Lesson 2: Pythagorean Theorem

Selected Content Standards

Benchmarks Addressed:

G-5-M *Making and testing conjectures about geometric shapes and their properties*

G-7-M *Demonstrating the connection of geometry to the other strands and to real-life situations (e.g., applications of the Pythagorean Theorem)*

GLEs Addressed:

Grade 8

31. Use area to justify the Pythagorean theorem and apply the Pythagorean theorem and its converse in real-life problems (G-5-M) (G-7-M)

Lesson Focus

In this lesson, students will develop skills in use of the Pythagorean theorem. It includes the following aspects:

- Developing the concept of the Pythagorean theorem
- Using the Pythagorean theorem to complete indirect measurement.

GEE 21 Connection

The skills that will be addressed in this lesson include the following:

- Understand the origin of the Pythagorean theorem
- Demonstrate the use of the Pythagorean theorem

Translating Content Standards into Instruction

A. It is important that students understand why $a^2 + b^2 = c^2$

1. Start with the addition of positive numbers. Remind students that distance is always a positive number and that the Pythagorean theorem deals with the distance from one point to another. Show students how each addend in the addition problem is smaller than the sum.

   2+3=5 7+6=13 9+10=19

2. Each leg is smaller than the hypotenuse; thus, when thinking of the formula

a^2	+	b^2	=	c^2
one can think of it as				
(leg)²	+	(leg)²	=	(hypotenuse)²
One can also express it in terms of small, medium, and large				
(small side)²	+	(medium side)²	=	(large side)²

3. Using the puzzle provided in **Student Worksheet 1,** measure the sides of the small square and the sides of the large square. Label the sides of one square as *a* and the other as *b*. The square whose sides have length *a* will be referred to as square A. The square whose sides have length *b* will be referred to as square B. From the measurements compute the area of each square. Be sure students recognize that the shape between the square forms a right triangle.
4. Have the students cut out the squares and cut along the dark lines inside the large square. There should be 5 pieces, if cut properly.
5. Have the students assemble all pieces into one square. Be sure they have a cover sheet to cover their puzzles, when complete. Do not end the project until all students have assembled a new square.
6. Discuss with the students that if the area of the square A is a^2 and the area of the square B is b^2, then the areas of the new square with side length *c* should be c^2 since it was assembled from the pieces of squares A and B. Measure the new square to determine if this is so. Remind students that their ruler measurements are approximations, so answers may not be exact.
7. Then write the equation:
   area of square A + area of square B = area of square C.
8. Show students the formula will NOT work unless you square the sides. This can be done by drawing a right triangle, measuring each leg, and then measuring the hypotenuse. The sum of the legs alone will not equal the hypotenuse.
9. Then remind students of the previous demonstration where the sides of the triangle were squares that added up to the same area as the large square.

B. Students should learn to manipulate the Pythagorean theorem no matter what information is given them. Use **Teacher Blackline #2** to demonstrate the following concepts.
   1. Encourage students to always begin by writing the formula $a^2 + b^2 = c^2$, then substitute the values given.
   2. Solve various problems for *a*, *b*, and *c*. Explain that while *a* and *b* are interchangeable, *c* must always be the longest side (the hypotenuse).
   3. When solving for *a*, *b* or *c* in the formula, students need to be reminded that the last step will be finding the square root. Be sure students are aware of the square root key on the calculator and how to use it. This function varies by manufacturer.
   4. Use **Student Worksheet 2** for practice and application problems.

Sources of Evidence about Student Learning

A. Have students solve equations to determine the lengths of all sides of right triangles.
B. Have students "square up" four pieces of the wood using precise measurements on the diagonal.

