objects
- Review geometry terminology
- Create a device for estimating/measuring

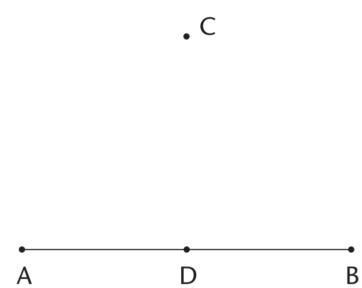
Use Patty Paper to construct measurement device instead of following the instructions in #1 and 2 on p.193.

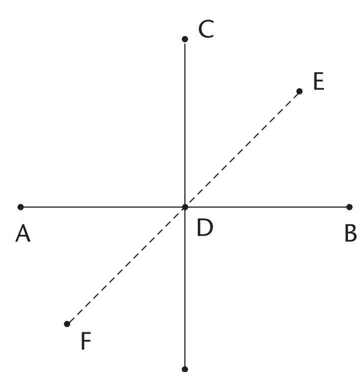
Patty paper can be purchased at Big Eric's on Lady Hammond Drive in Halifax (or any restaurant supply). One thousand sheets will cost between $10 and $15. The sheets are square, 13 cm on a side, waxy on one side and rough on the other. Pencil works well on the rough side but doesn't erase well. Each student will need one sheet. It can be kept in a binder as patty paper is hole-punched to fit perfectly in a regular 3 ring binder.

Lead students through the following activity. Emphasize correct language and symbolism as needed in the naming of angles and segments. Be sure to engage in discussion about how students know what the angle measurements are without actually using a protractor to measure. Other relevant terms should also be mentioned and discussed. For example, CD is the perpendicular bisector of AB. Examine the words perpendicular (intersecting at 90°) and bisector (dividing into two equal parts).

Instructions for students:

- Draw a straight line segment roughly in the middle of the patty paper. Label it AB.
- Fold the segment so that A lands exactly on B. Make a sharp crease in the paper. Draw a line in the crease. Label D where the lines intersect and C elsewhere on the line. Identify the angles that are formed and indicate their measure. Example: m<CDB=90°. Why?
- Name two 180° angles. Are there others?
- Construct, by folding, a 45° angle.
- Label points E and F on the angle bisector.





- Use your diagram to find and name at least one angle with each of the following measures.
  a) 45°
  b) 135°
  c) 225°
  d) 270°
  e) 315°
  f) 360°

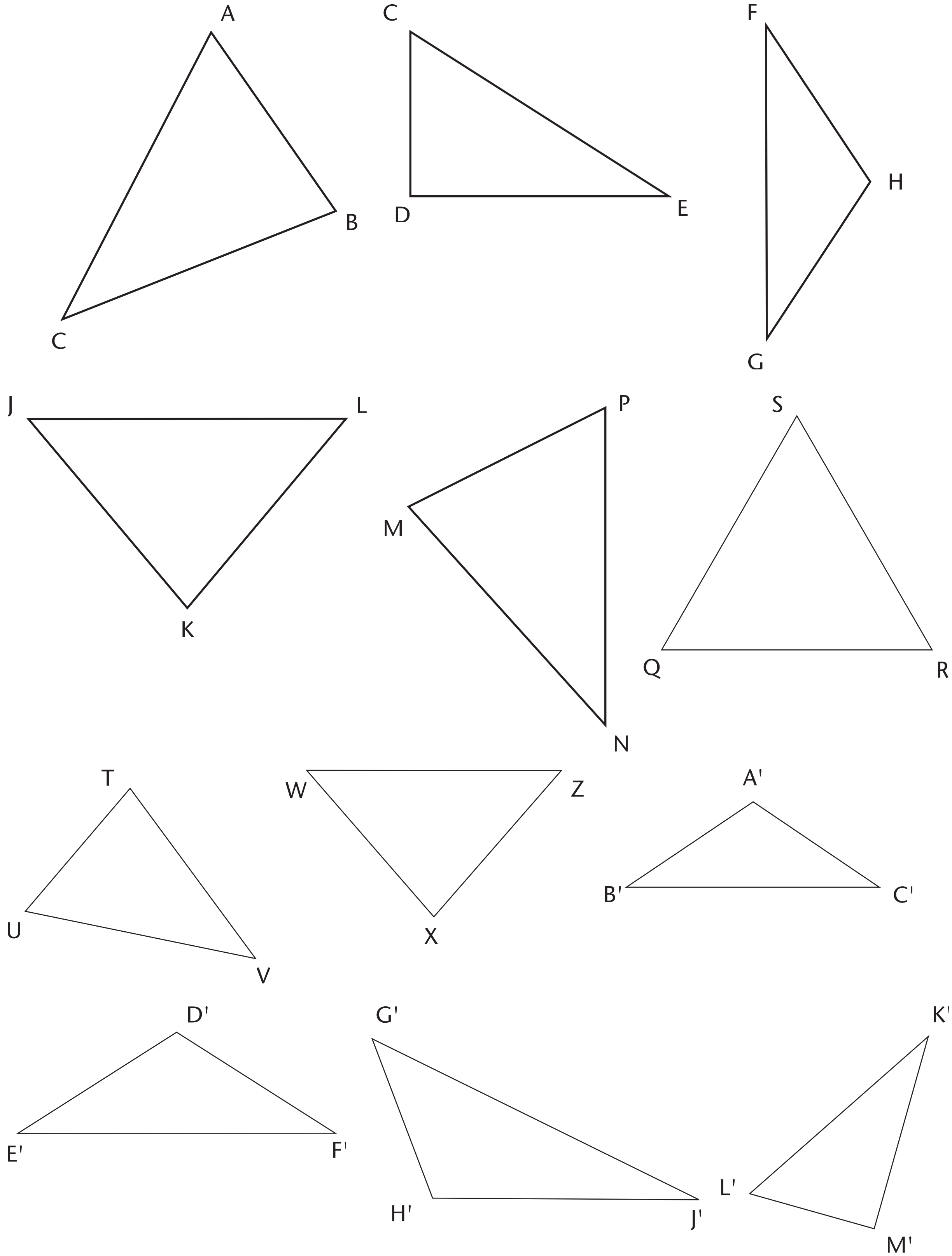
Students may wish to indicate angle measurements on the diagram for reference. Arcs may be used. The patty paper provides the necessary models to be used in #3, p.193 (Nexus)

Review the terminology associated with angles. Students include definitions in their personal "dictionary". These terms, as well as others that arise throughout the chapter, could be posted on a "word wall".

- Acute angle
- Obtuse angle
- Right angle
- Straight angle
- Reflex angle

Classifying Triangles

Measuring the sides and angles carefully.



Classifying Triangles

By Sides

Equilateral triangles have three equal sides.
- Which triangles are equilateral?

Isosceles triangles have two equal sides.
- Which triangles are isosceles?

Scalene triangles have all sides different.
- Which triangles are scalene?

By Angles

In an **acute** angled triangle, all angles are less than 90°.
- Which triangles are acute angled triangles?

In an **obtuse** angled triangle, one angle is more than 90°.
- Which triangles are obtuse angled triangles?

In a **right** angled triangle, one angle is 90°.
- Which triangles are right angled triangles?

Can you find?
- An acute isosceles triangle?
- An obtuse isosceles triangle?
- An obtuse scalene triangle?
- A scalene right triangle?
- An isosceles right triangle?
- An equilateral right triangle?

If you can't find an example of each of these, try to draw one. If you can't draw it, explain why it is impossible.

Comparing Triangles

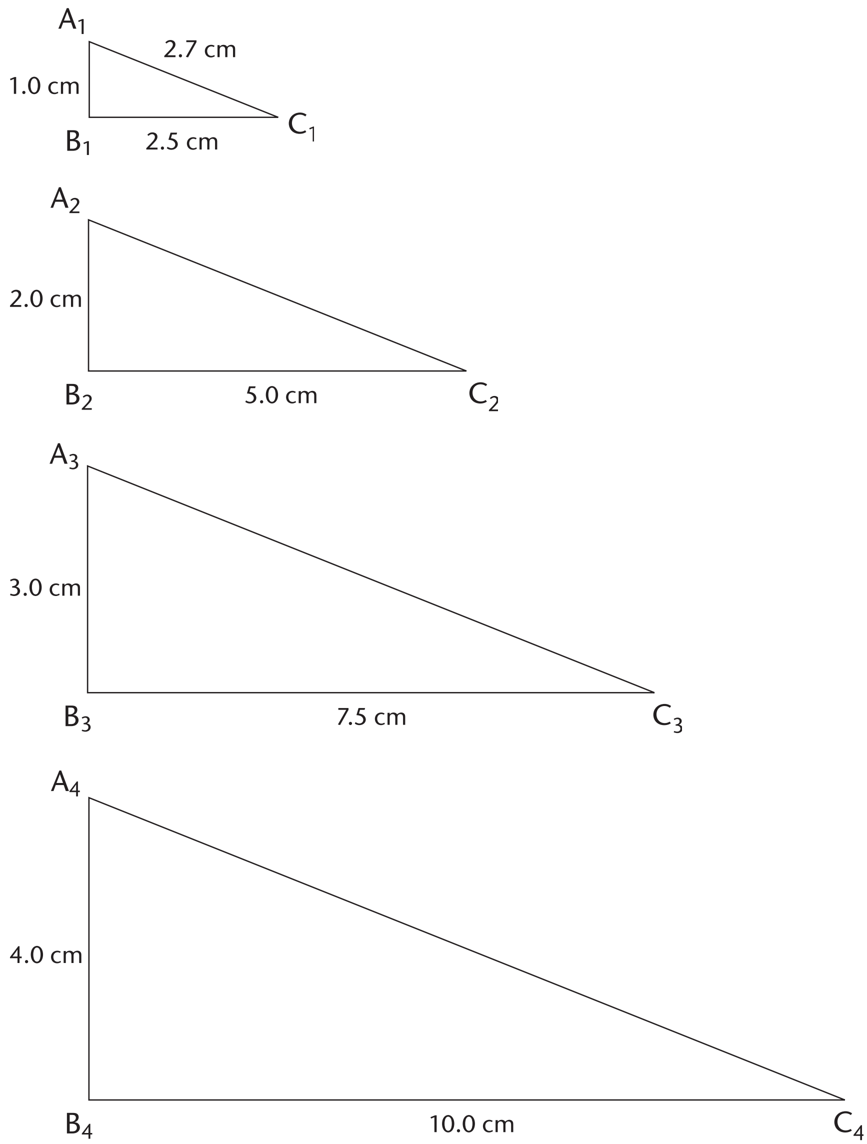
Congruent Triangles
- Have the same size
- Have the same shape

⇒
- Angles have the same measure
- Sides have the same length

Similar Triangles
- Have the same shape

⇒
- Angles have the same measure
- Corresponding sides are proportional

More Patterns in Similar Triangles



Each triangle contains a right angle at B.

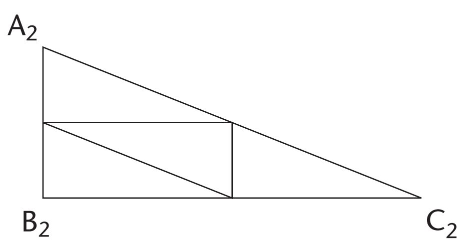
1. How can you be sure the triangles are similar?
2. Determine the lengths of A_2C_2, A_3C_3, and A_4C_4
3. Verify that corresponding angles are equal.

4. Complete the table

	Ratio of sides compared to $\triangle A_1B_1C_1$	Perimeter	Ratio of perimeter compared to $\triangle A_1B_1C_1$	Area	Ratio of Area compared to $\triangle A_1B_1C_1$
$\triangle A_1B_1C_1$					
$\triangle A_2B_2C_2$					
$\triangle A_3B_3C_3$					
$\triangle A_4B_4C_4$					

5. If the sides of $\triangle A_5B_5C_5$ are five times as long as the sides of $\triangle A_1B_1C_1$, predict:
   - Perimeter of $\triangle A_5B_5C_5$
   - Area of $\triangle A_5B_5C_5$

6. Look again at $\triangle A_2B_2C_2$. Segments are drawn joining the midpoints of the sides of $\triangle A_2B_2C_2$. How can you use this to explain the patterns you found in the table above?



Section 5.2

Day 8:

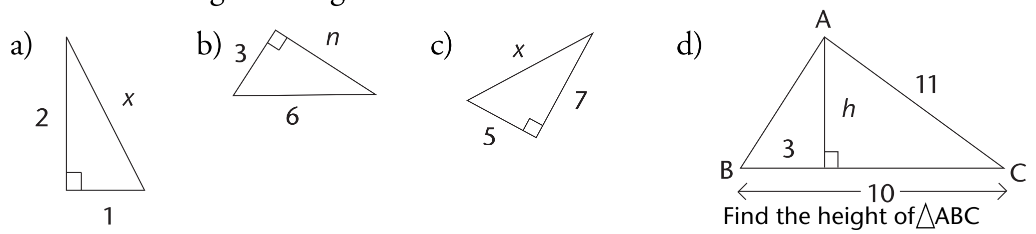
Introduce the learning

- discuss the outcomes (section 5.2)
- show samples and discuss what the learned product looks like
- plan with students - set standards, and time lines
- reform base groups (not mandatory)

Begin the pretest (all students write independently)

Pretest:

Find the missing side lengths ...



Have the students mark each others' papers according to the following rubric. Record nothing on the paper except the mark. The rubrik is used for placement in ability groups for this section and can be used as an indicator of skills to be revisited.

Rubrik (out of 4):

1 point	any attempt but shows little knowledge or understanding
2 points	shows knowledge of the Pythagorean theorem, but not the importance of the hypotenuse (understanding)
3 points	shows knowledge and understanding of the Pythagorean theorem
4 points	everything correct (minor calculation error(s) ok)

Do 5.2 Lesson 1 with everyone.

Groups 1

- do Activity 1, questions 1 and 2, p.170 (Nexus)

Groups 2

- do #3, p.171 (Nexus)

Assign a reading assignment (see Appendix) on Pythagorus to all - have students write a report on what they read.

Day 9:

Groups 1

- ask students to compare their results in their groups
- have them follow the discussion in 5.2 Lesson 2 activity.

Groups 2

- compare your results with group members
- share results with groups 1

Large group

- hands on the internet (or demo if need be)
- let the students explore and have fun with these three websites (work in pairs)
  - http://www.usna.edu/MathDept/mdm/pyth.html
  - http://www.ies.co.jp/math/java/geo/pythagorus.html
  - http://matti.usu.edu/nlvm/nav/frames_asid_164_g_4_t_3.html?open=instructions

Groups 1

- do Investigation 2, parts A to C, p.222 (text)
- activity on pretest corrections:
  - give student three solutions to (a) or (b), each has errors ... one with hypotenuse used incorrectly, one just forgetting to square root at the end. Have students find and correct errors.
  - do the same for question (c)
  - do the same for question (d)
- From the study of the solutions with errors, students should discuss when to square root or not, and if so, to how many decimal places.
- They should be able to now do each of the questions correctly - so, do so.
- do #7, p.224 (text)

Groups 2

- do Method 3, p.223 (text)
- do #7, p.224 (text)

Assign #8, p.224 (text) ... give the students about a week to finish this, but remind them each day to complete this, and to work independently.

Day 10:

Groups 1

- prepare to present Inv 1 parts A to C
- compare and correct you results for #7, p.224 (text)

Groups 2

- prepare to present Method 3, p.223
- prepare to present #7, p.224 (text)

Large group

- In a large group have some from Groups 1 present steps A to C to the whole class
- lead the class through a discussion of steps D, E, and F
- Groups 1 now present Method 3
- give the class a quiz that will show that they have learned from doing #7, p.224

Quiz:

(Hand in for formative marking)

1) Copy from your notebooks the conclusions that you wrote about (i) and (ii).
2) Give the students the measures of three sides of a triangle and ask them to determine if the triangle is a right triangle or not.

Groups 1

- do 5.2 Lesson 3 activity
- Read and discuss in groups Focus E, p.224 (text)
- do 9 to 12, pp.224–225 (text)

Groups 2

- Read Focus E, p.224 (text)
- Do #9 to 13, pp.224–225 (text)
- continue with "chpt project" p.225

Day 11

In Base groups

- go over questions 9 to 12, pp.224–225 (text)
- teacher randomly assigns a group to present one of the problems to the whole class
- presentations and discussions (further the conversation about when to square root, and when not to)

Groups 1

- do Activities 2, 3, and 4, pp.172–175 (Nexus)
- tell them some will be marked to count

Groups 2

- do the "challenge yourself", p.225 (text)
- do Activity 3 and 4, pp.173–175 (Nexus) - marked to count

5.2 Lesson 1 Teacher notes

It is important to provide students with a visual representation for square numbers and to repeatedly reinforce this connection.

Begin the lesson by instructing students to create a square with an area of 9 u^2. This can be done using geoboards, grid paper or dot paper.

Questions:

- What makes it a square?
- Verify that the area is 9 $units^2$.
- Why do you think we refer to multiplying a number by itself as "squaring" the number?
- Nine is called a "square number". Give three more examples of square numbers.
- Create the squares with areas equal to these square numbers.
- Do you think you can construct a square with area exactly 10 u^2? Why?

We will revisit this last question. (suggestion: post it in the classroom)

Before assigning the measuring activity, be sure students are familiar with the hypotenuse of a right angled triangle. It is the longest side; it is opposite the right angle.

5.2 Lesson 2 Teacher Notes

Discussion questions:

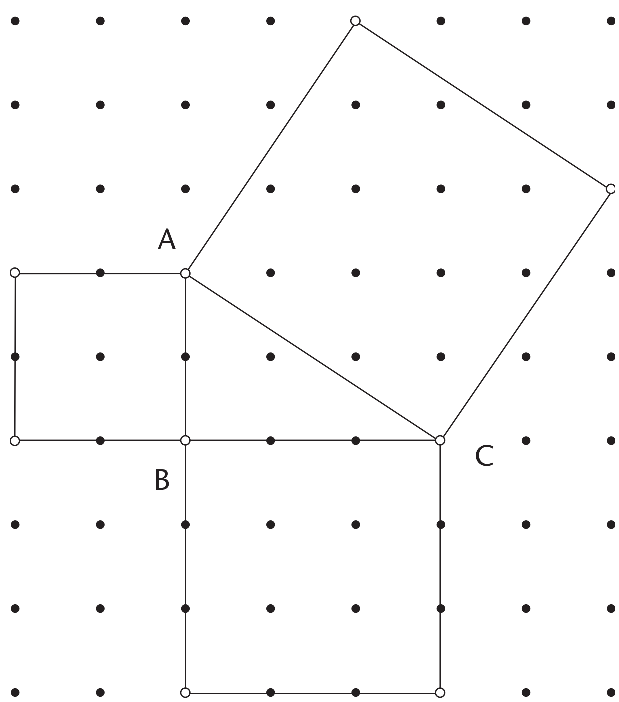
- Look at the first table. Were there any consistent patterns?
- Look at the second table. Were there any consistent patterns?
- Create the word equation suggested in 1c. Does it fit every case in the table?
- Can we be absolutely sure that it will always be true? Have we covered every case? (ie Is this a proof?)

Consider possibilities:

- GSP demonstration (pyth_thm_1.gsp) Open the file. Click on Show objects. Right click on each square in turn and select Area. Compare the areas of the squares.
- Visit websites:
  - http://www.usna.edu/MathDept/mdm/pyth.html
  - http://java.sun.com/applets/archive/beta/Pythagoras/
  - http://www.ies.co.jp/math/java/geo/pythagoras.html
  - http://matti.usu.edu/nlvm/nav/frames_asid_164_g_4_t_3.html?open=instructions
- Pythagorean Theorem proof handout for cutting and pasting in scribbler – to be completed for homework

Closing Activity (Investigation 2, p.222 text)

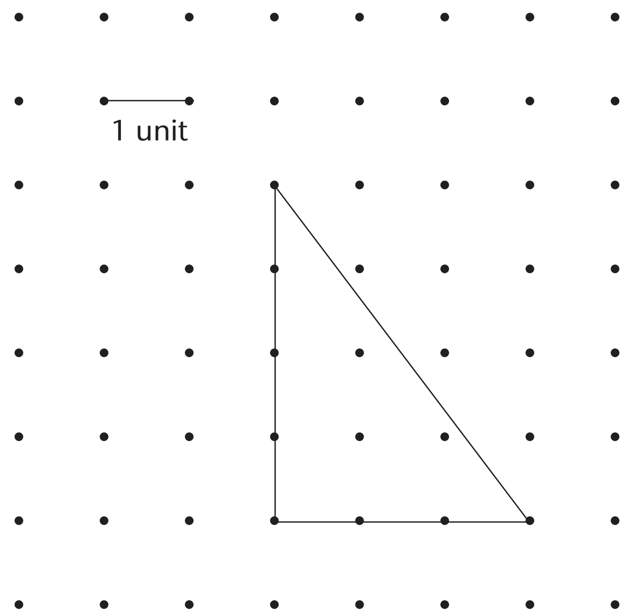
Explain the diagram. Give as much detail as possible.



