Nova Scotia (PLANS) Program of Learning Assessment for Nova Scotia

Nova Scotia Examinations
Mathematics 12 and Advanced Mathematics 12
2009–2010

Information Guide

Nova Scotia

Education

Evaluation Services

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Introduction

The purpose of this *Information Guide* is to provide teachers with information about the Nova Scotia Examinations (NSE) in Mathematics 12 and Advanced Mathematics 12. There is currently no NSE in Mathematics Foundations 12. Teachers are encouraged to share the contents of the guide, particularly the sample questions and answers, with their students.

Overview of NSE Mathematics 12/Advanced Mathematics 12

The NSE in Mathematics 12/ Advanced Mathematics 12 is designed to evaluate student achievement in relation to the outcomes for Mathematics 12/ Advanced Mathematics 12 prescribed by the Atlantic Canada Mathematics Curriculum. The examination results contribute 30% to students' final course mark.

All students registered in Mathematics 12/ Advanced Mathematics 12 are eligible to write the corresponding NSE. Students who have an Individual Program Plan (IPP) in Mathematics, and therefore engage with a different set of mathematics outcomes, do not participate in the examination.

The NSE Mathematics Advisory Group, comprised of senior high school mathematics teachers representing all school boards in Nova Scotia, assists in the development of examinations. The advisory group, under the guidance of department staff, follows the examination development procedures outlined in the Nova Scotia Examination Development Model. As well, an examination review team, comprising experienced senior high mathematics teachers, reviews and approves the final examination forms for each administration. All processes, examination development, administration, marking, and reporting are facilitated by the Evaluation Services Division of the Department of Education.

The mathematics examination is constructed according to precise specifications. Questions are written to match outcomes in the curriculum and these are field-tested with students in Nova Scotia schools. Field-test results are then analyzed.

Examination questions, either in selected response or constructed response format, are used to evaluate students' mathematics thinking and understanding in relation to quadratics, exponentials, geometry, and probability.

The examination is scored by Mathematics 12/ Advanced Mathematics 12 teachers at regional marking sessions led by Evaluation Services Division staff. Standards for marking are set by the Advisory Group. Results are published in the Minister's Report to Parents.

Curriculum Links and Rationale

> The document for the Atlantic Canada Mathematics Curriculum, *Mathematics 12 and Advanced Mathematics 12*, articulates the curriculum for both of these courses. The document provides the teachers of each mathematics course with information to plan for instruction. Teachers must carefully follow the curriculum as contained therein to design learning experiences for their students.

The NSE in Mathematics 12 and Advanced Mathematics 12 are designed to reflect the tables of specifications in this guide (see page 3). The tables are aligned to reflect the Atlantic Canada Mathematics Curriculum. The outcomes listed in the appendices of this guide have been used to construct both examinations. Appendix B outlines those outcomes that may be addressed on the Mathematics 12 examination and Appendix C outlines those that may be addressed on the Advanced Mathematics 12 examination. Note that the examinations may not assess all outcomes listed in the appendices.

Some examination questions will assess students' understanding of an individual outcome, while other questions will assess a number of outcomes. The examinations are comprised of a variety of question types including selected response and constructed response, requiring both short and extended answers. Questions are developed to assess students' performance at different cognitive levels (low, moderate, and high complexity).

The Information Guide will be revised as needed to reflect changes in the examination process. Teachers will be notified as soon as possible when any changes occur. A copy of this guide is posted on the Evaluation Services Division website http://plans.ednet.ns.ca .

Tables of Specifications

Examination Construction

Nova Scotia Examinations in Mathematics 12 and Advanced Mathematics 12 are constructed in accordance with tables of specifications and the Nova Scotia Assessment Development Model. They include questions (items) that have met the following criteria:

- ✓ rigorous content review by the provincial mathematics examination advisory group for alignment with outcomes as listed in the appendices and for possible bias and construction flaws;
- ✓ field-testing under monitored conditions in Mathematics 12 and Advanced Mathematics 12 classrooms;
- ✓ statistical analysis of the students' responses following the field-testing to determine levels of difficulty, validity, and reliability of each question.

Specification Tables

The following table provides the approximate weightings of each unit on the examinations and is based on the recommendations for time allotment found in the Atlantic Canada Mathematics Curriculum for Mathematics 12 and Advanced Mathematics 12.