C. Given a variety of triangles, determine which are right triangles, using the Pythagorean theorem.

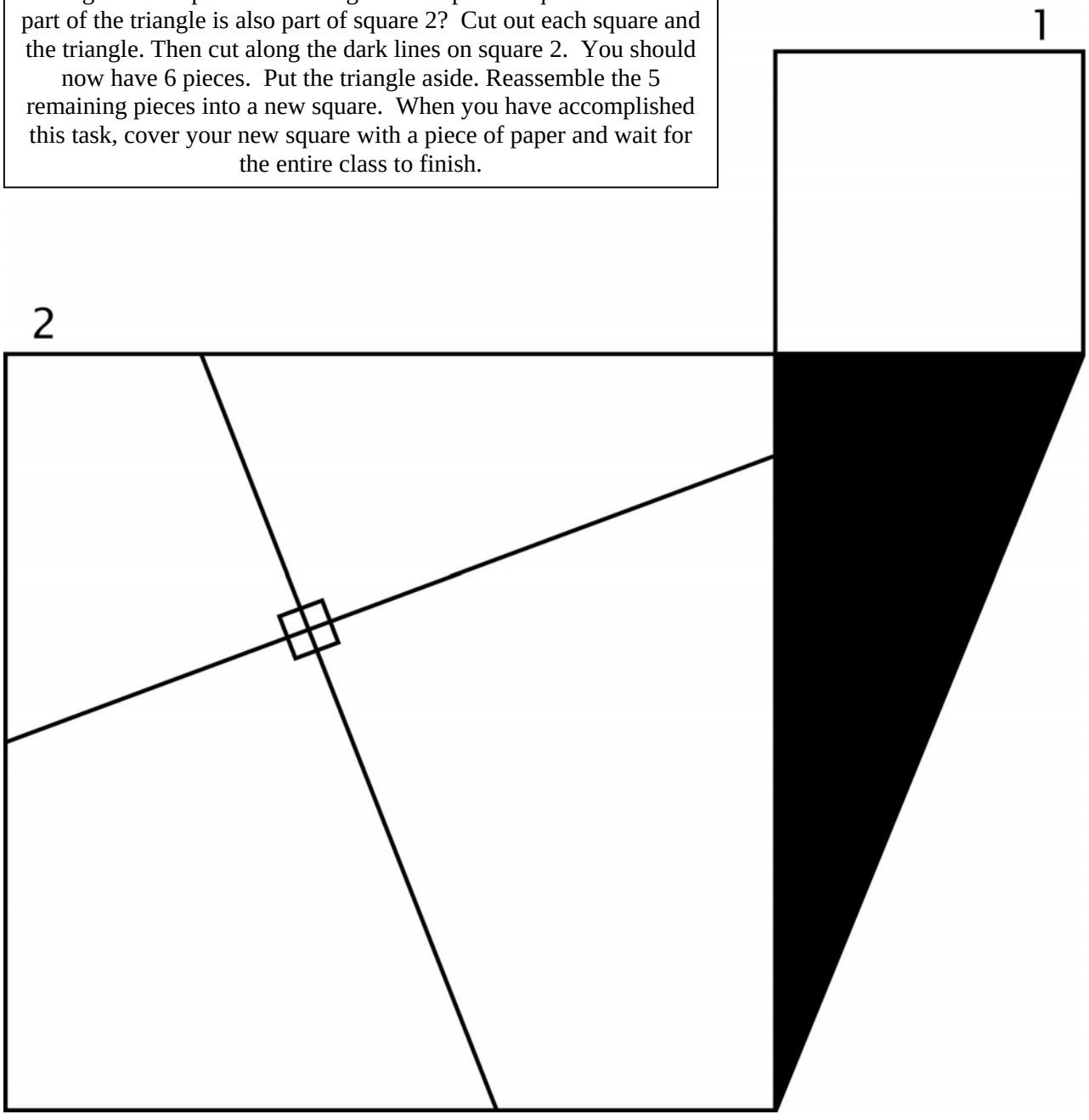
GEE 21 Connection

See attachment at the end of this unit for sample questions related to the GEE 21.

Attributes of Student Work at the "Got-It" level

Students recognize that the distance from home plate to 2nd base is the hypotenuse of a right triangle and can determine that distance on a baseball field knowing the length of a baseline. They can also determine the length of a baseline given the hypotenuse.

In the puzzle shown, name the dark shape between the two squares. While it is obviously a triangle, we cannot be sure it is a right triangle. Notice the location of each square in relation to the triangle. What part of the triangle is also part of square 1? What part of the triangle is also part of square 2? Cut out each square and the triangle. Then cut along the dark lines on square 2. You should now have 6 pieces. Put the triangle aside. Reassemble the 5 remaining pieces into a new square. When you have accomplished this task, cover your new square with a piece of paper and wait for the entire class to finish.