5.2 Lesson 3 Teacher Notes

Opening Activity:

Give students a copy of the diagram.



Question:

How long is each side of the triangle?

To verify that the hypotenuse is indeed 5 units long, use dot paper as your ruler and show that it is 5 units long.

3-4-5 is a Pythagorean Triple.

What makes a set of numbers a Pythagorean Triple?

See MM 1 p.224 Focus E. Record the definition.

Ask students to suggest a possible Pythagorean triple. Check it.

Example: 2-3-4

If this is a Pythagorean triple, 4 would be the hypotenuse.
So we want to know if $2^2 + 3^2 = 4^2$
Left side = LS = $2^2 + 3^2 = 4 + 9 = 13$
Right side = RS = $4^2 = 16$
LS $\neq$ RS

Section 5.3

Day 12

Before beginning the discussion about the new section, ask students in Groups 1 to hand in:

- #2 and 3, p.172 (Nexus)
- #2b) from Activity 3 (Nexus)
- #3 and #7 Activity 4 (Nexus)

Students in Groups 2 are to hand in:

- #2c) from Activity 3 (Nexus)
- #2, 7 and 8 Activity 4 (Nexus)

Now begin the new section ... Introduce the learning

- discuss the outcomes (section 5.3)
- show samples and discuss what the learned product looks like
- plan with students - set standards, and time lines
- reform base groups (not mandatory)

Begin the pretest (all students write independently)

Pretest

1) Describe in your own words what the difference is between rational numbers and irrational numbers.
2) a) Give an example of a rational number that is not expressed as a fraction or decimal.
   b) Can a rational number be negative? Explain.
   c) Classify the following as rational or irrational:
      i) -2
      ii) $\frac{1}{2}$
      iii) 0.732
      iv) $0.73\overline{2}$
      v) $3\sqrt{2}$
      vi) $4\sqrt{49}$
      vii) 0.7323223324...
3) Estimate the value for:
   i) $\sqrt{23}$
   ii) $\sqrt{82}$
   iii) $\sqrt{120}$

4) Determine the simplified answer (no decimals), if possible:
   i) $2\sqrt{3} + 3\sqrt{27}$
   ii) $3\sqrt{10} \times 4\sqrt{6}$

Exchange the papers and have the students mark them. Teacher leads the correcting by asking students to circle the errors, count the errors and write that number at the top of the page beside the student's name. Hand in to teacher.

While the teacher reviews the pretests and reforms Groups 1 and Groups 2 based on them, the students will rewrite or edit their answers to the first question, and 2a and 2b.

- Everyone will do 5.3 Lesson 1
- do #2, and 3 p.162 (Nexus)
- do #3, and 4 p.96 (MIA)

Day 13

Groups 1
- Review answers to #2, 3 (Nexus)
- review answers to #3, 4 (MIA)
- do #4 – 17, pp.162–65 (Nexus)

Groups 2
- Review answers to #2, 3 (Nexus)
- review answers to #3, 4 (MIA)
- do Investigation 3, p.226 (text)
- do #1, 2, 3, and 5, pp.227–8 (text)

Day 14

Groups 1
- do 5.3 Lesson 3
- do #1 to 6, pp.227–8 (text)

Groups 2
- Section 4.3, pp.102–104 (MIA)
- do 3b, 4, and 6, p.228 (text)
- Focus G - prepare to teach

Day 15

Groups 1

- do 5.3 Lesson 4
- do #9, p.230 (text)
- page 204 (Algebra with Pizazz)

Groups 2

- prepare, then teach the whole class Focus G
- do #7 to 9, p.230 (text)
- page 204 (Alg w Pizazz)

Day 16

Groups 1

- do 5.3 Lesson 5
- do #12, p.231 (text)
- Pizzaz p.210

Groups 2

- read Focus H, p.230 (text)
- do #10 to 14, pp.231–2 (text)
- Pizzaz p.210

Day 17

Discuss #8, p.224 (text) - due today - have some students present their work.

Groups 1

- do 5.3 Lesson 6
- pizazz, p.209

Groups 2

- MIA #8, p.107
- MIA #3, 4, p.109
- MIA: #10, p.110
- chpt project, p.232 (text)

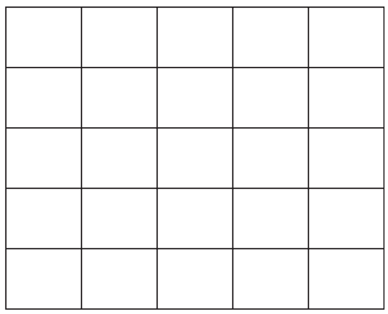
Day 18

Groups 2 - some students or pairs of students should present (make it interactive) to the whole class their findings with the MIA activities #8, 3 and 4, and #10.

Test on the Section - marks will count

5.3 Lesson 1 Teacher Notes

Ask students to draw a square with area 10 u^2 on the grid provided (no calculator). Consider their responses. Ask why they drew the square as they did?



Direct the questioning/discussion to establish the following:

- A 3 by 3 square has an area of 9 u^2
- A 4 by 4 square has an area of 16 u^2
- A square with area 10 u^2 will be larger than the 3 by 3 but smaller than the 4 by 4
- It will be only a little larger than the 3 by 3 square

Find the length of the side of the square by estimating. Students are allowed to use the calculator but only to check their guesses by multiplying, or better, by squaring. Many students see square rooting as somehow the opposite of multiplying. The intent is to establish that square root is the inverse of squaring.

Use a table to record results. The activity might take a form similar, but not identical, to the following.

First guess: side is 3.3 $\rightarrow$ enter 3.3 and corresponding area (10.89) in table. This estimate is too high.

Second guess: 3.1 $\rightarrow$ enter 3.1 and corresponding area (9.61). This estimate is too low. Leave room in the table to insert guesses where needed to close in on the desired area.

Ask for a guess to bring us closer. Students recognize the need find a number between 3.1 and 3.2. This requires using hundredths. Try student suggestions. For example, 3.15 gives an area of 9.9225.

Trying 3.16 gives an area of 9.9856

But 3.17 gives an area too large.

Again the need arises to find a number between 3.16 and 3.17. This necessitates going to thousandths. Encourage students to look at the area values to decide whether to guess closer to 3.16 or 3.17.

Length of side	Area of square
3.1	9.61
3.15	9.9225
3.16	9.9856
3.17	10.0489
3.3	10.89

Although this activity takes some time and many students will know the single keystroke that will provide the answer, encourage them to “play along” at least until the digit in the thousandths place is determined before resorting to the square root function on the calculator. At this point, ask the question “What does the square root key on the calculator do?”

Caution – the definition of square root in Nexus is simplistic (and inaccurate). Use the definition in the margin notes on p.226 of MM 1 or the following:

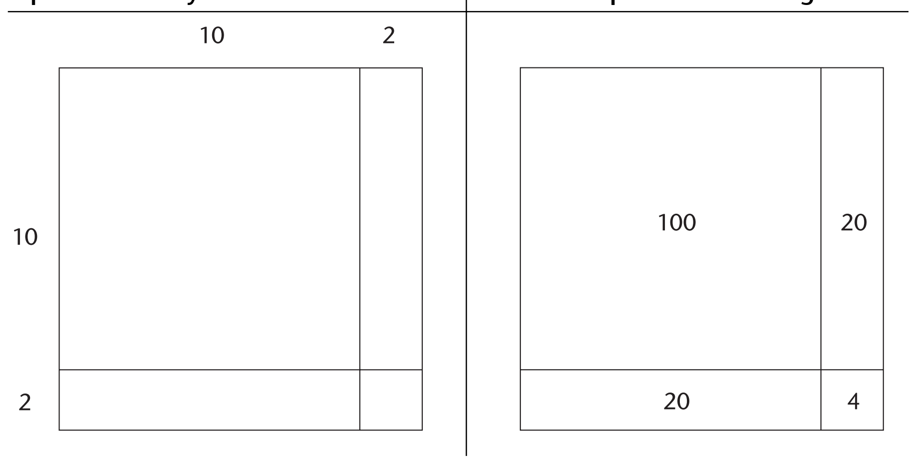
- is read as “square root of n” or “radical n”
- denotes the principal square root of n
- is the positive number, which, when you square it, gives n

Students work in pairs to complete Nexus p.162 #2,3.

Discuss mental math strategies – example – ask students to predict the final digit before trying to determine the squares in #2.

In #3 – a visual representation is also possible to assist in the mental computation.

If the dimensions of the square are 12 by 12:	**Then the total area is made up of the following:**

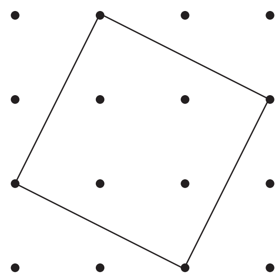


5.3 Lesson 3 Teacher Notes

Opening activity:

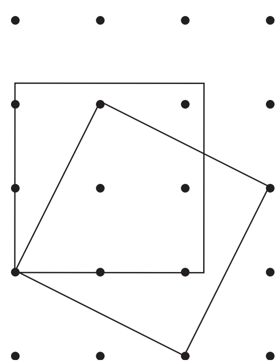
Provide the diagram and ask the questions:

- What do you see?
- What information can you give?

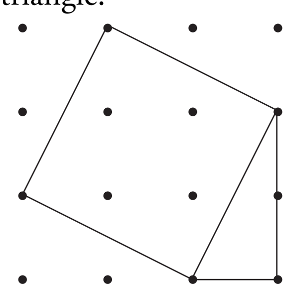


Possible responses:

- Square. How can you be sure? (should lead to a discussion of counting horizontal and vertical distances)
- Area is less than 9 u^2 (because it fits inside a 3 by 3 square)
- Area is 4 u^2 because it looks like you can turn it. If this suggestion is voiced, and students are unable to argue why it cannot be 4, provide the following diagram. Is there a way to tell that the slanted segments are more than 2 units long? (think Pythagoras)

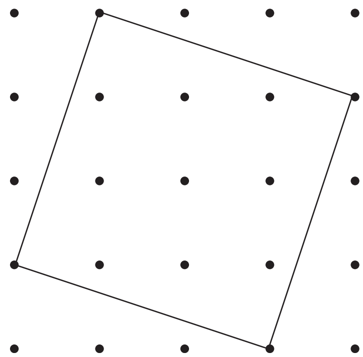


If students have difficulty determining the area of the square, show this diagram. Recognize that the square is built on the hypotenuse of the right triangle so its area must be the sum of the area on the other two sides of the triangle.

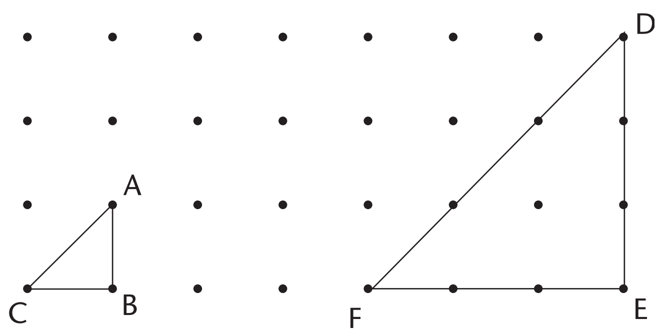


Ask again: Is it possible to build a square with an area of 10 u^2?

Practice: MM 1 p227–8 #1–6. In 3b clarification of slope may be needed. It may be enough at this time to talk simply about counting up and over.



5.3 Lesson 4 Teacher Notes



Find the length of AC in $\triangle$ABC.

$1^2 + 1^2 = h^2$

$1 + 1 = h^2$

$h = \sqrt{2}$

Look at $\triangle$DEF. How does the length of DF compare to the length of AC? (It is three times as long?) Why? (The triangles are similar and DE=3×AB and EF=3×BC). We found the length of AC to be $\sqrt{2}$.

So it seems that DF = $3 \times \sqrt{2}$

What result do we get if we use the Pythagorean theorem to find the length of DF.

$3^2 + 3^2 = h^2$

$9 + 9 = h^2$

$18 = h^2$

$h = \sqrt{18}$

It must be true then that $\sqrt{18} = 3\times\sqrt{2} = 3\sqrt{2}$ if we use the same convention as in algebra where 3w means "three times w".

The expressions look very different! Notice that 18 can be factored to give 9 × 2. Nine is a perfect square. Its square root is a whole number.

$$\begin{aligned}\sqrt{18} &= \sqrt{9 \times 2} \\ &= \sqrt{9} \times \sqrt{2} \\ &= 3 \times \sqrt{2} \\ &= 3\sqrt{2}\end{aligned}$$

Write 18 as a product of two factors (under the root sign) so that one factor is a perfect square.

Write as a product of roots.

Simplify.

The radical is in simplified form. This number is exact. It is read as "three square root two" or "three roots of two"

If the number under the radical is a multiple of a square number, the square root can be simplified.

Examples:

Simplify

$$\begin{aligned}\sqrt{75} &= \sqrt{25 \times 3} \\ &= \sqrt{25} \times \sqrt{3} \\ &= 5 \times \sqrt{3} \\ &= 5\sqrt{3}\end{aligned}$$

Simplify

$$\begin{aligned}5\sqrt{24} &= 5 \times \sqrt{4 \times 6} \\ &= 5 \times \sqrt{4} \times \sqrt{6} \\ &= 5 \times 2 \times \sqrt{6} \\ &= 10\sqrt{6}\end{aligned}$$

Practice: Algebra with Pizzazz p.204 (require that students show steps in scribbler).

5.3 Lesson 5 Teacher Notes

Operating on Radicals: Addition and Subtraction

Teacher led class discussion:

In $\triangle$DEF from Lesson 4, we saw that the length of DF was
$\sqrt{2} + \sqrt{2} + \sqrt{2} = 3\sqrt{2}$

This is much like $x + x + x = 3x$

Simplify:
- $4\sqrt{2} + 5\sqrt{2}$
- $7\sqrt{5} - 3\sqrt{5}$

Explain why this makes sense.

What if? $\sqrt{2} + \sqrt{3}$ This is different than the questions above.
- Is there a way to simplify?
- Check your ideas with your calculator.
- Think of a situation in algebra where you could not combine terms to give a single term.

Work through examples in MM 1 p231 #12.

Practice: algebra with Pizzazz p210 What Should You Do If Nobody Will Sing With You?

5.3 Lesson 6 Teacher Notes – More About Multiplying Radicals

Based on what we have seen with radicals, decide whether each of the following is True or False.

1. $\sqrt{2} + \sqrt{3} = \sqrt{5}$	True or False
2. $\sqrt{2} \times \sqrt{3} = \sqrt{5}$	True or False
3. $\sqrt{7} - \sqrt{3} = \sqrt{5}$	True or False
4. $\sqrt{5} + \sqrt{3} = 2\sqrt{5}$	True or False
5. $3\sqrt{5} \times 2\sqrt{2} = 6\sqrt{10}$	True or False

Ask students to check each using a calculator and four decimal places.

Example #1:
$\sqrt{2} + \sqrt{3} = 1.4142 + 1.7321 = 3.1463$
$\sqrt{5} = 2.2361$
$\sqrt{2} + \sqrt{3} \neq \sqrt{5}$

Use this opportunity to refresh the rules for adding and subtracting radicals.

In a similar manner, use the calculator to verify each statement. Then discuss the connections to previous learning.

Look at numbers 2 and 6 above. It looks as though, in the order of operations, the square rooting and multiplying operations are interchangeable when these are the only operations involved. Make up another example to test this out. Why does it make sense that this is true?

The justification should include suggestions like:

- When we multiply x times y we can write the answer as xy
- When we simplified radicals we saw that, for example, $\sqrt{12} = \sqrt{4 \times 3}\sqrt{4} \times \sqrt{3} = 2\sqrt{3}$
- Multiplication is both commutative and associative. Students may relate to this better using an example. Ask them to multiply as quickly as they can without a calculator: $2 \times 17 \times 5$. The way the question is written suggests the following: $2 \times 17 \times 5 = (2 \times 17) \times 5 = 34 \times 5 =$

But we know that the order and groupings will not affect the answer so:
$2 \times 17 \times 5 = 2 \times 5 \times 17 = (2 \times 5) \times 17 = 10 \times 17 =$

Working with multiplying radicals, this suggests:

$$\begin{aligned} 3\sqrt{5} \times 4\sqrt{2} &= 3 \times \sqrt{5} \times 4 \times \sqrt{2} \\ &= 3 \times 4 \times \sqrt{5} \times \sqrt{2} \\ &= 12 \times \sqrt{10} \\ &= 12\sqrt{10} \end{aligned}$$

ˇ Insert operation signs
- Use commutative property to change the order.
- Use the associative property to group the factors more conveniently.
- Multiply
- Write in simplified form

This is provided as a justification, not as a suggested method for each question. A one step solution is sufficient. Students should recognize the thinking which underlies it, however.

Student practice: Pizzazz p.209 “What Do You Call a Group of Factory Foremen Who Sing While Drinking Tab Cola and Eating Crab Apples?”

Section 5.4

Day 19

Now begin the new section ... Introduce the learning

- discuss the outcomes (section 5.4)
- show samples and discuss what the learned product looks like
- plan with students - set standards, and time lines
- reform base groups (not mandatory)
- If the class has access to Geometer's Sketch Pad - do 5.4 Lesson 1
- do Investigation 4, p.233 (text) - base groups - measurement important
- do 5.4 Lesson 2
- do #1, 3, 6, 7, 8, pp.234–235 (text)
- homework: #4, 5 p.234 (text), and
- #1, and 3 to 8, pp.211–12 (Nexus)

Day 20

Base groups - compare and review the homework - teacher observes and gives help to those having trouble.

- groups continue work on doing teacher-prepared questions like #4 and 5, p.234, but using more than just the 60° angle measure from the chart.
- those having little trouble can be pulled out of the groups and be given the following task:
  - Read and discuss Focus I, pp.235–6 (text), and do #9, 10, 11, pp. 235–6 (text), and Pizazz p.227
- teacher works with the rest of the groups until ready to move on, and with them does ...
- do 5.4 Lesson 3, and Pizazz, p.227.

Day 21

Base Groups

- mental math - practice the identification of trig ratios - teacher uses overhead, and shows various pictures of triangles - students have to identify which trig ratio to use (also... the opposite, adjacent, and hypotenuses)
- prepare to present questions from the homework
- quiz on errors (see Appendix)
- assessment activity (pg 71 Pilmer)
- do #12 to 16, pp.236–37 (text)
- as students work, observe and work with students who need help

Day 22

Base Groups

- Pizzaz p.228
- Nexus p.215, #1, 2, 3
- text p.239, #18 to 20
- discuss "how to write a solution" (see 5.4 Lesson 7)
- do Pizazz, p.229.

Day 23

Base groups

- discuss p.229 Pizzaz
- do p.230 Pizzaz
- everyone: do #21 p.239 (text) - see p.319–325 (MIA) for more of this type
- enrichment : #22 to 25 p.240 (text)

5.4 Lesson 1 Teacher Notes

This is a Geometer's Sketchpad activity. If you have never used Geometer's Sketchpad with students, this is a great place to start. The activity has very little chance to go off in the wrong direction and allows for a quick realignment. It is important to allow students to "play" a little first.

Students need access to **triangle.investigation.gsp**

They will also need a copy of Triangle Investigation Instructions and Triangle Investigation Table. It is recommended that students work in pairs at the computer but each student will need a copy of the table for their own use.

If computers are networked, this can be saved in a location where students have access, but cannot change the original. This means that if students get muddled at any time during the investigation, they can simply reload the original gsp page and discard whatever mess they got into. There is no need to keep any sides constant as they change the measure of <A, although there is a tendency to do so. If they try to do this, students will create a triangle that is too large to manage as the size of <A gets large. Either instruct them to try to make the triangle small enough to fit on the screen, or reload the gsp page and keep the triangle small while they increase the size of <A.

It will take about 20–30 minutes for students to complete the first three columns of the table, measuring and recording the side lengths (#1 – 4 on the instruction sheet). Be sure they are recording the lengths correctly. A brief discussion identifying hypotenuse, opposite and adjacent before setting them to work is a good idea.

It may be necessary to review with students how to set up the calculations of the ratios. They can complete the table for homework. They will not complete steps 5 and 6 (comparing and conclusion) until the following class.

Encourage students to accept and work with their own results, even though they may be different from other students. (This is what creates the aha!)

Triangle Investigation Data Table

A	Length of hypotenuse	Length of side opposite ∠A	Length of side adjacent to ∠A	Ratios (calculate to 3 decimal places)		
				$\frac{\text{opposite } \angle A}{\text{hypotenuse}}$	$\frac{\text{adjacent } \angle A}{\text{hypotenuse}}$	$\frac{\text{opposite } \angle A}{\text{adjacent } \angle A}$
0°						
5°						
10°						
15°						
20°						
25°						
30°						
35°						
40°						
45°						
50°						
55°						
60°						
65°						
70°						
75°						
80°						
85°						
90°						

Triangle Investigation Instructions

Experimenting:

You will be using the Selection Arrow. You are allowed to drag vertices and segments only. Do not add/or erase anything. Experiment to see how you can change the triangle.