Table 1

UNIT	***Math 12 Percentage of the examination***	***Advanced Math 12 Percentage of the examination***
Quadratics	30-40 %	30-35 %
Exponential Growth	30-40 %	30-35 %
Circle Geometry	10-15 %	20-25 %
Probability	10-15 %	10-15 %

Table 2 outlines the construction of each examination according to question format, including selected-response and constructed-response questions. The selected-response questions offer the student four choices, three of which are plausible distractors, and one that is the correct response. Constructed-response questions may require the solution of a problem or a written response at any of the three cognitive levels.

Table 2

Question Format	***# of Questions***	***Percentage of Examination***	***Cognitive Levels***
Selected response (multiple choice)	35	~ 35%	1 and 2
Constructed response (short answer and extended response)	14 - 18	~ 65% *	1, 2, and 3

* The exam does not necessarily add to 100 points.

Table 3 outlines the construction of each examination according to three levels of question complexity: low complexity (level 1), moderate complexity (level 2), and high complexity (level 3).

Table 3

Cognitive Level	**Approximate weighting**
Low complexity (level 1)	30%
Moderate complexity (level 2)	50%
High complexity (level 3)	20%

Explanation of Cognitive Levels

Questions on the NSE are developed to assess students' performance at three cognitive levels. Cognitive levels indicate the type of intellectual process required to respond to each question. This guide includes the marking scheme for constructed-response questions so that you may familiarize yourself with scoring as it is done at regional sessions.

Low Complexity Questions (Level 1)

Low complexity questions will require students to recall and recognize previously learned concepts and principles. Students may demonstrate the use of routine procedures to solve a problem. Students are not be required to develop an original method for solving a problem. This level may include recognition or recall of terminology, formulae, algorithms, graphs, geometric figures, properties, and theorems. Questions at this level include key words such as: *identify*, *compute*, *recall*, *recognize*, *find*, *use*, *what*, *list*, *define*, and *name.*

The following are some examples of what a low-complexity question might require a student to do:

- recall or recognize a fact, term, or property
- recognize an example of a concept
- compute a sum, difference, product, or quotient
- perform a specific procedure
- solve a one-step word problem
- retrieve information from a graph, table, figure or function

Examples:

Selected-response question

Given $y = \mathrm{a}x^2 + \mathrm{b}x + \mathrm{c}$. The value of $-\frac{\mathrm{b}}{2\mathrm{a}}$ represents {outcome C31}

a) the minimum or maximum value

✓ b) the '*x*' value of the vertex

c) the coordinates of the vertex

d) the *y*-intercept

In this example the student needs to recall or recognize a fact.

Constructed-response question

Given the function $y = -2x^2 + 4x - 5$.

{outcomes C9, C31}

(a) Write the function in standard or transformational form. (2 points)

$y + 5 = -2x^2 + 4x$

$y + 5 = -2(x^2 - 2x)$

$y + 5 - 2 = -2(x^2 - 2x + 1)$

$y + 3 = -2(x - 1)^2$

$-\frac{1}{2}(y + 3) = (x - 1)^2$

Points awarded:

- 0.5 pt for algebraic manipulation
- 1 pt for completing the square
- 0.5 pt for final answer

Show your work above and write your conclusion or final answer in the box below.

$-\frac{1}{2}(y + 3) = (x - 1)^2$

(b) What is the vertex of the parabola? (1 point)

Final Answer: $(1, -3)$

(c) What is the equation of the axis of symmetry? (1 point)

Final Answer: $x = 1$

The above example (a) requires a student to perform a specific procedure while parts (b) and (c) require the student to retrieve information from a given function.

Moderate Complexity Questions (Level 2)

Moderate-complexity questions require students to identify and understand how the parts of a question are connected to the task of solving the problem or question. A question at this level might be a problem that is typical of, but not identical to, ones studied in class and it requires that students identify and use the appropriate algorithm. At this level of complexity, students are asked to translate, interpret, or extrapolate. Translation refers to the ability to communicate the problem and its solution. Interpretation involves making inferences, generalizations, or summaries. Extrapolation would require the student to estimate or predict the solution from given information by identifying trends and tendencies. Questions at this level include key words such as: *classify, organize, estimate, interpret, predict, infer, translate, generalize, summarize, problem solve,* and *apply*.