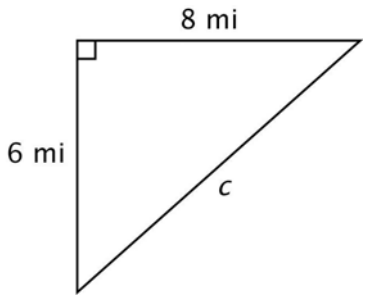


1) If one leg of a right triangle is 12 and the other leg is 16, what is the length of the hypotenuse in this right triangle?

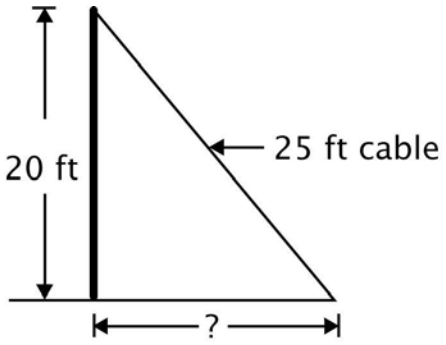
2) Find the missing measure if a and b are the legs of the right triangle and c is the hypotenuse, with $a = 11$ and $c = 61$.

3) The measures of three sides of a triangle are given. Determine whether a triangle with sides 9, 40 and 41 is a right triangle. Explain your answer.

4) Find the missing side of the triangle.



5) A telephone pole support cable attaches to the pole 20 feet high. If the cable is 25 feet long, how far from the bottom of the pole does the cable attach to the ground?



1) Find the length of the hypotenuse of a right triangle, if one leg is 15 and the other leg is 8.

2) The legs of a right triangle have lengths a and b. The hypotenuse has length c. Find the unknown length for each triangle.
   (a) $b = 18,\ c = 82$ (b) $a = 12,\ c = 37$

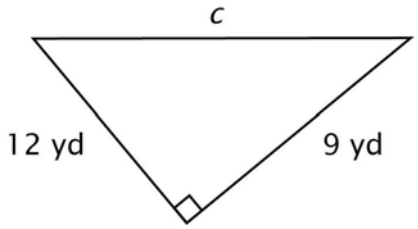
3) The measures of three sides of a triangle are 9, 16, and 20. Determine whether the triangle is a right triangle. Explain your answer.

4) The size of a television screen is given by the length of the diagonal of the screen. What size is a television screen that is 21.6 inches wide and 16.2 inches high?

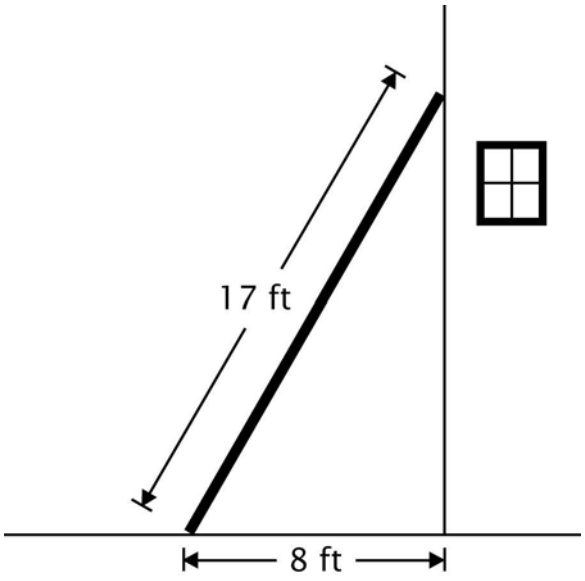
5) If the diagonal of a rectangle measures 60 inches and one side measures 48 inches, what is the length of the other side of the rectangle?

6) A disabled ship radios to shore for help. The Coast Guard determines that the ship is 16 miles east and 43 miles north of the station. What is the direct distance between the ship and the Coast Guard station? Round answer to the nearest whole number.

7) Find the missing side of the triangle.



8) Tara leaned a 17 foot ladder against the house. The bottom of the ladder is 8 feet from the house. How high up the side of the house is the top of the ladder?



Teacher Blackline #1

1) c = 20
2) b = 60
3) yes; The square of one leg, 9, plus the square of the other leg, 40, is equal to the square of the hypotenuse, 41.
4) 10 mi
5) 15 ft

Student Worksheet #2

1) 17
2) (a) a = 80 (b) b= 35
3) no; The sum of the square of the leg, 9, and the square of the other leg, 16, is 337. The square of the hypotenuse, 20, is 400. For this to be a right triangle the two sums must be equal.
4) 27
5) 36 in
6) approximately 46 miles
7) 15 yd
8) 15 ft