If a point or segment is highlighted, it is selected. To deselect, simply move the selection arrow to a blank space on the screen and right click.

Be sure you can answer each question.

- What kind of triangle is $\triangle$ ABC? How do you know?
- Move A. How does it move? How does the triangle change?
- Move B. How does it move? How does the triangle change?
- Move C. How does it move? How does the triangle change?
- Is there anything that remains the same through all your changes?

Measuring:

- Drag C up or down so that <CAB=10°.
- Record the lengths of AC (hypotenuse), BC (side opposite <A) and AB (side adjacent to <A) in your table.

Collecting Data:

- Move C up until <CAB=20°.
- Record the lengths of AC, BC and AB in your table.
- Repeat steps a, b, and c for = 30°, 40°, 50°, 60°, 70°, 80°, and 90°. (If you have time, repeat the steps for the other angles in your table.)

Calculations:

- Calculate the indicated ratios for each angle correct to 4 decimal places. Record them in your table.

Comparing:

- Compare your table with two or three classmates.
- Look at the side lengths for the same size angle. Are they the same?
- Look at the ratios of lengths for the same size angle. Are they the same?

Conclusions:

- What can you conclude?

5.4 Lesson 2 Teacher Notes

Students may take some comfort in comparing answers with a few friends before the lesson gets underway.

Full class discussion:

- Use a transparency of the table to record results.
- Ask one student to report lengths for <A = 10°. Record these in table. "Did everyone get the same results?" (most probably not)
- "Would you expect everyone to get the same lengths? Why?"
- Ask the same student to report ratios and record these.
- "Is this result surprising?"
- Repeat for <A = 20°, 30°, etc. At some point it becomes unnecessary to consider the original lengths measurements. Students are interested only in checking their ratio calculations. Complete the last three columns of the table. Not every student will have been successful completing all measurements and ratios. However, every student should complete the three columns of ratios as they are being recorded.

Do not at this point introduce the names of the ratios. Students need some time to digest their observations and consider them.

In class:

Journal entry:

- MM 1 pp.234–5 – 3, 6, 7, 8 (For #3 change the wording: Instead of "For the three triangles with the same reference angle" it should read "For all the different triangles with the same reference angle")
- MM 1 p.234 #4, 5

Homework:

Nexus pp.211–2 #1, 3, 4, 5, 6, 7, 8

5.4 Lesson 3 Teacher Notes

Discuss solutions for homework questions MM 1 p234 #4 and 5. Reflect on how the ratios makes it possible to get the required information.

Clearly the table of ratios help us to get information about the sides and angles in right triangles. Because these ratios are so useful, they have been "named".

The ratio $\frac{\text{opposite}}{\text{hypotenuse}}$ = sine of angle θ or $\frac{\text{opp}}{\text{hyp}} = \sin\theta$

The ratio $\frac{\text{adjacent}}{\text{hypotenuse}}$ = cosine of angle θ or $\frac{\text{adh}}{\text{hyp}} = \cos\theta$

The ratio $\frac{\text{opposite}}{\text{adjacent}}$ = tangent of angle θ or $\frac{\text{opp}}{\text{adj}} = \tan\theta$

These three ratios are called trigonometric ratios. Since the sides are named with reference to an angle, you must indicate the angle whenever you write a trig ratio.

Quick practice using definitions. Look at the table created in Lesson 3. From the table read the following:

- Sin 80°
- Tan 30°
- Cos 40°
- Tan 10°

Continue with this activity until students are connecting the ratio to the name.

Further practice connecting the ratio to the name: Algebra with Pizzazz p.227 "What Did Mrs. Margarine Think About Her Sister's Husband?"

5.4 Lesson 4 Teacher Notes

- Review names of trig ratios. Provide a "memory trick" to help students recall the connections.

  For example:

  $\text{Sin } \theta = \frac{\text{opp}}{\text{hyp}}$

  $\text{Cos } \theta = \frac{\text{adh}}{\text{hyp}}$ $\Rightarrow$ SohCahToa

  $\text{Tan } \theta = \frac{\text{opp}}{\text{adj}}$

- Assessment activity: Math Survival Kit Grade 10 p71
- Student practice: MM 1 p236–6 #9, 10, 12, 13, 14, 15, 16

Note: students are still using the table they created. There has been no mention of more comprehensive tables or calculators yet.

5.4 Lesson 5 Teacher Notes

Discussion of homework questions provides material for observations and conclusions.

See TR to help focus the discussion. Structure the responses to provide good notes for students to record.

Handout Algebra with Pizzazz p.228 "What Did the Prince Do Whenever He Found a Girl Who Might Be Cinderella?"

Notice the table. Compare it to the table constructed by the class.

Complete the activity.

5.4 Lesson 6 Teacher Notes

- See p.MM1 p238. Read and discuss.
- See p.336. Discuss the set up of the table.
- How is the table on p.336 similar to the table or ratios constructed by the class?
- Instruct students in the use of the calculator to determine trig ratios. If TI-83's are available, there is some wisdom in encouraging their use: it standardizes the instructions; the order of entering information is consistent with the way it is written on paper; mode does not change easily; students begin to become comfortable with them in preparation for their extensive use in later grades. On the other hand, it is desirable that students can complete homework assignments and this may be difficult unless they are allowed to use whatever calculator is available.
- Practice: Nexus p215 #1, MM1 p239 #18, 19, 20

5.4 Lesson 7 Teacher Notes

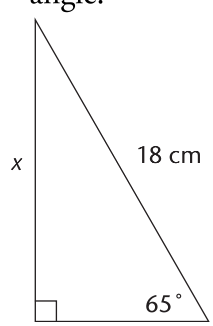
It is less confusing for students if the set up is consistent. The following format allows for the same initial set up whatever the given information.

Determining the Length of a Missing Side

Example 1

Determine the value of x

- Examine the diagram and label the sides with reference to the indicated angle.



- Identify the trig ratio that corresponds to the relevant information. Write it in definition form.
- Fill in the information from the diagram.
- Perform any calculations possible. In other words, do what you can to simplify. Use trig ratios to four places.
- Solve the equation, maintaining proper equation form. In this case, it is necessary to multiply both sides by the denominator, 18.
- Simplify.

Solution:

- Hypotenuse is 18.
- Since the 65° angle is indicated, x is opposite
- $\frac{opp}{hyp} = \sin\theta$
- $\frac{x}{18} = \sin 65°$
- $\frac{x}{18} = .9063$
- $18\left(\frac{x}{18}\right) = 18(.9063)$
- $x = 16.3$

Example 2

Determine the value of x

- Examine the diagram and label the sides with reference to the indicated angle.
- Identify the trig ratio that corresponds to the relevant information. Write it in definition form.
- Fill in the information from the diagram.
- Perform any calculations possible. In other words, do what you can to simplify. Use trig ratios to four places.
- Solve the equation, maintaining proper equation form. In this case, it is necessary to multiply both sides by the denominator, x.
- Simplify.
- To isolate x, divide both sides by the coefficient of x.
- Simplify.

Solution:

- Hypotenuse is x.
- Since the 40° angle is indicated, 20 is adj.
- $\frac{adj}{hyp} = \cos\theta$
- $\frac{20}{x} = \cos 40°$
- $\frac{20}{x} = .7660$
- $x\left(\frac{20}{x}\right) = x(.7660)$
- $20 = .7660x$
- $\frac{20}{0.7660} = \frac{0.7660x}{0.7660}$
- $x = 26.1$

Practice: Algebra with Pizzazz p.229 “What Do They Call the Big Grass Field on an Orbiting Satellite?”

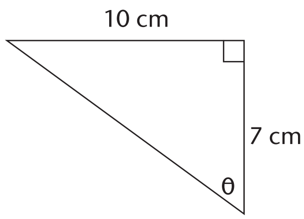
5.4 Lesson 8 Teacher Notes

This continues the modeling of solutions of problems involving trig ratios.

Determining the measure of a Missing Angle

Example 3

Determine the value of θ



- Examine the diagram and label the sides with reference to the indicated angle.
- Identify the trig ratio that corresponds to the relevant information. Write it in definition form.
- Fill in the information from the diagram.
- Perform any calculations possible. In other words, do what you can to simplify. Use trig ratios to four places.
- It is necessary to find the angle that has a tangent value equal to 1.4286.

Solution:

- Since θ is indicated, 10 is opp and 7 is adj.
- $\frac{\text{adj}}{\text{hyp}} = \tan \theta$
- $\frac{10}{7} = \tan \theta$
- $1.4286 = \tan \theta$
- $\theta = 55°$

In class practice: Algebra with Pizzazz p230 "Daffynition Decoder"

Section 5.5

Day 24

Begin the new section ... Introduce the learning

- discuss the outcomes (section 5.3)
- show samples and discuss what the learned product looks like
- plan with students - set standards, and time lines
- reform base groups (not mandatory) - based on observations for the past two or three days with this topic, reform groups 1 and 2as well.

Group 1

- review and compare answers to 230 pizzaz and #21, p.239 (text) - prepare to present
- present solutions - careful to examine how solutions are presented
- do #1 and 2, Activity 3 p.223 (Nexus) - discuss making diagrams
- do #3 to 11, p.225 to 227 (Nexus)

Group 2

- compare solutions within the groups to #22–25, p.240 (text)
- Work on Case Studies and Extensions, pp.249–50 (text)

Day 25

- all students prepare to present (in groups) selected (by the teacher) questions form the previous assignments - present to whole group.
- in base groups - do #3 to 9, pp.242–3 (text)
- in base groups - do #15 and 16, p.244 (text)

Day 26

Base Groups

- compare and review solutions to problems form yesterday - teacher might select certain problems for groups to present to whole class
- do #10 to 13, p.243 (text)
- do #17 to 20, p.245 (text)

Day 27

Groups 1

- independently copy solutions from #10, 11b), and 18 onto paper to hand in. Teacher will mark these to inform students of progress

Groups 2

- independently copy solutions from #12, and 19, pp.242 and 245 (text)

In Base groups

- do Investigation 5, p.246 (text) - use 5.5 Lessons if you think it to be helpful - then do #21 and 22, same page.
- large group discussion re: the investigation
- do #23 to 25, p.247 (text)

Homework:

- Groups 1: #26 and 29, pp.247–8 (text)
- Groups 2: #27, 28, and the "Challenge Yourself", pp.247–8 (text), and complete the "Chapter Project", p.248 (text)

Day 28

- Whole group presentations of the homework from last night (not the project)
- in base groups, review for the chapter test - do #1 to 20, pp.255–6 (text).

Day 29

- In base groups compare solutions to last night's homework
- Test
- Groups 2 students should pass in their "Chapt Projects".

5.5 Lesson 1 Teacher Notes

Activity 3 in Nexus provides students with practice creating diagrams from words.

As an in class activity, students are able to check their diagrams with each other and the teacher before they apply trig ratios to solve the problems. Discuss angle of elevation and angle of depression, Nexus p223.

Assign and then use student diagrams to model a solution for Nexus p225 #3, 4, 5

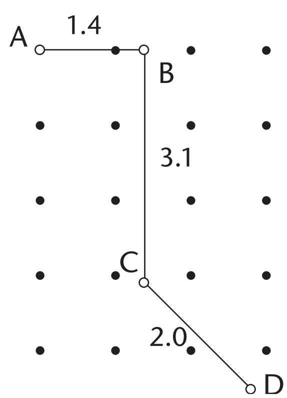
Use class time to create diagrams – assign Nexus p226 #6, 7

5.5 Lesson 4 Teacher Notes

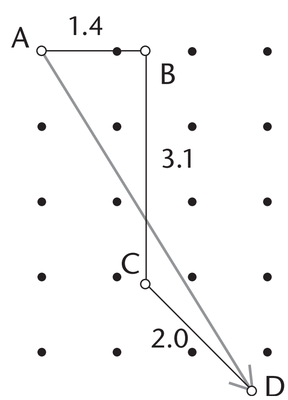
See TR for suggestions. Students should create scale diagrams for any bearing problem. Dot or rectangular graph paper facilitates accurate sketching. Students have had some practice drawing bearing angles and vectors in Focus C.

The following shows a possible progression through the problem.

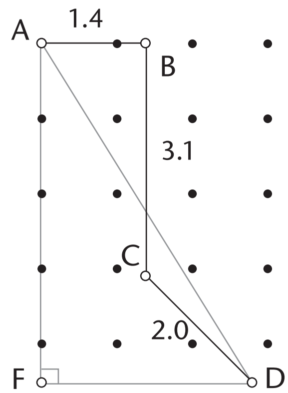
Use the instructions to create a scale diagram.



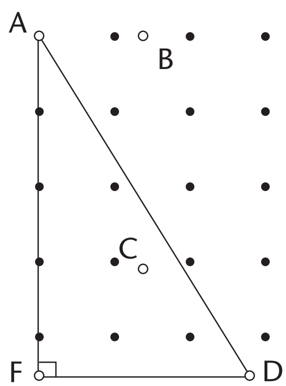
The resultant vector begins at A and ends at D. It is called $\overrightarrow{AD}$ To describe it we need to know its length and direction



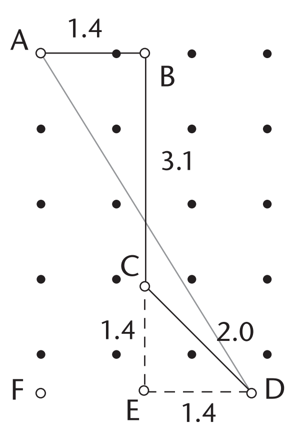
If we create a right triangle with AD as the hypotenuse, we can use trigonometry.



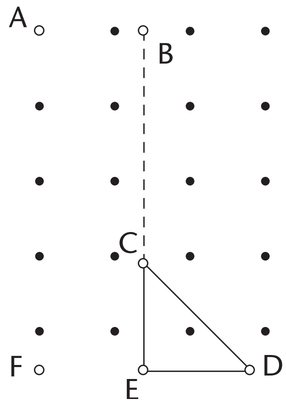
AD is the hypotenuse of △AFD. We don't know the measures of the sides or angle directly. Look for other right triangles in the diagram.



When BC is extended to meet FD at E, the right triangle CED is formed. Determine the measure of <ECD and <CDE. Use this information to find the lengths of CE and ED. What kind of triangle is △CDE?



Add this information to the diagram. Determine the lengths of AF and FD. It is also possible to determine the measure of <FAD now that all sides are known. The bearing angle may then be determined.



Practice: MM1 P247 #23

SECTION 2: LANGUAGE AND MATHEMATICS

The ability to read, write, listen, speak, think, and communicate visually about concepts will develop and deepen students' mathematical understanding. It is through communicating about mathematical ideas that students articulate, clarify, organize, and consolidate their thinking. Writing and talking about mathematics makes mathematical thinking visible. Language is as necessary to learning mathematics as it is to learning to read.

As teachers, you must assist students as they learn how to express mathematical ideas, explain their answers, and describe their strategies. You can encourage students to reflect on class conversations and to "talk about talking about mathematics" *(Principles and Standards for School Mathematics,* p. 128). You should ask questions that direct students' thinking and present tasks that help students think about how ideas are related.

Principles and Standards states that instructional programs should enable all students to

- organize and consolidate their mathematical thinking through communication
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- analyse and evaluate the mathematical thinking and strategies of others
- use the language of mathematics to express mathematical ideas precisely

As students progress through the grades they should be developing more fluency in their ability to talk about mathematics and their learning. Their development of language skills is not the primary goal, but students require frequent opportunities to use language to make sense of mathematical ideas through sharing of problem solving strategies and modelling. Students learn through discourse. Communication should include sharing ideas, asking questions, and explaining and justifying ideas. As well, students need experience in writing about their mathematical conjectures, questions, and solutions. "Teachers need to explicitly discuss students' effective and ineffective communication strategies" *(Principles and Standards for School Mathematics,* p. 197).

As teachers, you need to refine your own listening, questioning, and paraphrasing techniques, both to direct the flow of learning and to provide models for student dialogue. By asking questions that require students to articulate and extend their thinking, you are able to assess students' understanding and make appropriate instructional decisions.

As students progress into high grades they should be expected not only to present and explain a strategy used to solve a problem but also to compare, analyse, and contrast the usefulness, efficiency, and elegance of a variety of strategies. Students must be encouraged to provide explanations of mathematical arguments and rationales, as well as procedural descriptions or summaries. The use of oral and written communication provides students opportunities to

- think about solving problems
- develop explanations
- acquire and use new vocabulary and notation
- explore various ways of presenting arguments
- critique and justify conjectures
- reflect on their own understanding as well as that of others

"Teachers should build a sense of community ... so students feel free to express their ideas honestly and openly, without fear of ridicule" *(Principles and Standards for School Mathematics,* p. 269).

Classroom Strategies

According to provincial curricula, students must relate to and respond to a wide variety of experiences using many communicative forms of representation to express their work. Teachers need to utilize strategies that will allow students to process what they are learning, organize their ideas in ways that are meaningful to them, and to reflect on and extend their learning.

The integration of the five mathematics strands from the *Atlantic Canada Mathematics Curriculum* (Number Concepts/Number and Relationship Operations, Patterns and Relations, Shape and Space, Data Management, and Probability) with the three language strands from the *Atlantic Canada English Language Arts Curriculum* (Speaking and Listening, Reading and Viewing, and Writing and Other Ways of Representing) provides the richness and diversity in curriculum and instruction that will enable students to confidently explore mathematical concepts and ideas . The essence of the three language strands can be found in the *Atlantic Canada English Language Arts Curriculum: Grade 10* on pp.119 – 152. In this document, *Outcomes Framework to Inform Professional Development for Teachers of Reading in the Content Areas* identifies the expectations of all content teachers with respect to literacy.

To help teachers deliver the prescribed curriculum, this section identifies classroom strategies that support teachers as they develop the vocabulary of mathematics, initiate effective ways to maneuver informational text, and encourage students to reflect on what they have learned. Along with a suggestion of how to use the strategy is a mathematical application. When attempting to use these strategies, teachers are reminded of the suggestions found in *Secondary Science: A Teaching Resource*

- start small—try something that feels comfortable and build competence with time
- network with others
- take advantage of professional development opportunities

At this time it is important to stress that when teachers use these strategies in the instructional process or as assessment tasks, the expectations for students must be made explicitly clear. The student's understanding of the mathematics involved and maintaining the integrity of the curriculum are still the foremost concern.

Name of Strategy	Before	During	After	Assessment
1. Concept Circle	X	X	X	X
2. Frayer Model	X	X	X	X
3. Concept Definition Map	X	X	X	X
4. Word Wall	X	X	X	
5. Three Read		X	X	
6. Graphic Organizer	X	X	X	X
8. K-W-L	X		X	X
7. Think-Pair-Share	X	X		
9. Think Aloud	X	X		
10. Academic Journal Mathematics		X	X	
11. Exit Cards			X	X

1. Concept Circle

A concept circle is a way for students to conceptually relate words, terms, expressions, etc. As a before activity, it allows students to predict or discover relationships. As a during or after activity, students can determine the missing concept or attribute, or identify an attribute that doesn't belong.

The following steps illustrate how the organizer can be used.

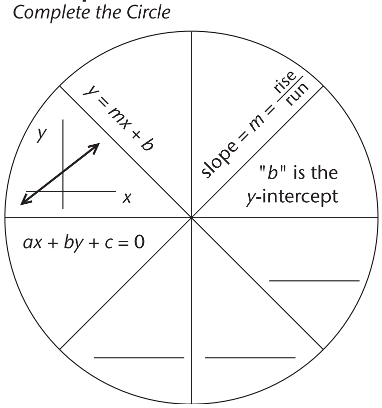
- draw a circle with the number of sections needed
- choose the common attributes and place them in the sections of the circle
- have students identify the concept common to the attributes

This activity can be approached in other ways.

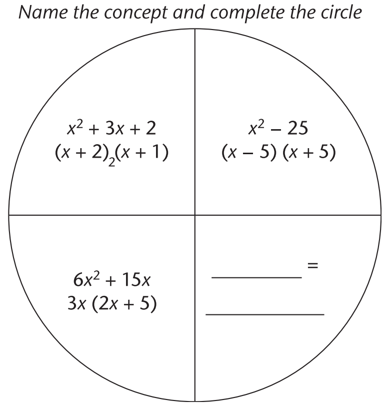
- supply the concept and some of the attributes and have students supply the missing attributes
- insert an attribute that is not an example of the concept and have students find the one that does not belong and justify their reasoning
- give students a completed circle with errors and have them make corrections.

Concept Circle

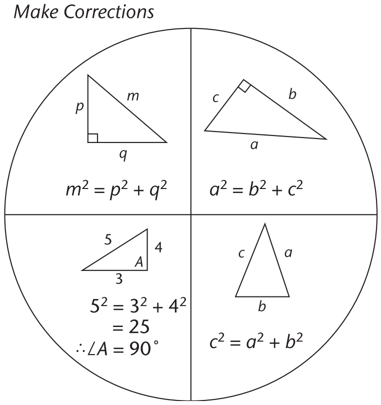
Concept: Linear Relations
Complete the Circle



Concept: ________________
Name the concept and complete the circle



Concept: Pythagorean Theorem
Make Corrections

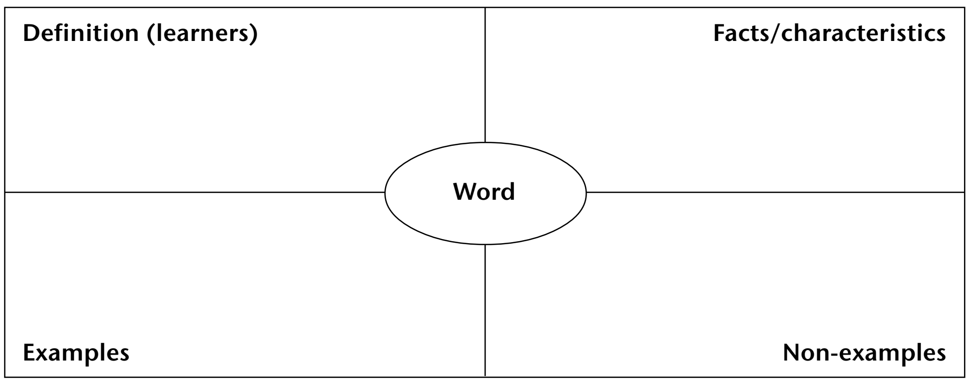


2. The Frayer Model

The Frayer Model is a graphic organizer used to categorize a word and build vocabulary. It prompts students to think about and describe the meaning of a word by

- giving a definition
- describing the main characteristics
- providing examples and non-examples of the word or concept

It is especially helpful to use with a concept that might be confusing because of its close connection to another concept. The following is a rendition of the template found in the Appendix.



The following steps illustrate how the organizer can be used.

- Display the template for the Frayer Model and discuss the various headings and what is being sought.
- Model how to use this map by using a common word or concept. Give students explicit instructions on the quality of work that is expected
- Establish the groupings (e.g., pairs) to be used and assign the concept(s) or word(s)
- Have students share their work with the entire class

This is an excellent activity to do in poster form to display in class. Each group might do the same word or concept or a different word or concept could be assigned.

The Frayer Model

Essential Characteristics

- a rectangular arrangement of numbers in rows and columns

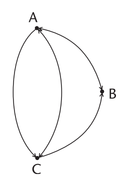
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots \\ b_{11} & b_{12} & b_{13} & \dots \\ c_{11} & c_{12} & c_{13} & \dots \end{bmatrix}$$

Nonessential Characteristics

- number of rows and columns might be the same (square matrix) or might be different
- only some matrices can be multipled
- a network can be modeled with a matrix
- as digraph can be drawn to represent a matrix, and vice versa

Matrix

Examples



$$\begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$$

$$\underset{2 \times 3}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix}} \times \underset{3 \times 2}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}} = \underset{2 \times 2}{\begin{bmatrix} 1 & 1 \\ 2 & 5 \end{bmatrix}}$$

Nonexamples

a) $\begin{matrix} 2 & 3 \\ 1 & 5 \end{matrix}$

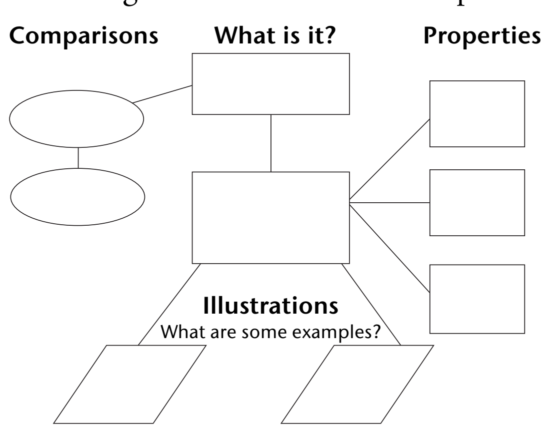
b) $\frac{1}{2}$

c) $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 5 & \\ 3 & & \end{bmatrix}$

d) $\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = ?$

3. Concept Definition Mapping

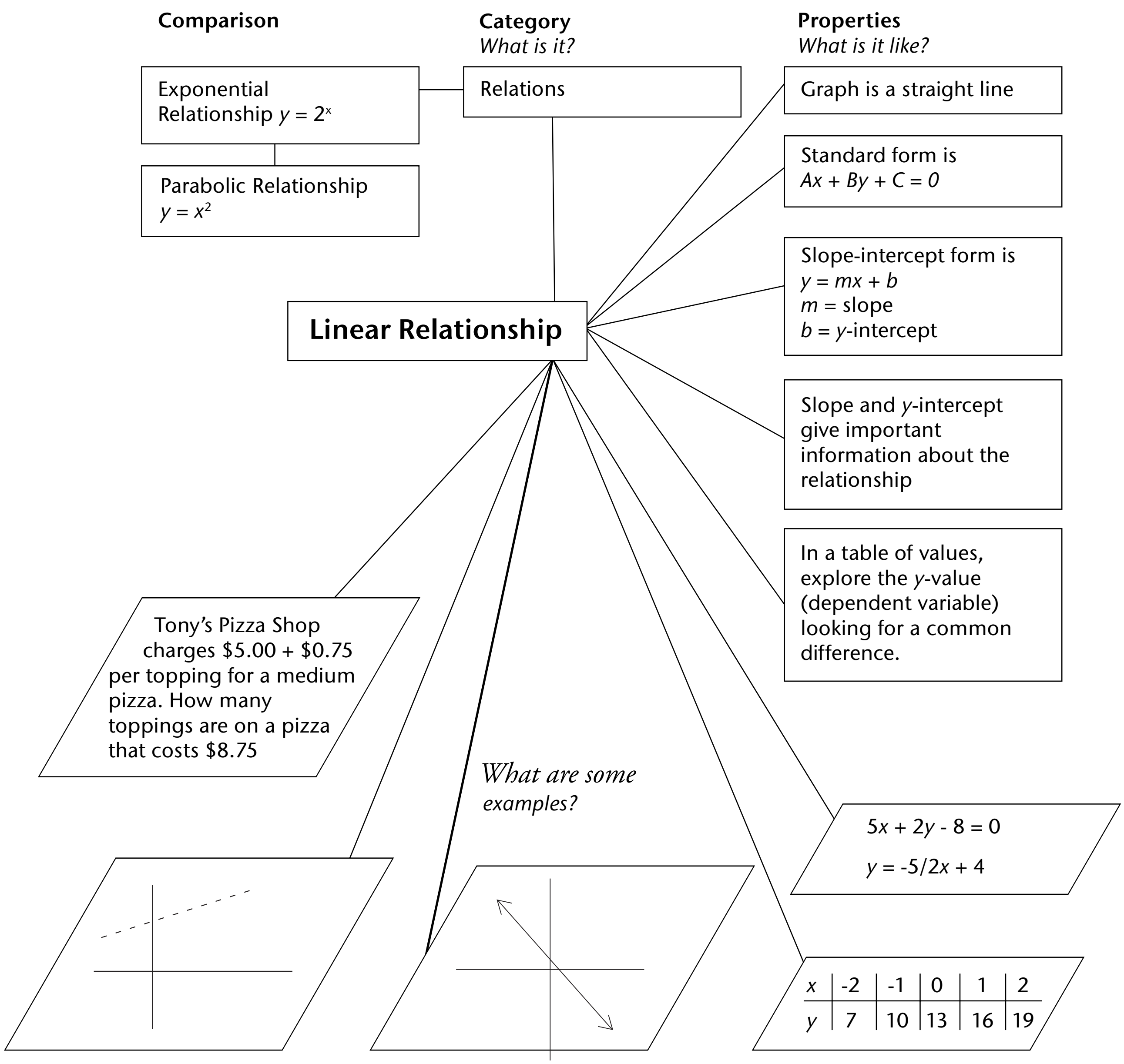
The purpose of a concept definition map is to prompt students to identify the main components of a concept, show the interrelatedness, and build vocabulary. Information is placed into logical categories, allowing students to identify properties, characteristics, and examples of the concept. The following is a rendition of the template found in the Appendix.



The following steps illustrate how the organizer can be used.

- Display the template for the concept definition map.
- Discuss the different headings, what is being sought, and the quality of work that is expected.
- Model how to use this map by using a common concept.
- Establish the concept(s) to be developed.
- Establish the grouping (e.g., pairs) and materials to be used to complete the task.
- Complete the activity by having the student write a complete definition of the concept.

Encourage students to refine their map, as more information becomes available.



4. Word Walls

A mathematics word wall is based upon the same principle as a reading word wall found in many classrooms. It is an organized collection of words that is prominently displayed in the classroom and helps teachers teach the language of mathematics. A word wall could be dedicated to a concept, big idea or unit in the mathematics curriculum. Words are printed in bold, block letters on a card and then posted on the wall or bulletin board.

Depending upon the grade level, illustrations placed next to the word on the wall would add to the students' understanding. Students may also elaborate on the word in their journals by illustrating, showing an example, and using the word in a meaningful sentence or short paragraph. Students can each be assigned a word and its illustration to display on the word wall. Room should be left to add more words and diagrams as the unit or term progresses.

As a new mathematical term is introduced to the class, students can define and categorize the word in their Mathematics Journals under an appropriate unit of study. Then the word can be added to the Mathematics Word Wall so students may refer back to it as needed. Students will be surprised about how many words fall under each category and how many new words they learn to use in mathematics.

Note: The word wall is developed one word at a time as the new terminology is encountered.

To set up a word wall:

1. Determine the key words that students need to know or will encounter in the topic or unit.
2. Print the word in large block letters and add the appropriate illustrations.
3. Display the card when appropriate
4. Regularly review the words as a warm-up or refresher activity.

5. Three Read Strategy

Using this strategy, found in the Nova Scotia Department of Education's document Towards a Coherent Mathematics Program (2002), the teacher encourages students to read the problem three times before they attempt to solve it. There are specific purposes of each reading.

First Read

The students try to visualize the problem in order to get an impression of the overall context of the problem. They do not need specific details at this stage, only a general idea so they can describe the problem in broad terms.

Second Read

The students begin to gather facts about the problem to make a more complete mental image of the problem. As they listen for more detail, they focus on the information to determine and clarify the question.

Third Read

The students check each fact and detail in the problem to verify the accuracy of their mental image and to complete their understanding of the question.

During the Three Read strategy, the students discuss the problem including any information needed to solve the problem. Reading becomes an active process that involves oral communication among students and teachers and also written communication as teachers encourage students to record information and details from their reading as well as to represent what they read in other ways with pictures, symbols, or charts. The teacher facilitates the process by posing questions that ask students to justify their reasoning, to support their thinking, and to clarify their conclusions.

6. Graphic Organizer

A graphic organizer can be of many forms: webs, charts, diagrams etc. Graphic organizers use visual representations as an effective tool to do such things as

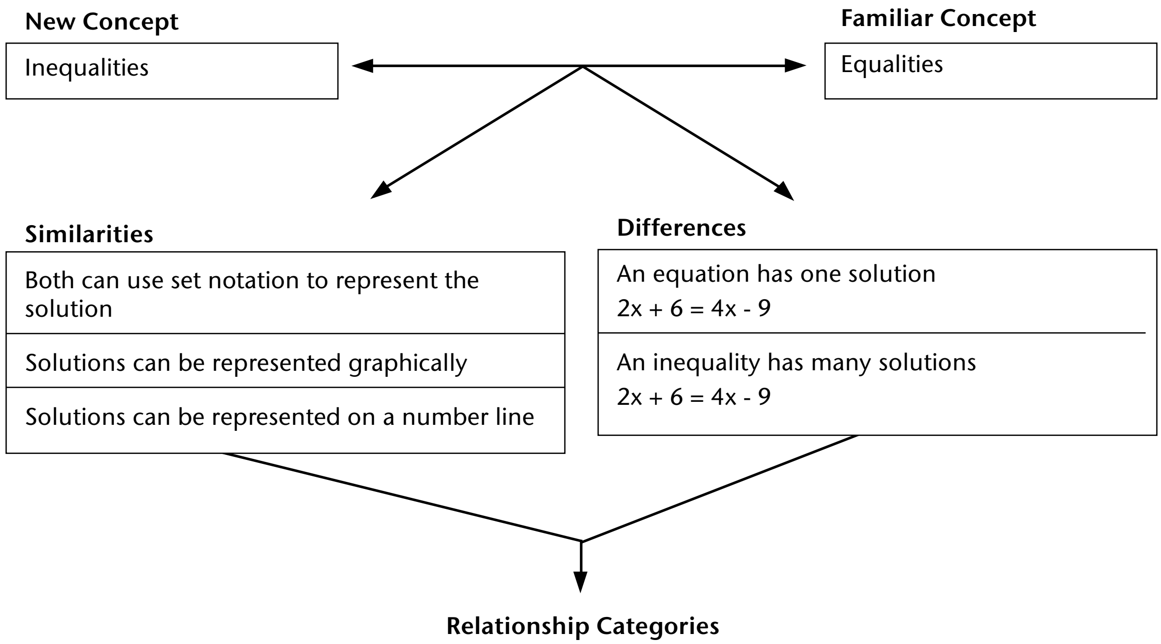
- activate prior knowledge
- make connections
- organize
- analyse
- compare and contrast
- summarize

The following steps illustrate how the organizer can to be used.

- Present a template of the organizer and explain its features.
- Model how to use the organizer, being explicit about the quality of work that is expected.
- Present various opportunities in the classroom and as assessment tools for students to use graphic organizers

Students should be encouraged to used graphic organizers on their own as a way of organizing their ideas and work.

Analogy Graphic Organizer



7. K-W-L (Know/Want to Know/Learned)

K-W-L is an instructional strategy that guides students through a text and uses a three column graphic organizer to consolidate the important ideas. Students brainstorm what they know about the topic and record it in the K column. They then record what they want to know in the W column. During and after the reading, students record what they have learned in the L column. The K-W-L strategy has several purposes.

- obtain a student's prior knowledge of a topic
- give a purpose to the reading
- help a student monitor his/her comprehension

Know	Want to Know	Learned

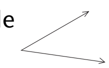
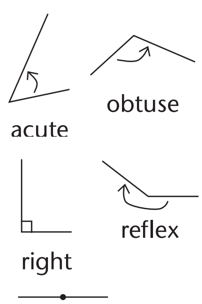
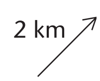
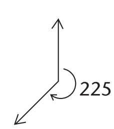
The following steps illustrate how K-W-L can be used.

- Present a template of the organizer to the students, explain its features, and be explicit about the quality of work that is expected.
- Ask them to fill out the first two sections: what they know and what they want to know before proceeding.

- Check the first section for any misconceptions in thinking or weaknesses in vocabulary.
- Have the students read the text, taking notes as they look for answers to the questions they posed.
- Have students complete the last column to include the answers to their questions and other pertinent information.
- Discuss this new information with the class and address any questions that were not answered.

Variations on this strategy are explained in *Secondary Science: A Teaching Resource*, pp.2.9–2.15.

K-W-L Concept (Bearings)

Know	Want to Know	Learned
• bearing can be represented by an angle • angles are the union of two rays at a common endpoint called the vertex of the angle  • angles can be acute, obtuse, or right, also reflex and straight  • angles are measured in degrees ... there are 360° in a circle • to measure an angle, use a protractor and measure the number of degrees from one side of the angle (start at zero) to the other.	• what is a bearing? • how do we measure bearings? • how are they used? • what is a vector? • how do vectors relate to bearings?	• bearing is the angle of direction meausred from the north • bearings are oftenused along with distance • vectors are often used for direction and distance - the direction is indicated by the arrowhead on the vector, or by a given bearing, and the distance is a related number • vectors can be representatinos of bearings, examples:  vector: 3km east  vector: 2km northeast  bearing: 225°  bearing: 2km, 225°

8. Think-Pair-Share

Think-Pair-Share is a learning strategy designed to encourage students to participate in class and keep them on task. It focusses students' thinking on specific topics and provides the students with an opportunity to collaborate and engage in meaningful discussion (Van de Walle, 2000).

- First, teachers ask students to think individually about a newly introduced topic, concept, or problem. This provides essential time for each student to collect their thoughts and focus their thinking. (read Focus C, p.216 Nelson, and complete KWL for learning vectors.)
- Second, each student pairs with another student and together the partners discuss each other's ideas and points of view and review your KWL with another student. Students are more willing to participate because they don't feel the peer pressure that is involved when responding in front of the class. Teachers ensure that sufficient time allows each individual student to voice his or her views and opinions. Students use this time to talk about personal strategies, compare solutions, or to test ideas or understandings (in this case) with their partners. This forces students to make sense of the problem or information in terms of prior knowledge.
- Third, each pair of students shares with the other pairs of students in a large group discussion. In this way, each student has the opportunity to listen to all the ideas and concerns discussed by the other pairs of students. Teachers point out similarities, overlapping ideas, or discrepancies among the pairs of students and facilitate an open discussion to expand upon any key points or arguments they wish to pursue.

9. Think Aloud

Think-aloud is a self-analysis strategy that allows students to gain an insight into the thinking process of a skilled reader as he/she works through a piece of text. Thoughts are verbalized and meaning is constructed around vocabulary and comprehension. It is a useful tool for such things as brainstorming, exploring text features, and constructing meaning when solving problems. When used in mathematics, it can reveal to teachers the strategies that are part of a student's experience and those that are not.

The think aloud process will encourage students to use the following strategies as they approach a piece of text.

- connect new information to prior knowledge
- develop a mental image
- make predictions and analogies
- self-question
- revise and fix-up as comprehension increases

The following steps illustrate how to use a think aloud.

- Explain that reading in mathematics is important and requires students to be thinking and trying to make sense of what they are reading.
- Identify a comprehension problem or piece of text that may be challenging to students, have students read it quietly while the teachers reads it aloud.
- While reading, teachers can model the process by verbalizing what they are thinking, what questions they have, and how they would approach a problem.
- This process should then be modelled a second time but have a student read the problem and do the verbalizing.
- Once students are comfortable with this process, then a student should take a leadership role.

10. Academic Journals—Mathematics

An academic journal in mathematics is an excellent way for students to keep personal work and other materials that they have identified as being important for their personal achievement in mathematics. The types of materials that students would put in their journal would be

- strategic lessons—lessons that they would identify in being pivotal as they attempt to understand mathematics
- examples of problem-solving strategies
- important vocabulary

Teachers are encouraged to allow students to use these journals in an assessment. This way it will emphasize to the student that the material that is to be placed into their journal has a purpose. Also, teachers would probably only mark these journals on the basis of how students are using them and whether or not they have the appropriate entries.

Math Journals

The goal of writing in mathematics is to provide students with opportunities to explain their thinking about mathematical ideas and then to re-examine their thoughts by reviewing their writing. Writing will enhance students' understanding of math as they learn to articulate their thought processes in solving math problems and learning mathematical concepts.

11. Exit Cards

Exit cards are a quick tool for teachers to become better aware of a student's understanding. They are written student responses to questions that have been posed in class or solutions to a problem-solving situation. They can be used at the end of a day, week, lesson, or unit. An index card is given to each student with a question that promotes understanding on it and the student must complete the assignment before they are allowed to "exit" the classroom. The time limit should not exceed 5–10 minutes and the student drops the card into some sort of container on the way out. The teacher now has a quick assessment of a concept that will help teachers as they plan instruction.

Grade 10 Mathematics Language

Number Sense

- Absolute Value
- Continuous data
- cube Root
- Discrete data
- Inequality
- Integers
- Irrational numbers
- Matrices
- Matrix
- Perfect Square
- Perfect Cube
- Principal square root
- Pythagorean Triple
- Radical
- Radicand
- Rational numbers
- Real numbers
- Set notation
- Set of real numbers
- Solution set
- Square root
- Subset of real numbers
- Undefined

Operations

- Algebraic expression
- Additive Inverse
- Associative
- Binomial
- Commutative
- Distributive
- Exponent laws
- Identity
- Identity property
- Integral exponents
- Inverse
- Inverse property
- Matrix operations
- Monomial
- Multiplicative Inverse
- Polynomial
- Product matrix
- Rational numbers
- Scalar
- Scalar multiplication
- Square roots
- Trinomial

Patterns & Relationships

- Adjacency Matrix
- common Factoring
- Degree of equations
- Difference of Squares
- Digraphs
- Dimensions (of a matrix)
- Domain
- Edge
- Element (of a matrix)
- Equation
- Even vertex
- Exponential relation
- Factoring
- Factors
- Function
- Identity
- Intersection Point
- Linear
- Major Diagonal (of a matrix)
- Mapping Notation
- Modeling
- Network
- Network Graph
- Odd vertex
- Parabolic curve
- Perfect Square
- Quadratic
- Range
- Rise
- Run
- Sequence
- Slope of a line
- Slope
- Slope y-intercept form of equation
- Square matrix
- Tree diagram
- Verify
- Vertical Line Test
- X-intercept
- Y-intercept
- Single-variable equation
- Independent variable
- Dependent variable

Measurement

- Accuracy
- Angle of Elevation
- Bearing angle
- Capacity
- Cone
- Congruent triangles
- Cosine
- Cylinders
- Economy rate
- Lateral Face
- Lateral Surface
- Metric conversion
- Precision
- Prisms
- Pyramid
- Rates
- Ratios
- Resultant Vector
- scale Factor
- Similar triangles
- Sine
- Slant Height
- Slopes
- Sphere
- Tangent
- Trigonometric Ratios
- Vector
- Volume

Geometry

- Adjacent Side
- Altitude
- Angle Bisector
- Apothem
- ASA/SAS/SSS
- Centre of Gravity
- circumcentre
- Circumscribed Circle
- Coincident
- Collinear
- Congruence
- Congruent triangles
- Deductive reasoning
- Dilatations
- Exterior Angle
- Heptagon
- Hexagon
- Hypotenuse
- Incentre
- Inscribed Circle
- Inductive reasoning
- Interior Angle
- Line Symmetry
- Median (of a triangle)
- Minimum sufficient conditions
- Octagon
- Opposite Side
- Order of Rotational Symmetry
- Orientation of image
- Pentagon
- Perpendicular Bisector
- Polygon
- Pythagorean Theorem
- Reflections
- Regular Polygon
- Rigidity
- Rotations
- Rotational Symmetry
- Similarity
- Transformations
- Translations

Data Management

- Bin
- Box-and-whisker plot
- Broken-Line Graph
- Controlled variables
- Correlation – strong and weak
- Correlation Coefficient
- Curve of best fit
- Data analysis
- Dependent variable
- Dispersion
- Distribution
- Extrapolate
- Extremes – upper and lower
- Frequency Polygon
- Frequency Table
- Histogram
- Independent variable
- Interpolate
- Line of best fit
- Mean
- Median
- Median-median Line
- Mode
- Negative correlation
- Normal distribution
- Outlier
- Positive correlation
- Prediction
- Precision
- Quartile – upper and lower
- Range
- Regression
- Scatter plot
- Significant digits
- Standard Deviation
- Stem-and-Leaf Plot
- Summary Point

SECTION 3: ASSESSMENT AND EVALUATION

Introduction

Assessment and evaluation are essential to student success in mathematics. The purpose of assessment is manifold: Assessment yields rich data to evaluate student learning, the effectiveness of teaching, and the achievement of the prescribed curriculum outcomes. However, assessment without evaluation is insufficient, as the collection and reporting of data alone are not entirely useful unless the quality of the data is evaluated in relation to the outcomes. To this end, teachers use rubrics, criteria, marking keys, and other objective guides to evaluate the work of their students.

Assessment

Assessment is the process of collecting information about student learning (for example, through observation, portfolios, pencil-and-paper tests, performance). Assessment is the gathering of pertinent information.

Evaluation

Evaluation follows assessment by using the information gathered to determine a student's strengths, needs, and progress in meeting the learning outcomes. Evaluation is the process of making judgments or decisions based on the information collected in assessment.

Assessments *for* Learning

Assessments are designed with a purpose. Some assessments are designed by teachers as assessments "for" learning. The purpose of these assessments is, in part, to assist students in their progress towards the achievement of prescribed curriculum outcomes. In such assessments, the tasks used by teachers should inform students about what kinds of mathematical knowledge and performances are important.

As well, assessments for learning help teachers to know where their students are on the learning continuum, track each student's progress, and plan what "next steps" are required for student success. Following assessments for learning, teachers help students toward the achievement of a mathematics outcome by providing them with further opportunities to learn. In this way, such assessments take a developmental perspective and track students' growth through the year, and should not count towards the assigning of a mark.

Assessment as Learning

Some assessments for learning are designed specifically to encourage student involvement and provide students with a continuous flow of information concerning their achievement. When students become involved in the process of assessment, it becomes assessment "as" learning. Assessment techniques such as conversation, interviews, interactive journals, and self-assessment help students to articulate their ideas and understandings and to identify where they might need more assistance. Such techniques also provide students with insight into their thinking processes and their understandings. This kind of assessment is used not only to allow students to check on their progress, but to advance their understandings, to encourage them to take risks, to allow them to make mistakes, and to enhance their learnings. This kind of assessment also helps students to monitor and evaluate their own learning, to take responsibility for their own record keeping, and to reflect on how they learn, and should not be included in their mark.

Teachers should keep in mind that such assessment practices may be unfamiliar to students at first, and that the emphasis on their being actively involved and thinking for themselves will be a challenge for some students. Such practices, however, enable teachers and students, together, to form a plan that ensures students are clear about what they have to do to achieve particular learning outcomes.

Assessments *of* Learning

Assessments "of" learning provide an overview of a student's achievement in relation to the outcomes documented in the Atlantic Canada mathematics curriculum that form the basis for the student's learning requirements. When an assessment of learning achieves its purpose, it provides information to the teacher **for the grading of student work** in relation to the outcomes.

Final assessments of learning should be administered after the student has had the fullest opportunity to learn the intended outcomes in the mathematics program. Assessments of learning check for a student's achievement against the outcomes. It should be noted that any assessment for learning that reveals whether a student has met the intended outcome can also be considered assessment of learning, and the evaluation of that assessment may be used to report on the student's achievement of the outcome.

Assessments "as," "for," and "of" learning are what teachers do in a balanced classroom assessment process.

Alignment

Assessments serve teaching and learning best when teachers integrate them closely with the ongoing instructional/learning process, when assessments are planned in advance, and when both formative and summative assessments are used appropriately. The nature of the assessments used by the teacher must be appropriate to and aligned with curriculum, so that students' progress is measured by what is taught and what is expected. When learning is the focus, curriculum and assessment become opposite sides of the same coin, each serving the other in the interest of student learning and achievement. Assessments, therefore, should inform classroom decisions and motivate students by maximizing their confidence in themselves as learners. For this reason, teachers need to be prepared to understand the fundamental concepts of assessments and evaluation.

Choosing and using the right kinds of assessments are critical, and teachers need to be aware of the strengths and weaknesses of their assessment choices. As well, employing a variety of appropriate assessments improves the reliability of their evaluation and can help to improve both teaching and learning.

Assessment as and for learning are the foundation of classroom assessment activities leading to assessment of learning.

Planning Process: Assessment and Instruction

The Classroom Assessment Process

The following steps describe what the classroom assessment process might look like.

1. The teacher needs to have a clear understanding of the outcomes that are to be achieved and the multiple ways that students can be involved in the learning process. The teacher must
    - be able to describe what the student needs to achieve
    - collect and create samples that model what the learning looks like
    - decide what kinds of evidence the student can produce to show that he or she has achieved the outcome(s) (i.e., design or select assessment tasks that allow for multiple approaches)
    - select or create learning activities that will ensure student success
2. To introduce the learning, the teacher
    - discusses the outcomes and what is to be achieved
    - shows samples and discusses what the product of the learning should look like

- plans with students by setting criteria for success and developing time lines
- activates prior knowledge
- provides mini-lessons if required to teach/review prerequisite skills

3. After assigning the learning activity, the teacher
   - provides feedback and reminds students to monitor their own learning during the activity
     - Feedback to any student should be about the particular qualities of the work, with advice on how to improve it, and should avoid comparisons with other students.
     - The feedback should have three elements: recognition of the desired performance evidence about the student's current understanding; and some understanding of a way to close the gap between the first two.
   - encourages students to reflect on the learning activity and revisit the criteria
     - The discourse in the classroom is imperative—the dialogue should be thoughtful, reflective, and focussed to evoke and explore understanding.
     - All students should have an opportunity to think and express their ideas.
     - The teacher must ask questions.
     - The questions must require thought.
     - The wait time must be long enough for thinking to take place.
   - uses classroom assessments to build student confidence
     - Tests given in class and exercises given for homework are important means for providing feedback.
     - It is better to have frequent short tests than infrequent long ones.
     - New learning should first be tested within about a week.
     - Quality test items, relevance to the outcomes, and clarity to the students are very important.
     - Good questions are worth sharing. Collaboration, between and among teachers, is encouraged.
     - Feedback from tests must be more than just marks.
     - Quality feedback from tests has been shown to improve learning when it gives each student specific guidance on strengths and weaknesses.
     - Instruction should be continuously adjusted, based on classroom assessments.

- encourages students to self-assess, review criteria, and set new goals to help them take responsibility for their own learning
  - Students should be taught self-assessment so that they can understand the purposes of their learning and understand what they need to do to improve.
  - Students can reflect in their learning journals.

4. The teacher uses cumulative assessment.
   - Eventually, there is a time when students must be able to demonstrate what they have learned, what they understand, and how well they have achieved the outcomes.
   - Assessment should reflect the outcomes and should focus on the students' understanding, as well as their procedural skills.

Assessing Students' Understanding

What does it mean to assess students' understanding?

Students should be asked to provide evidence that they can

- identify and generate examples and non-examples of concepts in any representation (concrete, context, verbal, pictorial, and symbolic)
- translate from one representation of a concept to another
- recognize various meanings and interpretations of concepts
- identify properties and common misconceptions
- connect, compare, and contrast with other concepts
- apply concepts in new/novel/complex situations

What does it mean to assess students' procedural skills?

Students should be asked to provide evidence that they can

- recognize when a procedure is appropriate
- give reasons for steps in a procedure
- reliably and efficiently execute procedures
- verify results of procedures analytically or by using models
- recognize correct and incorrect procedures

What follows is a stem problem and 5 questions that address the 5 bullets above. This addresses outcome 10^{+}C1 and 10D8, 10D14, 10E7 and 10E9

Marla is going to clean the outside of her bedroom window which is on the second floor of her house, 7.5 m above the ground. She has a 9 m ladder and positions it against the house so that it just reaches the bottom of the window. How far from the house on level ground is the foot of the ladder?

1. How would you find the answer to this problem?
2. Bobby's solution to this problem starts like this:

    92 – 7.52 = x2.

    Why did Bobby do this?
3. Solve the problem.
4. The following is Mary's solution. Has she made any errors or omissions? Explain how you know:

    92 + 7.52 = x2

    18 + 15 = x2

    33 = x2

    x = about 5.5
5. Provide three possible student responses to the above problem and ask students to identify any correct or incorrect procedures, and explain their decisions.
    a) a correct algebraic response using symbols
    b) an algebraic response using area of the triangle incorrectly (A = .5(9)(7.5)) instead of the Pythagorean Theorem
    c) a correct response using a scale diagram

Questioning

When we teach, we ask questions. Are we planning the kinds of questions that we want to ask? The effective use of questions may result in more student learning than any other single technique used by educators.

When designing questions, avoid those that can be answered with "yes" or "no" answers. Remember the levels. Questions written at the knowledge level require students to recall specific information. Recall questions are a necessary beginning, since critical thinking begins with data or facts. There are times when we want students to remember factual information and to repeat what they have learned. At the comprehension level, the emphasis is on understanding rather than mere recall, so we need to ask open-ended questions that are thought provoking and require more mental activity. Students need to show that concepts have been understood, to explain similarities and differences, and to infer cause-and-effect relationships.

Comprehension-level questions require students to think, so we must establish ways to allow this to happen after asking the question.

Ask students to discuss their thinking in pairs or in small groups—the respondent speaks on behalf of the others.

Give students a choice of different answers—let the students vote on the answer they like best—or ask the question to the class, then after the first student responds, without indicating whether that was correct or not, redirect the question to solicit more answers from other students.

Ask all students to write down an answer, then select some students to read a few responses.

When redirecting questions (asking several students consecutively without evaluating any) and after hearing several answers, summarize or have the class summarize what has been heard. Or have students individually create (in writing) a final answer to the question now that they have heard several attempts. Lead the class to the correct or best answer to the question (if there is one). Inattentive students who at first have nothing to offer when asked, should be asked again during the redirection of the question to give them a second chance to offer their opinions.

Discourse with the Whole Group

Sometimes it is best to begin discussion by asking more divergent-level questions, to probe all related thoughts and bring students to the awareness of big ideas, then to move towards more convergent questions as the learning goal is being approached. Word the questions well and explain them in other words if students are having trouble understanding. Direct questions to the entire class. Handle incomplete or unclear responses by reinforcing what is correct and then asking follow-up questions. There are times when you want to restate correct responses in your own words, then ask for alternative responses. Sometimes it is important to ask for additional details, seek clarifications, or ask the student to justify a response. Redirect the question to the whole group if the desired response is not obtained. Randomize selection when many hands are waving. Ask a student who is always the first to wave a hand to ask another student for an answer, then to comment on that response. As the discussion moves along, interrelate previous students' comments in an effort to draw a conclusion. It is particularly important to ask questions near the end of the discussion that help make the learning goal clear.

Questioning is a way of getting to assess student progress and an important way to increase student learning. As well, it is a way to get students to think and to formulate and express opinions.

Examples of Effective Questions:

Critical Thinking • Why did ... ? • Give reasons for ... • Describe the steps ... • Show how this ... • Explain why ... • What steps were taken to ... ? • Why do you agree (disagree) with ... ? • Evaluate the result of ... • How do you know that ... ?	**Cause/Effect Relationship** • What are the causes of ... ? • What connection exists between ... ? • What are the results of ... ? • If we change this, then ... ? • If these statements are true, then what do you think is most likely to happen ... ?
Comparison • What is the difference ... ? • Compare the ... • How similar are ... ? • Contrast the ...	**Problems** • What else could you try ... ? • The diagrams in the problem suggest the following relationship: x 1 2 3 4 5 y 1 3 6 10 15 • What would you say is the y-value when ... ?
Personalized • Which would you rather be like ... ? • What would you conjecture ... ? • Which do you think is correct ... ? • How would your answer compare ...? • What did you try ... ? • How do you feel about ... ? • What would you do if ... ? • If you don't know ... how could you find out?	**Descriptive** • Describe ... • Tell ... • State ... • Illustrate ... • Draw (sketch) ... • Define ... • Analyze ...

Levels of Questioning

In recent years, the Nova Scotia Department of Education has administered elementary and junior high mathematics assessments. In an attempt to set standards for these assessments, committees of teachers prepared tables of specifications. These committees also decided that there would be a blend of various complexity levels of questions and determined what percentage of the whole assessment should be given for each level of question. They also agreed that the assessments would use a combination of selected response questions and constructed response questions, some of which might require extended responses, others short responses.

Level 1: Knowledge and Procedure (low complexity)

Key Words

- identify
- compute
- recall
- recognize
- find
- evaluate
- use
- measure

– Level 1 questions rely heavily on recall and recognition.
– Items typically specify what the student is to do.
– The student must carry out some procedure that can be performed mechanically.
– The student does not need an original method of solution.

The following are some, but not all, of the demands that items of "low complexity" might make:
– recall or recognize a fact, term, or property
– recognize an example of a concept
– compute a sum, difference, product, or quotient
– recognize an equivalent representation
– perform a specified procedure
– evaluate an expression in an equation or formula for a given variable
– solve a one-step word problem
– draw or measure simple geometric figures
– retrieve information from a graph, table, or figure

Level 2: Comprehension of Concepts and Procedures (moderate complexity)

Key Words

- classify
- organize
- estimate
- explain
- interpret
- compare

- Items involve more flexibility of thinking and choice.
- Questions require a response that goes beyond the habitual.
- The method of solution is not specified.
- Questions ordinarily have more than a single step.
- The student is expected to decide what to do using informal methods of reasoning and problem-solving strategies.
- The student is expected to bring together skills and knowledge from various domains.

The following illustrate some of the demands that items of "moderate complexity" might make:

- make connections between facts, terms, properties, or operations
- represent a situation mathematically in more than one way
- select and use different representations, depending on situation and purpose
- solve a word problem involving multiple steps
- compare figures or statements
- explain and provide justification for steps in a solution process
- interpret a visual representation
- extend a pattern
- retrieve information from a graph, table, or figure and use it to solve a problem requiring multiple steps
- formulate a routine problem, given data and conditions
- interpret a simple argument

Level 3: Applications and Problem Solving (high complexity)

Key Words

- analyse
- investigate
- formulate
- prove
- explain
- describe

- Questions make heavy demands on students.
- The students engage in reasoning, planning, analysis, judgment, and creative thought.
- The students must think in an abstract and sophisticated way.

The following illustrate some of the demands that items of "high complexity" might make:

– explain relations among facts, terms, properties, or operations
– describe how different representations can be used for different purposes
– perform a procedure having multiple steps and multiple decision points
– analyse similarities and differences between procedures and concepts
– generalize a pattern
– formulate an original problem
– solve a novel problem
– solve a problem in more than one way
– justify a solution to a problem
– describe, compare, and contrast solution methods
– formulate a mathematical model for a complex situation
– analyse the assumptions made in a mathematical model
– analyse or produce a deductive argument
– provide a mathematical justification

Recommended Percentages for Testing

Level 1: 25–30%

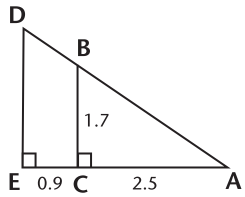
Level 2: 40–50%

Level 3: 25–30%

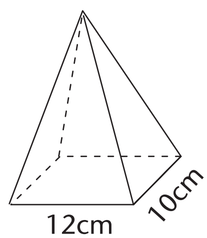
Sample Questions

Level 1

1. The mapping rule for a reflection in the y-axis is
   a) $(x,y) \rightarrow (-x,y)$ b) $(x,y) \rightarrow (x,-y)$
   c) $(x,y) \rightarrow (-x,-y)$ d) $(x,y) \rightarrow (y,x)$
2. If the triangle ABC is similar to the triangle ADE find the length of DE



3. If the vertical height of this pyramid is 15cm, determine the volume.



4. Solve $\frac{1}{2}(x+6) = \frac{2}{3}x-9$
5. What is the slope and y-intercept of the line $2x - 4y + 12 = 0$
6. Find the product of $(2x+1)(x-3)$
7. Find the factors of $x^2 - 8x + 12$

Level 2

1. In class, your group performed an experiment and collected the following data. The experiment was to determine the distance a ball bearing would roll once it hit the floor after being released from various heights on a elevated ramp.
   a) Plot the graph and describe its shape.
   b) Why is it impossible to obtain a slope for this graph?
   c) If the ball is dropped from a height of 15 cm, how far will it roll?
   d) If the ball rolled 50 cm how high would it have been on the ramp?
2. A reflection in the y-axis can be represented by a mapping rule. Use the mapping rule to determine the equation of the image of the line $y = 2x + 1$.
3. Marla and Bob are working together to complete their math project. Marla concluded that the area of the object being studied was 10,000 mm^2. Bob said that they should record the answer in cm^2, and wrote: 1000cm^2. Show, using labeled diagrams, that Bob's answer cannot be correct.

4. While exploring transformations on graph paper, Marla transformed the line y=x to the image y=2x. Graph these two lines and explain how the graph shows what transformation has taken place.
5. Illustrate how algebra tiles can be used to determine the factors of $4x^2 - 6x$.
6. Tom had a winning pop bottle cap, but in order to claim the prize, he had to answer the skill-testing question: $\frac{1}{2} - \frac{3}{4}\left[3 - 2\left(\frac{-3}{4} - \frac{2}{3}\right) - 5 \div -\frac{1}{4}\right]$. He said the answer was . Will he win the prize?
7. Determine which of these table of values represents a linear relationships and explain why

a)

x	-2	-1	0	1	2
y	10	8	6	4	2

b)

x	-2	-1	0	1	2
y	48	24	12	6	3

Level 3

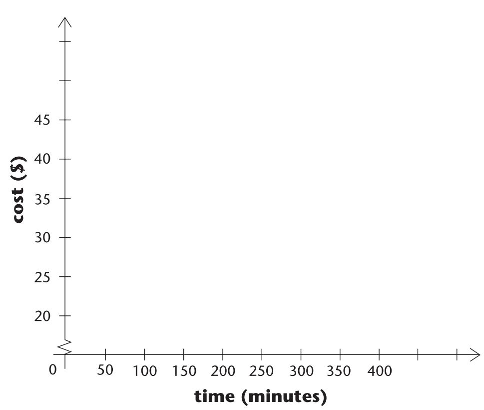
1. The slope of a whell-chair ramp must, by regulation, be 5°. You have to construct a ramp to reach a door 3m above the ground. Would a 20m ramp be acceptable?
2. The radius of the eart is 6378.1 km. The radius of the sun is 695500 km. Determing how many times greater the volume of the sun is than the volume of the earth. Express your anwer in scientific notation.
3. Your parents have bought you a cell phone but you are responsible for paying the monthly payments. Here are the thre options.

Plan A: \$20 per month inclueds 200 free minutes of airtime and 8¢ for each additional minute.

Plan B: \$30 per month includes 150 minutes and 5¢ for each additional minute.

Plan C: \$40 per month includes unlimited time.

Use the axis provided to determine under what circumstances each of the other three plans would be the best choice. Justify your answer.

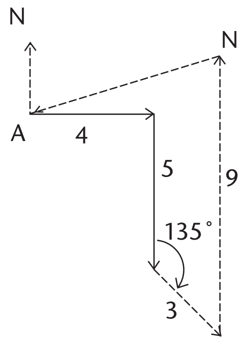


Scoring Open-ended Questions by Using Rubrics

Students should have opportunities to develop responses to open-ended questions that are designed to address one or more specific curriculum outcomes.

Example of an "open-ended" question:

Make up a problem and solve it given the information in this diagram.



Open-ended questions allow students to demonstrate their understandings of mathematical ideas and show what they know with respect to the curriculum, and they should lead to a solution.

Often, responses to open-ended questions are marked according to a rubric that allows the teacher to assign a level of achievement towards an outcome based on the solution written by the student.

For each individual open-ended question, a rubric should be created to reflect the specific important elements of that problem. Often these individual rubrics are based on more generic rubrics that give examples of the kinds of factors that should be considered. Details will vary for different grade levels, but the basic ideas apply for all levels.

How do you begin thinking about a rubric? Consider this. Let us say that you asked your students to write a paper on a particular mathematician. They handed in their one-page reports, and you began to read them. As you read, you were able to say, Hey, that one is pretty good, this one is fair, and this third one needs a lot of work. So you decide to read them all and put them into the three piles: good, fair, and not so good. A second reading allows you to separate each of the three piles into two more piles: top of the level and bottom of the level. When done, you have six levels, and you could describe the criteria for each of those six levels in order to focus more on the learning outcome. You have created a rubric.

Some rubrics have criteria described for six levels, some five levels, and some four levels. Some people like four levels because it forces the teacher to distinguish between acceptable performance, and below (not acceptable), there is no middle—you either achieve a level 2 (not acceptable work) or level 3 (acceptable).

The first example that follows includes the criteria for a generic four-level rubric, found in the NCTM booklet Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions (Stenmark 1991). In choosing to use this rubric, a teacher would change the generic wording to fit the given problem or open-ended situation. Some teachers like to assign a name to the different achievement levels. For example, they may call the "top level" responses, exceptional; the "second level," good; the "third level," not quite; and the "fourth level," needs more work. Many schools simply assign a number rating, usually the top level receiving a 4, then going down, 3, then 2, then 1. Students strive for a level 3 or 4. Some schools assign letters to the categories, giving A to the top level, then B, C, and D.

Achievement Level	**Criteria**
Top Level	• contains complete response with clear, coherent, unambiguous, and elegant explanation • includes clear and simple diagram • communicates effectively to an identified audience • shows understanding of the question's mathematical ideas and processes • identifies all the important elements of the question • includes examples and counter-examples • gives strong supporting arguments • goes beyond the requirements of the problem
Second Level	• contains a good solid response with some of the characteristics above, but not all • explains less elegantly, less completely • does not go beyond requirements of the problem
Third Level	• contains a complete response but the explanation may be muddled • presents arguments but they are incomplete • includes diagrams, but they are inappropriate or unclear • indicates understanding of the mathematical ideas but they are not expressed clearly
Fourth Level	• omits significant parts or all of the question and response • has major errors • uses inappropriate strategies

A second example of a rubric (on the following page) that allows for six achievement levels. This example can be found in the booklet *Assessment Alternatives in Mathematics*, prepared by the Equals staff and the Assessment Committee of the California Mathematics Council *Campaign for Mathematics*, under the leadership of Jean Stenmark.

Achievement Level	Criteria
6. Exemplary Response	• complete response, with clear, coherent, unambiguous, and elegant explanation • clear and simplified diagrams • communicates effectively • shows understanding of the mathematical ideas and processes • identifies all the important elements • may include examples and counter-examples • presents strong supporting arguments
5. Competent Response	• fairly complete response, with reasonably clear explanations • may include an appropriate diagram • communicates effectively • shows understanding of the mathematical ideas and processes • identifies the most important elements of the problem • presents solid supporting arguments
4. Satisfactory (minor flaws)	• completes the problem satisfactorily but explanation might be muddled • incomplete arguments • diagram inappropriate or unclear • understands the underlying mathematical ideas • uses mathematical ideas effectively
3. Nearly Satisfactory (serious flaws)	• begins problem appropriately, but fails to complete or may omit parts of the problem • fails to show full understanding of the mathematical ideas and processes • may make major computational errors • may misuse or fail to use mathematical terms • response may reflect inappropriate strategy
2. Incomplete	• explanation is not understandable • diagram may be unclear • shows no understanding of the problem situation • may make major computational errors
1. Ineffective Beginning	• words do not reflect the problem • drawings misrepresent the problem situation • copies part of the problem but without attempting a solution • fails to indicate which information is appropriate to the problem
0. No Attempt	• no evidence of anything meaningful

The top two levels of the above rubric have been titled “Demonstrated Competence”, and will get a 6 and a 5. The next two levels called “Satisfactory” will get a 4 and a 3, while the bottom three levels, “Inadequate Response” receive a 2, 1, or 0.

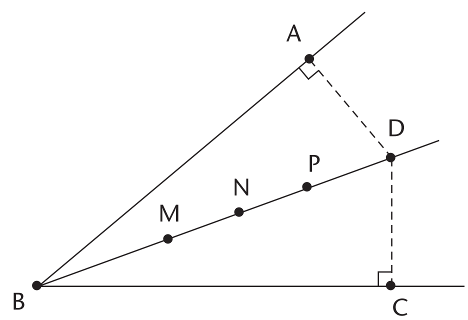
A third kind of rubric that is becoming more popular these days includes more than one domain when assigning the levels. For example, in the next generic rubric the solution attempt will follow the criteria for four levels of achievement, but in four domains: problem solving, understanding concepts, application of procedures, and communication. The teacher assigns levels of achievement for each domain.

Expectation	Level 1	Level 2	Level 3	Level 4
Problem Solving	• shows none or very little understanding of the mathematical ideas and processes required • uses no strategies	• shows little understanding of the mathematical ideas and processes required • inappropriate strategy or strategies attempted	• shows understanding of the mathematical ideas and processes required • uses some appropriate strategies	• shows thorough understanding of the mathematical ideas and processes required • uses appropriate strategies
Understanding Concepts	• demonstrates no understanding of the mathematical concepts required in the problem situation	• demonstrates little understanding of the mathematical concepts required in the problem situation	• demonstrates some understanding of the mathematical concepts required in the problem situation	• demonstrates a thorough understanding of the mathematical concepts required in the problem situation
Application of Procedures	• a few calculations and/or use of skills and procedures may be correct, but inappropriate for the problem situation	• makes several errors in calculations and/or use of some appropriate skills and procedures	• may make a few errors in calculations and/or use of skills and procedures	• uses accurate calculations and appropriate skills and procedures
Communication	• no appropriate arguements • no appropriate diagrams • improper use of words and terms • incomplete arguments	• diagram inappropriate or unclear • may misuse of fail to use mathematical words and terms • fairly complete response, with reasonably clear explanations and appropriate use of words and terms • may include an appropriate diagram	• communicates effectively • presents some supporting arguments when appropriate • complete response, with clear, coherent, unambiguous, and elegant explanation and/or use of words and terms	• clear and simplified diagrams • communicates effectively • presents strong supporting arguments when appropriate

The use of rubrics for assessing open-ended problem situations allows the teacher to indicate how well the student has demonstrated achievement of the outcomes for which the assessment item has been designed in each of these important domains. In reporting to the student or the parent, the teacher can make very clear what it is that the student has not accomplished with respect to full achievement of the outcome(s). Over time as the outcome(s) is assessed again, progress or lack thereof becomes very clear when the criteria is clearly indicated.

Achievement levels can be changed into percentage marks (if that is the desire) by adding together the achievement levels obtained, dividing by the maximum levels obtainable, and changing that ratio to a percent. For example, Freddie received a level 3, 4, 3, 2, 3, 4, 3, 3, 3, and 2 during the term when open-ended problem solving opportunities were assigned. He could have obtained a grade of 4 each time, so his ratio is 30 out of a possible 40, giving him a 75 percent mark.

Example of a Rubric for a Level 3 question



- Given BD bisects angle ABC and M,N, and P are points anywhere on . Draw perpendiculars from M, N and P, to each of and .
- Measure the length of the perpendicular segment.
- What conjecture might you state about any point on the bisector of an angle in relation to the side of the angle?
- Describe how you could test your conjecture using D.

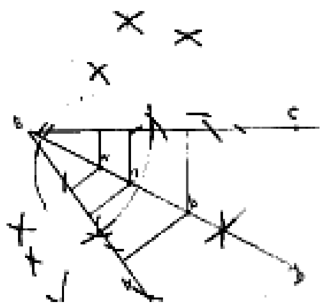
This activity involves the use of a straight edge, and a compass. Students will have to accurately construct the given diagram, draw perpendiculars, from given points, with a compass and measure their lengths. Students will the be asked to conjecture a relationship, then that it and apply it to solve a problem.

Expectation	Level 1	Level 2	Level 3	Level 4
Problem Solving	Demonstrates little or no understanding of the requirements to make a conjecture or solve the problem. Makes significant errors or omits steps which leads to no conjecture or wrong conjecture.	Demonstrates some understanding of the requirements to make a conjecture or solve the problem and/or makes significant errors in procedures so that a wrong conjecture is made or that a correct conjecture cannot be made.	Demonstrates a good understanding of the requirements to make a conjecture and to solve the problem. Makes a small error or errors, but still makes a correct conjecture and applies it correctly.	Demonstrates a thorough understanding of the requirements to make a conjecture and to solve the problem – draw the diagram accurately – measure accurately – states a correct conjecture tests the conjecture correctly – uses the conjecture to solve the problem correctly for point R.
Understanding Concepts	Demonstrates little or no conceptual understanding. Cannot make a correct conjecture.	Demonstrates some conceptual understanding but does not or cannot make a correct conjecture.	Demonstrates a good understanding of the concepts needed to make the conjecture and/or solve the problem, but has made a small conceptual error like, making the right angle on BD, or not using the three angles to solve the problem.	Demonstrates a thorough understanding of the concepts required. – angle bisector – perpendicular to a line – measures precisely – makes a conjecture – applies the conjecture
Application of Procedures	Makes major errors that leads to making no conjecture or an incorrect conjecture. Cannot apply the conjecture to solve the problem.	Makes many small errors and some major errors, or makes incorrect conjecture based on errors in procedures.	Makes a few small errors but is able to make a correct conjecture. Possible errors: – bisector off – perpendicular not quite correct – small measurement errors – errors in testing the conjecture.	– is able to draw an angle and bisect it – is able to place points on aline – is able to draw a perpendicular to line – meausre accurately – makes the expected conjecture – tests the conjecture – applies the conjecture to locate R understands the word “describe”

Communication	Major errors in following directions – leads to no conjecture, and/ or no solution to the problem.	Major errors in following directions leading to an incorrect conjecture, and/or major error in describing the location of R.	Small errors in following directions but still states a correct conjecture, and/or small error in describing location of the point R.	– follows directions completely – uses results to formulate a correct and meaningful conjecture. – describes the location of the point R accurately and completely.

Typical Student Sample 1:

Draws the diagram:



Measures and records:

from M : 1cm 1cm

N : 1.4cm 1.4cm

P : 2.2cm 2.2cm

Makes a conjecture:

Conjecture :

Any point on the bisector of an angle is the same (perpendicular) distance to the arms of the angle.

Tests the conjecture:

I would construct the perpendiculars from D to $\overline{BA}$ and $\overline{BC}$

I would measure the lengths of the 2 constructed perpendiculars

If the lengths are equal then my conjecture must be true

Applies the conjecture to solve the problem:

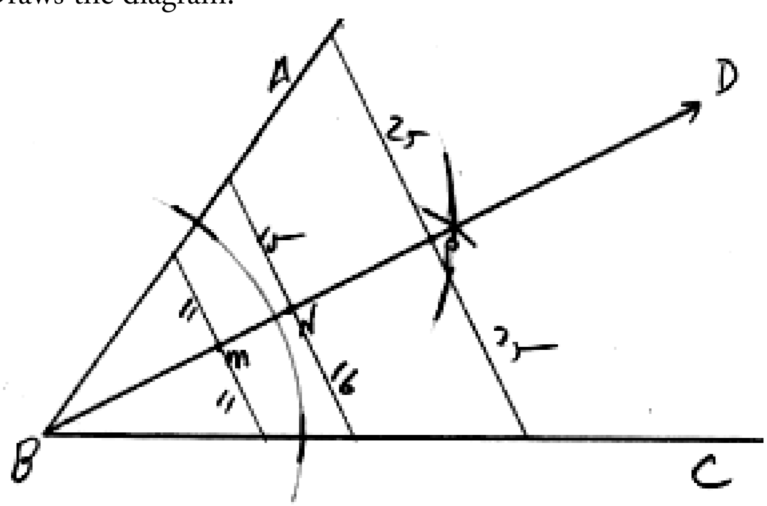
In order for the point R to be equidistant to two sides of the triangle it would have to be on the angle bisector of the angle formed by those two sides. To be equidistant to a different pair of sides, it would have to be on the angle bisector of the angle formed by those two sides. So, now, R is on two of the angle bisectors of 2 of the angles, so, it must be at their intersection point. Thus, to be equidistant to all three sides, R must be the point of intersection of the three angle bisectors.

Scoring:

- Problem Solving: [4] Shows complete understanding of what the activity was all about and all the components for a complete solution.
- Understanding Concepts: [4] Shows understanding of all the concepts involved in the activity.
- Application of Procedures: [4] All procedures followed correctly.
- Communications: [4] All communication opportunities completed successfully. Reasoning well explained.

Typical Student Sample 2:

Draws the diagram:



Measures and records:

See numbers marked on the diagram.

Makes a conjecture:

points on angle bisectors are the same distance away from the two sides of the angle.

Tests the conjecture:

I would measure from D to the two sides to see if they are the same distance.

Applies the conjecture to solve the problem:

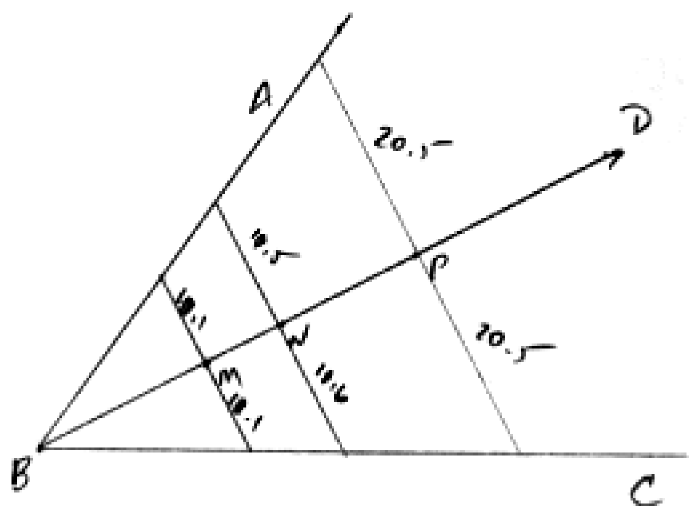
R would have to be at the spot where the three angle bisectors intersect.

Scoring:

- Problem Solving: [4] Shows understanding of what the activity was all about and all the components for a complete solution.
- Understanding Concepts: [3] Error made when constructing the perpendiculars to the lines.
- Application of Procedures: [4] All procedures followed correctly even though a conceptual error was made when constructing the perpendiculars.
- Communications: [3] All communication opportunities completed. Reasoning could be presented better.

Typical Student Sample 3:

Draws the diagram:



Measures and records:

See diagram

Makes a conjecture:

points on the bisectors are on the same line as points on the angle.

Tests the conjecture:

just lay your ruler along the lines

Applies the conjecture to solve the problem:

put R in the middle of the triangle

Scoring:

- Problem Solving: [1] Shows little understanding of what the activity was all about (no construction, right angles in the wrong places, incorrect measuring, and no sense of what the conjecture was about) and how to get a solution.
- Understanding Concepts: [2] Seems to understand what an angle bisector is, although not constructed, shows some sense of getting distance to the arms of the angle. Errors made in measuring lines, and not constructing the proper perpendiculars.
- Application of Procedures: [1] No required procedures followed correctly. Didn't seem to have a sense of what the conjecture should be about or how to apply it.
- Communications: [2] All communication opportunities completed although not necessarily correct. Not much reasoning presented.

Assessment Techniques

Observations

All teachers learn valuable information about their students every day. If teachers have a systematic way of gathering and recording this information then it will allow the teacher to provide valuable feedback about student progress to both students and parents. First, teachers must decide what they are looking for and which children will be observed today. Then the recording method can be decided. John A. Van de Walle's *Elementary and Middle School Mathematics; Teaching Developmentally, 4th Edition,* provides ideas on pp. 67–67. For example:

Super	
• clear understanding • communicates concepts in multiple representations • shows evidence of using ideas without prompting	
On Target • understands, or is developing well • uses designated models	
Not Yet • some confusion • misunderstands • only models ideas with help	

Use for several days during the development of a topic or during an activity, and record names of observed students and their achievement of learning efforts.

A generic observation checklist for being very specific as to what outcomes are being achieved and to what extent, as the learning develops, may be useful to accompany the above checklist.

Name: ______________________			**Grade:** __________	
	Not Yet	**OK**	**Super**	**Comments**
Topic - Concept specifics from associated outcomes on the conceptual development related to the topic to be observed				
Topic - Procedures specifics from associated outcomes on the procedural aspects of the topic to be observed				

Specifically, the following checklist shows what this may look like at the grade 10 level, when developing the topic of relations

Name: ________________________			Grade: ______________	
Fractions/Decimals	**Not Yet**	**OK**	**Super**	**Comments**
• represent patterns and relationships in a variety of formats				
• predict and justify unknown values using represented patterns and relationships.				
• interpret graphs that represent linear data				
• interpret graphs that represent non-linear data3				

Name:			Grade:	
Operations with fractions and decimals	**Not Yet**	**OK**	**Super**	**Comments**
• add decimals involving tenths and hundredths				
• subtract decimals involving tenths and hundredths				
• add simple fractions with common denominators using models				
• solve word problems involving adding and subtracting decimals (to hundredths)				
• create word problems involving adding and subtracting decimals (to hundredths)				
• estimate sums and differences of decimals				

A focussed checklist can be kept for each student during the development of particular topics.

Other examples can be seen in the NCTM booklet, *Mathematics Assessment; Myths, Models, Good Questions, and Practical Suggestions,* p.33.

Daily Observation Sheet

Name:			
Date	**Activity**	**Observed Behaviour**	**Progress Suggestions**

Conversations and Interviews

Interviews provide more information concerning a particular student and what he or she knows, and provides the opportunity for the teacher to learn more about how a child thinks. Start an interview with questions that a child can be successful with, then ask the child to explain how the answers were obtained, and/or why he/she thinks they are correct. Perhaps ask the child to explain this work to a child in a lower grade level.

- be accepting, but neutral
- avoid cuing or leading the child
- wait silently, don't interupt
- use prompts like "show me ..", and " tell me ..."
- avoid confirming requests for "am I doing this right?", and "is this ok?"

Examples of checklists can be seen in the NCTM booklet, *Mathmatics Assessment; Myths, Models, Good Questions, and Practical Suggestions,* p.33.

Student:	**Week:**
Observation and/or Interview Notes	
Tell me how you solved the problem.	
Is there another way to solve the problem?	
Where did you get stuck?	

Journal/Log Entries

Writing in mathematics is a very important way to help students clarify their thinking. Have students write explanations, justifications, and descriptions.

Here are some writing prompts for journals or logs:

- I think the answer is ...
- I think this is so, because ...
- write an explanation for a student in a lower grade, or for a student who was absent when this was taught
- what do I understand?... what don't I understand? ...
- I got stuck today because ...
- how do you know you are right? ...
- summarize concepts by drawing pictures of things that represent the concept, and things that don't represent the concept

The best assessment of these writings would be to respond to the students' writing with the intention to develop a written conversation that might lead to clarifying misconceptions, and giving better explanations about concepts or mathematical ideas being developed in the classroom, or about how to study, or prepare for tests, and so on.

The teacher need only keep a checklist of who has submitted (and when) journal or log entries, and to whom he/she has responded.

Self–Assessment

In the NCTM booklet, *Mathematics Assessment; Myths, Models, Good Questions, and Practical Suggestions,* it says that "self-evaluation promotes metacognition skills, ownership of learning, and independence of thought."

When students assess themselves the teacher and the student will be able to see:

- signs of change and growth in attitudes, mathematical understandings, and achievement
- alignment of how well students are performing with how well they think they are performing
- a match between what the teacher thinks the student can do, and what the student thinks the student can do.

Some prompts:

- how well do you think you understand this concept?
- what you believe ... how you feel about ...
- how well do you think you are doing ?
- when you work with a group what are your strengths ... what are your weaknesses ...

Self-assessment checklists can be developed for each specific activity in which the teacher wants the students to perform self-assessment. What follows are some suggested yes, no, not sure type questions from which more specific checklists can be developed. These suggestions come from the NCTM booklet, *Mathematics Assessment; Myths, Models, Good Questions, and Practical Suggestions,* put together by Jan Stenmark, University of California. See pp.58 and 59 for other ideas.

1. Sometimes I don't know what to do when I start a problem.
2. I like mathematics because I can figure things out.
3. The harder the problems, the better I like to work on them.
4. I usually give up when a problem is really hard.
5. I like the memorizing part of mathematics the best.
6. There is more to mathematics than just getting the right answer.
7. I think that mathematics is not really useful in everyday living.
8. I would rather work alone than with a group.
9. I like to do a lot of problems of the same kind rather than have different kinds all mixed up.
10. I enjoy mathematics.
11. There's always a best way to solve a problem.
12. I liked mathematics when I was younger, but now it's too hard.
13. Put an 'x' on this scale where you think you belong:

I am not good at mathematics	|———————|	I am good at mathematics

SECTION 4: TECHNOLOGY

Integrating Information Technology and Mathematics

The public school program includes "technological competence" as one of the essential graduation learnings. This means that technological activities and familiarity with technology and its impact on society are considered essential for inclusion in every curricular area. The Nova Scotia Department of Education has developed a document entitled *The Integration of Information and Communication Technology within the Curriculum* (Nova Scotia Department of Education 2005). This document provides a foundation for the use of information technologies within classrooms in Nova Scotia.

Many other technological tools can be used as part of curricular activities to help satisfy the essential graduation learning requirement for "technological competence." For the purposes of this section of the resource, however, only information technologies (computers, software, and Internet) will be discussed.

When planning lessons for students, it is important to consider the key-stage outcomes for the integration of technology. Where appropriate, lessons should include a technological approach or activities, as well as other approaches to facilitate student learning.

Technological Competence—What Is It?

Graduates will be expected to use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems.

Graduates will be able, for example, to

- locate, evaluate, adapt, create, and share information, using a variety of sources and technologies
- demonstrate understanding of and use existing and developing technologies
- demonstrate understanding of the impact of technology on society
- demonstrate understanding of ethical issues related to the use of technology in a local and global context

The Role of Technology—What Is It?

Just as computers and other technology play a central role in developing and applying scientific knowledge, they can also facilitate the learning of science. It follows, therefore, that technology should have a major role in the teaching and learning of mathematics.

Computers and related technology (projection panels, CD-ROM players, videodisc players, analog-digital interfaces, graphing calculators) have become valuable classroom tools for the acquisition, analysis, presentation, and communication of data in ways that allow students to become more active participants in research and learning. In the classroom, such technology offers the teacher more flexibility in presentation, better management of instructional techniques, and easier record keeping. Computers and related technology offer students a very important resource for learning the concepts and processes of mathematics through simulations, graphics, sound, data manipulation, and model building. These capabilities can improve mathematical learning and facilitate communication of ideas and concepts. It should be remembered that, although there is an emphasis on the use of computers as technological tools, there is a wide variety of other technological tools, that will also be of special benefit for a mathematics classroom. The following guidelines are proposed for the implementation of computers and related technology in the teaching and learning of mathematics.

Tutorial software should engage students in meaningful interactive dialogue and creatively employ graphs, sound, and simulations to promote acquisition of facts and skills, promote concept learning, and enhance understanding.

Simulation software should provide opportunities to explore concepts and models that are not readily accessible in the classroom, e.g., those that require

- expensive or unavailable materials or equipment
- hazardous materials or procedures
- levels of skills not yet achieved by the students
- more time than is possible or appropriate in real-time classrooms

Analog-digital interface technology could be used to permit students to collect and analyse data as scientists and statisticians do, enabling real-world mathematical experiences.

Databases and spreadsheets should be used to facilitate the analysis of data by organizing and visually displaying information.

Networking among students and teachers should be encouraged to permit students to emulate the way mathematicians work and, for teachers, to reduce teacher isolation. Using tools such as the World Wide Web should be encouraged, as it provides instant access to an incredible wealth of information on any imaginable topic.

Many software applications have a nearly universal use in classrooms as methods of acquiring and demonstrating knowledge. Students may use word-processing applications to write about their experiences. They might use software to create presentations as teaching tools for other students or as a way of presenting knowledge they have gained. Concept-mapping software can be used to create flowcharts, to visually show information, or to organize and plan other work. Students can take photographs and/or videos that demonstrate some aspect of their classroom learning and incorporate these into reports or create slide shows and videos of their activities. Specific software applications that address specific outcomes for various grades are available through the Nova Scotia School Book Bureau.

In order to effectively implement computers and other technology in mathematics education, teachers should

- know how to effectively and efficiently use the hardware, software, and techniques described above
- know how to incorporate microcomputers and other technology into instructional strategies
- become familiar with the use of computer applications such as management tools for grading and creating reports, inventories, and budgets
- exemplify the ethical use of computers and software
- seek to provide equitable access for all students

Information and Communication Technology—Why Use It?

When used properly, information and communication technology (ICT)

- facilitates students' communication, problem solving, decision making, and expression
- helps students to manipulate information so as to discover patterns and relationships and construct meaning
- helps students formulate, in the pursuit of knowledge, both conclusions and complex questions for further research
- permits students and teachers to access and share current learning content and information more immediately
- provides students with special needs fuller access to their learning environment
- allows Nova Scotians to develop and maintain competitive advantages in the global information economy

Learning Outcomes Framework

There are five strands in the learning outcomes framework for the integration of ICT within the curriculum.

1. Basic Operations and Concepts

The concepts and skills associated with the safe, efficient operation of a range of information technologies

2. Social, Ethical, and Human Issues

The understanding associated with the use of ICT that encourages in students a commitment to pursue personal and social good, particularly to build and improve their learning environments and to foster stronger relationships with their peers and others who support their learning

3. Productivity

The efficient selection and use of ICT to perform tasks such as

- the exploration of ideas
- data collection
- data manipulation, including the discovery of patterns and relationships
- problem solving
- the representation of learning

4. Communications

Specific, interactive technology use that supports collaboration and sharing through communication

5. Research, Problem Solving, and Decision Making (RPSD)

Students' organization, reasoning, and evaluation by their learning, which rationalize their use of information and communication technology

For further information, such as the key-stage outcomes for technological competence, refer to the *The Integration of Information and Communication Technology within the Curriculum* (Nova Scotia Department of Education 2005).

Using Software Resources in Mathematics Classrooms

A wide variety of software titles are useful in all classroom areas, including the mathematics classroom. These include word processors, databases, spreadsheets, graphics programs, presentation and/or multimedia production programs, video production programs, concept-mapping applications, and Internet browsers.

Many aspects of the mathematics curriculum can make use of basic software such as word-processing applications. Any written work, done either independently or in groups, could be produced on the computer, if this is appropriate. Much of this work could be done both at home and in school through the use of diskettes or e-mail. Data manipulation using tables and calculations and the collection of information, both qualitative and quantitative, can be made more efficient and effective through the use of databases and spreadsheets. Data charts can easily be converted into graphs through spreadsheet applications.

Several presentation and multimedia programs are available to schools, either as part of the Information Economy Initiative (IEI) projects or through the Authorized Learning Resources (ALR) list. These programs are excellent for allowing students to present information collected on a topic, to augment group reports on work done and research gathered, or for many other activities within the mathematics classroom. Software programs such as PowerPoint, HyperStudio, and Kid Pix Deluxe enable students to present their knowledge in creative and innovative ways. If students are presenting their knowledge and understanding through skits and dramatic activities, the use of video-editing software (iMovie for Apple computers or Pinnacle Studio for Windows computers) can greatly enhance their productions.

Concept-mapping software is an excellent method for collecting the information shared by students in brainstorming activities, as well as a quick way for students to display information collected on topics done in class. Outlining, thought webbing, and other organizers are easily created and used through these types of software. One such software program that is available in most schools in Nova Scotia is Inspiration.

Graphics programs can be used to create illustrations, cartoons, and diagrams. These can then be inserted into computerized reports, presentations, concept maps, etc. Very basic images can be created in software programs such as Paint (in the Accessories on Windows computers). More sophisticated images can be made using the drawing tools of HyperStudio or graphics programs such as Painter, Adobe Illustrator, or Adobe Photoshop.

Specific software programs for different grades and aspects of the mathematics curriculum are listed in Authorized Learning Resources. An annotated list of software titles is included at the end of this section.

Mathematics Software Titles

- ArcView
- Geometer's Sketchpad (v. 4)
- Graphing Equations (Green Globs)
- Hot Dog Stand: The Works
- Let's Do Math: Tools and Things
- Mighty Math Cosmic Geometry
- Secondary Mathlab Toolkit
- Tessellation Exploration
- Understanding Math, including the following modules:
  - Understanding Algebra
  - Understanding Equations
  - Understanding Exponents
  - Understanding Fractions
  - Understanding Graphing
  - Understanding Integers
  - Understanding Measurement & Geometry
  - Understanding Percent
  - Understanding Probability
- Zoombini's Logical Journey
- Zoombini's Mountain Rescue
- Inspiration
- word processor
- spreadsheet
- database
- Presentation software

Technological Tools for Mathematics

"Technology" does not equal "computers." There are many other technological tools that will provide students with excellent learning opportunities in mathematics. Some recommendations of technological tools that are available and can be used as part of curricular activities are listed below.

One per school (minimum):

- photocopier (activities such as congruency and scale)
- video camera
- still camera
- VCR and/or TV and videos for a list of videos available through the Nova Scotia Department of Education, see <http://lrt.EDnet.ns.ca/media_library/catalogues/math.html>
- computer and computer peripherals such as scanner, printer, and CD burner

One per classroom:

- overhead projector
- audio recorder/player for tape cassettes and CDs (if possible)
- overhead TI-83+ graphing calculator

One class set per classroom:

- TI-83+ Graphing calculator (grades 9 to 12)

Several versions of graphing calculator software are also available for computers. Most of these are available through the Internet.

Macintosh:

- Pacific Tech Graphing Calculator—a free version is included in the system of every Macintosh computer.

Windows:

- Microsoft Calculator—this can be viewed as either a standard or a scientific calculator
- Pacific Tech Graphing Calculator—available online

Using Calculators in Mathematics Classrooms

Calculators cannot replace the human brain. Students will always need to be able to read and understand problems, write appropriate equations, choose operations to be used, interpret the solution presented, and determine the appropriateness of the answer. Students must provide the important and complex thought processes required to do mathematics. Calculators can provide many varied instructional opportunities for teachers, allowing students to explore patterns and relationships. Calculators should not be used solely as an alternative to paper-and-pencil computation.

In early grades, the calculator can assist in helping the development of students' number sense. They can be used to carry out activities that allow students to construct mathematical relationships, especially in such areas as the understanding of 10 and 100 as abstract units. Problem solving may be more effective when students are allowed to use calculators to solve the arithmetical portions of a problem. They are more likely to use a calculator to perform a variety of exploratory computations that will help them understand the problem asked, while reducing the amount of time spent on paper-and-pencil computation. Calculators could help ensure that time in a mathematics classroom can be spent on concept development, problem solving, mental arithmetic, and estimation activities.

Graphing calculators are very useful in junior and senior high school classes. Students should be used to seeing and perhaps drawing statistic plots and graphs. Teachers would have students make scatter plots using TI-83+. In grade 10 students will be finding the line and curve of best fit to scatterplots using the TI-83+ to make scatterplots and histograms throughout unit 1 (Data Management) Linear and non-linear relationships can be easily understood when students can create visual images of them with a few keystrokes. Whether used in algebra, trigonometry, or calculus, these tools require student understanding and skill in the use of algebraic logic for the entry of expressions, function rules, equations, and inequalities. This means that students will require instruction in order to use these tools effectively. Students can spend their time concentrating on understanding, setting up, and choosing appropriate operations and equations. This can open the door for students to begin discovering topics and relationships on their own and to find alternative ideas and solutions for activities presented.

Using the Internet in Classrooms

The Internet is becoming one of the most powerful tools available for the gathering of current data and research results. Through the Internet, students can access many resources that may not be easily available in the typical mathematics classroom. They may access on-line articles and reports of current research. They can find and correspond with mathematicians regarding their work. There are literally millions of mathematics sites on the Internet, but teachers must be aware of the necessity for safety and critical examination of all sites used. A good starting point for information about Internet basics is to go to <http://www.EDnet.ns.ca/> and follow the links:

Educators → Classroom and Curriculum Resources → Curriculum-Related Websites → Integration of Information Technology → Search Engines

Sites used in schools should be evaluated, either by the teacher before the students use the site or by the students as part of the learning activity, with the teacher acting as a supervisor. A variety of websites have been evaluated on the Internet, and an overview of many of these can be found by going to <http://www.EDnet.ns.ca/> and following the links:

Educators —> Classroom and Curriculum Resources —> Curriculum-Related Websites —> Integration of Information Technology —> Evaluating Websites

Safety

There are many important issues surrounding the safety and well-being of students using the Internet. Predators "stalk" chat rooms and can also glean large amounts of information about children through school and classroom web pages. There are many inappropriate sites—pornographic and hate sites being only two types. Many websites are produced by people who have special interests and present themselves as credible and reasonable, but may provide information that is biassed and/or incorrect. Websites addressing safety issues, as well as critical thinking about the Internet, can be found by going to <http://www.EDnet.ns.ca> and following the links:

Educators —> Classroom and Curriculum Resources —> Curriculum-Related Websites —> Integration of Information Technology —> Internet Safety

Critical Awareness

Much information on the Internet today is of the same character as that provided in supermarket tabloids: sensational, biassed, and inaccurate. Students must have the skills and awareness to recognize sites that purport to contain serious information, but have unacceptable levels of unfounded information in them. The ability to recognize unbiassed information and to distinguish it from material that is unscientific is an important skill for students. Information on this topic can be found by going to <http://www.EDnet.ns.ca> and following the links:

Educators —> Classroom and Curriculum Resources —> Curriculum-Related Websites —> Integration of Information Technology —> Internet Safety

Educators —> Classroom and Curriculum Resources —> Curriculum-Related Websites —> Integration of Information Technology —> Evaluating Websites

Search Engines

When students search the Internet, they can spend over half their class time looking for useable information on a topic. It is much more efficient for the teacher to do a search prior to the class period and provide the students with several web addresses (URLs) that will give them information to get started. This will avoid the difficulty of students searching in such a general way that most of the sites returned in the search are not useful or may be completely inappropriate. For a site that provides many links to information about web searches go to <http://www.EDnet.ns.ca/> and follow the links:

Educators —> Classroom and Curriculum Resources —> Curriculum-Related Websites —> Integration of Information Technology —> Search Engines

The most common search engines also provide tips on searching, which can assist the user in ensuring that his or her search is more focussed. Every search engine does the searching a little differently, and it is probably best, for the beginner, to learn to use a common search engine, such as Google, <http://www.google.ca>.

The following example shows how you might search for information about river tank aquariums.

Using just the word **tank** in the search box could give you over 13 000 000 sites about Thomas the Tank Engine, storage tanks, fish tanks, armoured vehicles, septic tanks, stainless steel tanks, water tanks, and polyethylene tanks, to name a few.

A search for **aquarium** finds links for items such as Long Beach Aquarium, home aquarium, Monterey Bay aquarium, aquarium fish, aquarium supplies, fish tropical, and Baltimore Aquarium on over 5 000 000 sites.

Using the search engine to find the phrase **river tank** will narrow the search to links about topics such as tanks, snapping turtle, rivers, alligator snapping turtle, river tank system, monitor fishtank, river tank aquarium fish, and so on.

Search strings can also be used to narrow a search. A search string, typed into the search engine, with one space after each word or phrase can be tailored to a particular set of items. If a + is typed directly in front of a word or a phrase, most search engines will find only sites that contain that word and may contain any words included without the +. Here is a sample search string that is likely to find information specifically about setting up, using, and purchasing a river tank aquarium.

+ *aquarium* + *"river tank"* + *fish*

Many search engines allow more complex searches. The Advanced Search features of many of the popular search engines allow for Boolean searches, using AND, OR, AND NOT, or NEAR as instructions. Using parentheses around portions of a complex Boolean string can group items.

Many search engines also provide specific searches of a different type. Searches for pages that have certain items, links, images, text, or anchors can be made using special search commands typed into the regular search box:

- **link:URLtext** will find pages with a link to the page having the URL given
- **link:www.EDnet.ns.ca** finds pages with a link to any page on the Nova Scotia Department of Education's server

An easy reference guide to the advanced features of Google can be found at <http://www.google.ca/advanced_search>.

Alphabetical Software List for Mathematics, with Annotations

Title: ArcView

Grade Levels: 3–12

ArcView permits students to interpret data, create maps, do presentations, and explore many real geographic information system databases. From community studies in elementary grades to advanced problem solving using environmental or original data, many courses can make use of this software to provide experiences previously impossible. This is a professional-level program that is easy to learn and very powerful. It encourages student analysis and critical thinking. Several Nova Scotia municipalities use GIS data and students may have access to real data for their projects.

Title: Geometer's Sketchpad (V. 4)

Grade Levels: 7–12

This program can be used to construct a variety of figures. Once the figure is drawn, students or teachers can transform it with the mouse while preserving the geometric relationships of the construction. The geometric constructions lead to generalizations as you see which aspects of geometry change and which stay the same. This program brings geometry to life. The user can construct geometric figures and measure all attributes of the figure. As a figure is modified, the measurements reflect changes in the size and shape, which allow sstudents to construct their own learning. It is useful as a demonstration tool as well as a hands-on student activity.

Title: Graphing Equations (Green Globs)

Grade Levels: 7–12

This program offers a variety of practice with graphing equations. This program allows the student to draw the graphs of various functions and to determine the equations of graphs drawn by the program. The simulation included in this program is an excellent exercise for relating equations with points on the co-ordinate system and graphs.

Title: Hot Dog Stand: The Works

Grade Levels: 4–12

Students are challenged with running a simulated small business. The program provides an environment where students practise critical thinking, problem solving, and communication skills. It is an excellent vehicle for co-operative learning. Students take on the role of a vendor and are responsible for keeping records, determining prices, planning market strategies, making informed decisions based on their data analysis, and handling unexpected events. It provides tools for approximations, graphing, word processing, and sign creation. A calculator is also available. This is a good way to introduce practical entrepreneurial skills and to encourage independent learning.

Title: Inspiration

Grade Levels: P–12

Inspiration is a concept-mapping program. Concept mapping allows students, teachers, or anyone planning a project, event, or experience, to draw organizing diagrams. These can be various geometric shapes or objects with lines connecting as relational, logic, or time paths. Various shapes and styles make this program adaptable to almost any level. This program is easy to use and well designed. It makes good use of colours, graphics, and visual thinking and mapping skills. It encourages open-ended and co-operative learning activities and could be used in any subject area. It can also be used for problem solving, data collection, presentations, review, and assessment. Canadian symbols are provided for use with this software.

Title: Let's Do Math: Tools and Things

Grade Levels: 5–12

This program provides a variety of interactive tools for learning mathematics. It also contains a wide-ranging and well-organized database of approximately 900 terms and definitions. Although it is a searchable math dictionary, it uses a hyperlink format to allow individual exploration and has many interactive examples and activities. It is very easy to use with a good format and excellent diagrams.

Title: Mighty Math Cosmic Geometry

Grade Levels: 4–12

This program teaches basic geometry concepts and problem-solving skills. It contains sections on length, perimeter and area, surface area and volume, attributes of shapes and solids, constructions, transformations, and 2-D and 3-D coordinates, with many levels of difficulty. The sections on tessellations and transformations are very good. It is easy to navigate, especially for those with computer game experience. There are many activities to familiarize the students with geometry applications: a 3-D maze, assembling robots, creating tessellating patterns, making movies, etc.

Title: Secondary Mathlab Toolkit

Grade Levels: 7–12

This program contains sections on algebra, geometry, and advanced algebra. This toolkit-style program is both powerful and flexible. Some of the features of the program include a symbolic editor, assignment editor, coordinate graph window, algebra tiles, solver, calculator, matrix calculator, graphing calculator, probability editor, and a geometric figure generator. The symbol editor is easy to use. The spreadsheet will calculate values from linked equations. The linking to the graph maker is a very smooth process. Clicking on the graph of a function and moving it about causes the linked equation to instantaneously update, reflecting the changes in the graphed figure. Most secondary schools are likely to own copies of this title, as it was a title recommended for purchase during the IEI project. This program matches well with all motion geometry outcomes in GCO E at grades 4, 5, 6, and 7 levels. The teacher's binder is very helpful, but teachers should also refer to curriculum guides as well as other resources which are available, such as blackline masters and reference books.

Title: Understanding Math

Grade Levels: 4–12 (varies by title)

This series contains sections on concepts with graphic explanations, examples, interactive practice, and cumulative checks. The series is a comprehensive Canadian program in a tutorial format. It covers most of the math topics in the junior high curriculum. With good use of colour and animation, concepts are developed and checked as the lessons proceed. This would be especially useful for students who have missed time or who did not grasp a concept in class.

- Understanding Algebra 6–12
- Understanding Equations 7–12
- Understanding Exponents 6–12
- Understanding Fractions 4–12
- Understanding Graphing 6–12
- Understanding Integers 6–12
- Understanding Measurement & Geometry 5–12
- Understanding Percent 5–12
- Understanding Probability 5–12

Appendices

Appendix A: The Process Standards

Communication Standard

Instructional programs from pre-kindergarten through grade 12 should enable all students to organize and consolidate their mathematical thinking through communication; communicate their mathematical thinking coherently and clearly to peers, teachers, and others; analyse and evaluate the mathematical thinking and strategies of others; and use the language of mathematics to express mathematical ideas precisely.

Problem-Solving Standard

Instructional programs from pre-kindergarten through grade 12 should enable all students to build new mathematical knowledge through problem solving, solve problems that arise in mathematics and in other contexts, apply and adapt a variety of appropriate strategies to solve problems, and monitor and reflect on the process of mathematical problem solving.

Connections Standard

Instructional programs from pre-kindergarten through grade 12 should enable all students to recognize and use connections among mathematical ideas, understand how mathematical ideas interconnect and build on one another to produce a coherent whole, and recognize and apply mathematics in contexts outside of mathematics.

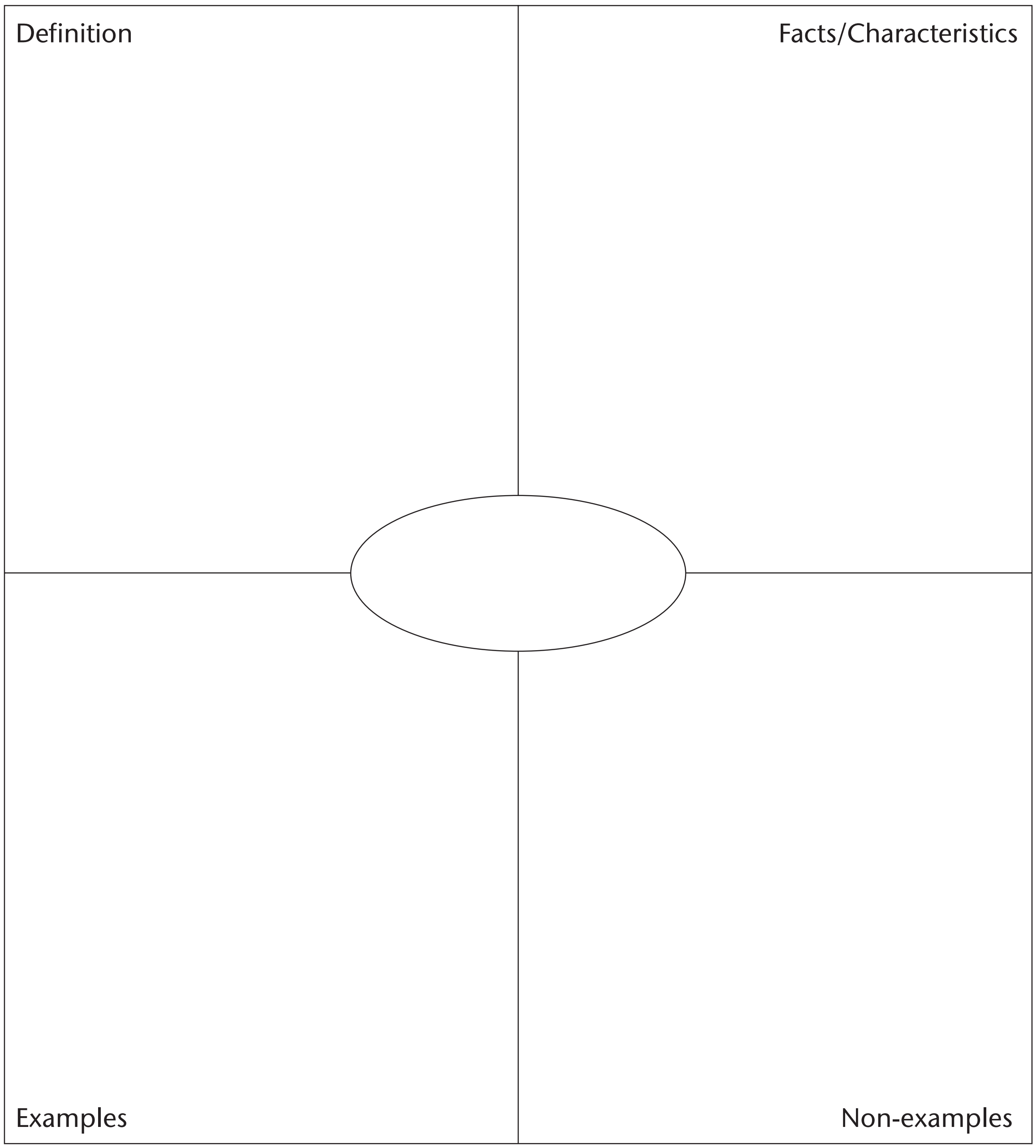
Reasoning and Proof Standard

Instructional programs from pre-kindergarten through grade 12 should enable all students to recognize reasoning and proof as fundamental aspects of mathematics, make and investigate mathematics conjectures, develop and evaluate mathematical arguments and proofs, and select and use various types of reasoning and methods of proof.

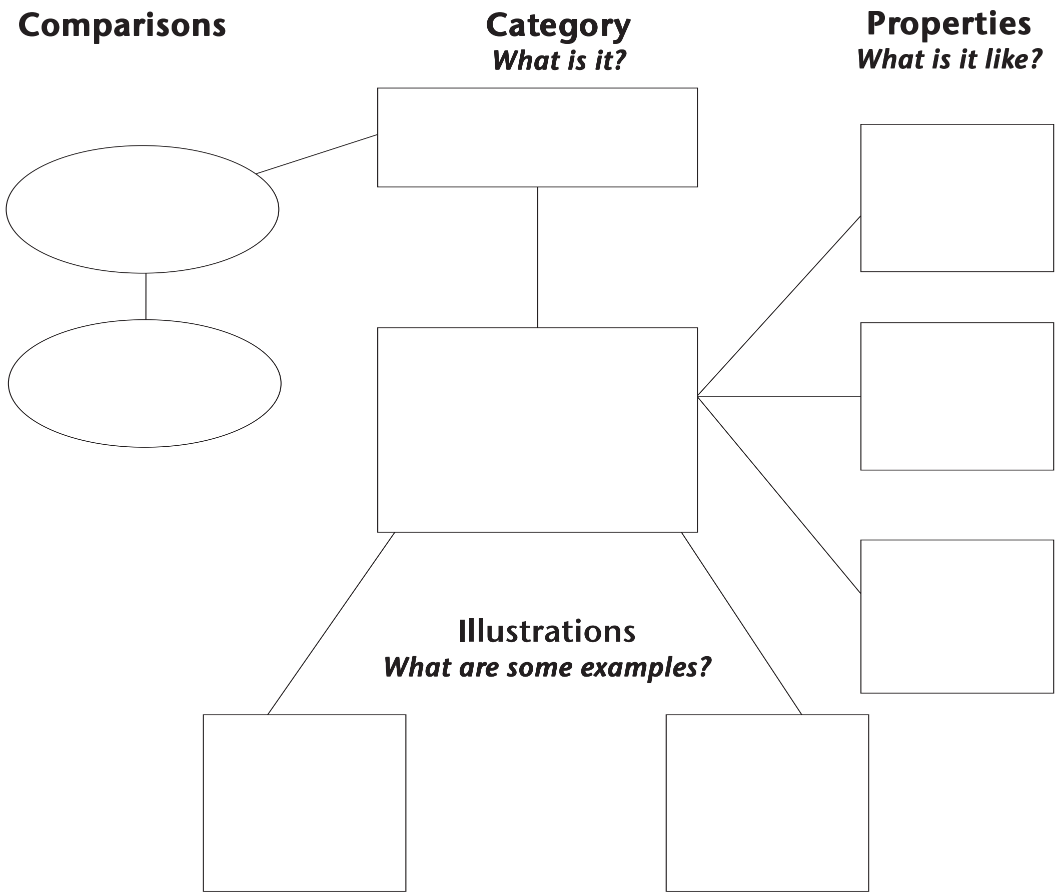
Representation Standard

Instructional programs from pre-kindergarten through grade 12 should enable all students to create and use representations to organize, record, and communicate mathematical ideas; select, apply, and translate among mathematical representations to solve problems; and use representations to model and interpret physical, social, and mathematical phenomena.

Appendix B: Frayer Model



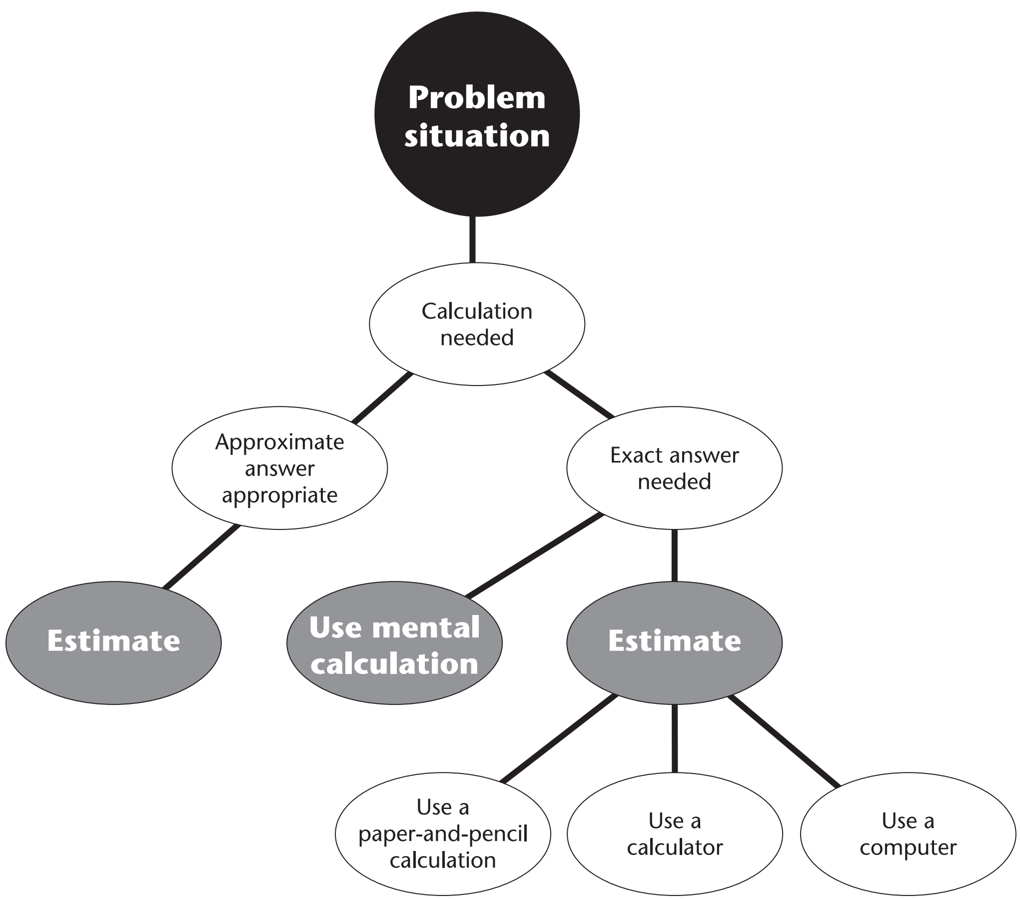
Appendix C: The Concept Map



Appendix D: What Is Mental Math?

Mental math is the mental manipulation of knowledge dealing with numbers, shapes, and patterns to solve problems. Both estimation and mental computation (often referred to as "mental math") are done "in one's head" and have been deemed important skills by the National Council of Teachers of Mathematics (NCTM). Computing mentally to find approximate or exact answers is frequently the most appropriate and useful of all methods.

Mental computation, estimation, pencil and paper, the use of manipulatives, computers, and calculators are all tools that are used to solve problems. Whenever a problem situation is encountered in mathematics, the first decision to make is whether it requires an exact or an approximate answer. If an approximate result is sufficient, an estimate can usually be arrived at mentally. If an exact answer is needed, an estimate could be obtained prior to formal calculations.



Teacher and student attention to mental computational estimation strategies helps students to develop number and operation sense, check reasonableness of solutions, recognize errors on calculator displays, and develop confidence when dealing with numbers. Students might need to estimate before calculating in order to judge the reasonableness of results obtained using other methods. A thorough understanding of, and facility with, mental computation also allows students to solve complicated multi-step problems without spending needless time figuring out calculations and is a valuable prerequisite for proficiency with algebra.

Estimation and mental math are not topics that can be isolated as a unit of instruction; they must be integrated throughout the study of mathematics. There are specific strategies and algorithms for mental computation that must be taught and practised regularly at the appropriate grade level. It is important to remember that

- skills in mental math take time to develop
- mental math is best developed in context
- a variety of strategies for one type of computation should be generated and shared by students
- students need to identify why particular procedures work; they should not be taught computational tricks without understanding
- students should be able to transfer, relate, and apply mental strategies to paper-and-pencil tasks
- helping students to make connections between the representations will help them develop mental images of shapes, patterns, and numbers and will assist them in solving problems

Mental computation strategies, both for exact and approximate answers, are often the most useful of all strategies; therefore they should be frequently revisited throughout the year. Every student of mathematics grades 1–9 is expected to engage in five minutes of mental math each day. These daily activities are part of a well-thought-out, coherent plan. As each additional strategy is introduced, enough practice must be given to allow for proficiency in the new strategy as well as all previous strategies. Strategies need to be carefully developed. Attention must be paid to instruction that promotes understanding, reinforcements that allow students to internalize their thinking, and assessments that determine if the skill has been acquired.

Appendix E: Outcomes Framework to Inform Professional Development for Mathematics Teachers

To be effective, professional development experiences for teachers must reflect and respond to their individual learning needs and build on their prior knowledge and experiences. It is recognized that mathematics teachers will be at different stages on the continuum of professional development required to expand their knowledge base and extend their repertoire of effective practices. It is also recognized that effective professional development is ongoing and requires long-range planning and long-term commitment. Substantial commitment is needed to ongoing professional development to ensure that educational practices reflect the requirements of the Atlantic Canada mathematics curriculum.

The following outcomes framework is intended to assist administrators, school staff, and individual teachers in planning and designing appropriate professional growth experiences, for themselves and for others, over time and in a range of contexts. It provides the "big picture" for professional development in mathematics education and reference points that may be helpful in determining the focus of professional development sessions and identifying individual teachers' learning needs for future professional growth opportunities.

Mathematics teachers need to have an understanding of concepts and strategies and the ways in which they are developed throughout the strands that make up the mathematics curriculum.

To do so, teachers need to
- know the curriculum outcomes prescribed for the grade level(s) they teach
- know the progression and development of outcomes before, during, and after the grade(s) they teach
- broaden and clarify their mathematical understandings of number concepts and operation, patterns and relation, measurement and geometry, and data management and probability
- broaden and clarify their mathematical understandings of the strategies students need to solve problems in number concepts and operations, patterns and relations, measurement and geometry, and data management and probability
- be aware of current research in mathematics education

Mathematics teachers need to have an understanding of the unifying ideas in mathematics and the ways they permeate the strands that make up the mathematics curriculum.

To do so, teachers need to

- model and emphasize aspects of problem solving, including formulating and posing problems
- model and emphasize mathematical communication using written, oral, and visual forms
- demonstrate and emphasize the role of mathematical reasoning
- represent mathematics as a network of interconnected concepts and strategies
- emphasize connections between mathematics and other disciplines and between mathematics and real-world situations
- model and value the five ways to represent a concept: contextually, concretely, pictorially, verbally, and symbolically

Mathematics teachers are expected to organize instruction to facilitate students' understanding and growth in mathematics.

To do so, teachers need to

- have a complete understanding of the pedagogy that is embedded throughout the mathematics curriculum
- use a repertoire of varied teaching strategies
- know how to access and utilize a range of resources to address diverse learning needs
- know how to design and select worthwhile tasks to support the mathematics curriculum
- have knowledge of the role of discourse in the mathematics classroom
- know how to group students effectively based on classroom dynamics and the nature of an activity
- know scope and sequencing for concept development, as well as an appropriate time frame to support this development
- be aware of current research regarding the ways students learn mathematics
- model and teach literacy skills relevant to mathematics
- value all ways of knowing and doing mathematics
- be able to work collaboratively to develop, implement, monitor, and evaluate programming for students with special needs
- be able to support teacher assistants working with students
- reflect on their own practice

Mathematics teachers are expected to assess the mathematical development of their students in relation to the specific curriculum outcomes (SCOs).

To do so, teachers need to

- understand the different purposes of assessment
- know when to use different assessment tools to determine a student's mathematical achievement in relation to the curriculum
- determine the cognitive levels indicated in an outcome and design assessment tasks that appropriately address that outcome
- know how to use assessment information to plan instruction and to challenge or support students as appropriate
- know how to record, organize, track, as well as communicate and report assessment information
- analyse and reflect upon a variety of information to make a professional judgment about student progress

Mathematics teachers need to have an understanding of how an appropriate learning environment supports the learning process.

To do so, teachers need to

- provide and structure the time necessary for students to explore mathematics and engage with significant ideas
- use physical space and materials in ways that facilitate students' learning of mathematics
- provide contexts that encourage the development of mathematical skill and proficiency
- respect and value students' ideas, ways of thinking, and mathematical dispositions
- expect and encourage students to work independently and collaboratively to make sense of mathematics
- expect and encourage students to take learning risks
- expect and encourage students to display mathematical competency
- extend opportunities for mathematics learning beyond the classroom to home and community

Mathematics teachers are expected to promote a positive mathematical disposition in their students.

To do so, teachers need to

- model a positive disposition to do mathematics
- demonstrate the value of mathematics as a way of thinking
- demonstrate the applications of mathematics in other disciplines, in the home, community, and workplace
- promote students' confidence, flexibility, perseverance, curiosity, and inventiveness in doing mathematics through the use of appropriate tasks
- be lifelong learners of mathematics

Appendix F: Outcomes Framework to Inform Professional Development for Teachers of Reading in the Content Areas

To be effective, professional development experiences for teachers, like all learning experiences, must reflect and respond to their individual learning needs, building on their prior knowledge and experiences. It is recognized that content area teachers will be at different stages on the continuum of professional development required to expand their knowledge base and extend their repertoire of effective practices and strategies that support students' development as readers. It is also recognized that effective professional development is ongoing and requires long-term commitment and long-range planning.

The following outcomes framework is intended to assist leadership teams, administrators, school staffs, and individual teachers in planning and designing appropriate professional growth experiences, for themselves and for others, over time and in a range of contexts. It provides the "big picture" for professional development in reading education and reference points that may be helpful in determining the focus of in-service sessions and identifying learning needs for future professional development opportunities for content teachers.

Content area teachers are expected to teach students to read for meaning.

To do this content area teachers need to know

- the features, structure, and patterns of various information texts and the corresponding demands on the reader
- the integration of various sets of knowledge that contribute to meaning making including
  - personal experience
  - literary knowledge
  - world knowledge
- the reading strategies of sampling, predicting, and confirming/self-correcting that proficient readers use as they identify words and comprehend text

Content area teachers are expected to assess the reading development of their students in order to plan effective instruction for them.

To do this content area teachers need to know

- the reading behaviours that characterize emergent, early, transitional, and fluent stages of development so that they can monitor students' growth as readers
- effective strategies for gathering information about students' understanding of concepts and ideas relevant to the subject area curriculum and the influence reading has on their level/degree of understanding
- planning strategies for whole-class, small-group, and individual instruction on the basis of the assessment of learners' reading development

Content area teachers are expected to organize instruction to facilitate students' reading growth.

To do this content area teachers need to

- assist readers of informational text throughout the reading process by supporting comprehension at all stages of reading
  - before (activating)
  - during (acquiring)
  - after (applying)
- model and teach literacy skills relevant to specific curriculum areas
- know classroom management and organizational strategies to facilitate whole-class, small-group, and individual instruction
- collaborate effectively with others who share responsibility for supporting students' reading development