The following are some examples of what a moderate-complexity question might require a student to do:

- make connections between facts, terms, properties, or operations
- solve a word problem requiring multiple steps
- compare figures or statements
- provide a justification for steps in a solution process
- interpret a visual representation
- extend a pattern
- retrieve information from a graph, table, or figure, and use it to solve a problem requiring multiple steps
- determine the formula for a relationship given data and conditions
- compute and solve using appropriate methods

Examples:

Selected-response question

Two dice are rolled. What is the probability that both dice will land on the same number?

{outcome G2}

a) $\frac{1}{36}$ b) $\frac{1}{18}$

✓ c) $\frac{1}{6}$ d) $\frac{1}{2}$

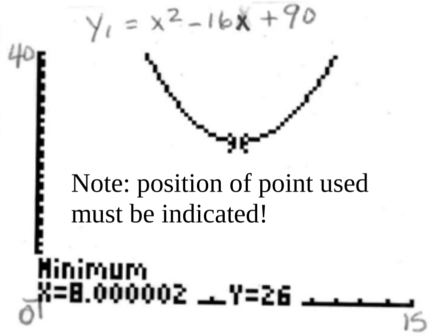
In this example, the student needs to solve using appropriate methods.

Constructed-response question

At the Halifax Airshow, a plane performs a power dive. The equation $h = t^2 - 16t + 90$ expresses the relationship between height, h, in metres and time, t, in seconds.

a) What is the minimum height the plane reaches during the dive? (2 points)

{outcomes C1, C23, C31}



Note: position of point used must be indicated!

Points awarded:
- 1.5 pts for graph with vertex clearly marked
- 0.5 pt for final answer

Show your work above and record your conclusion or final answer in the box below.

The plane reaches a minimum height of 26 m.

0.5 pt

b) When will the plane be at a height of 35 metres? (2.5 points)

{outcomes C1, C22, C23}

$$t^2 - 16t + 90 = 35$$
$$t^2 - 16t + 55 = 0$$
$$(t-11)(t-5) = 0$$
$$t - 11 = 0 \qquad t - 5 = 0$$
$$t = 11 \qquad t = 5$$

Points awarded:
- 0.5 pt for substitution of 35
- 1 pt for solving quadratic equation
- 1 pt for final answers

Show your work above and record your conclusion or final answer in the box below.

The plane will be at a height of 35m at 5s and at 11s.

In this example, the student needs to make a connection between (a) the minimum height of the plane and the vertex of the given function and (b) the height of the plane and the time needed to reach this height.

High Complexity Questions—Novel Problems (Level 3)

High-complexity questions include analysis, synthesis, and evaluation. At this level of questioning, students are required to think in an abstract and sophisticated way to reason, plan, analyse, judge and create. Questions at this level will often include key words such as: *analyse, investigate, formulate, prove, derive, explain,* and *describe.*

Note: There are no level 3 questions in the selected-response portion of the exam.

The following are some examples of what a high-complexity question might require a student to do:

- explain relations among facts, terms, properties, or operations
- analyse similarities and differences between procedures and concepts
- generalize a pattern
- formulate an original problem, given a situation or function
- solve a problem in more than one way
- explain and justify a solution to a problem
- describe, compare, and contrast solution methods
- formulate a mathematical model for a complex situation
- analyse the assumptions made in a mathematical model
- analyse or produce a deductive argument
- provide a mathematical justification
- use concepts taught at prior levels to solve a novel problem

Examples:

Constructed-response question

Is 2^x, 2^{x+2}, 2^{x+4} a geometric sequence? Explain your reasoning. (3 points)

{outcomes C4, C29}

$$\frac{2^{x+2}}{2^x} = 2^2 \qquad \frac{2^{x+4}}{2^{x+2}} = 2^2$$

Common Ratio ∴ georetric sequence

Points awarded:
- 2 pts for calculating ratios
- 1 pt for conclusion

This example requires students to use prior knowledge to solve a novel problem.

Create a problem that could be modelled by the equation $P = 5(2)^{\frac{t}{10}}$. (2 points)

{outcomes A5, C2}

Given an initial count of 5 widgets. If the widgets double every 10 days, what is the population, P, after "t" days?

Points awarded:

- 0.5 pt for initial amount
- 0.5 pt for doubling
- 0.5 pt for every "10 units"
- 0.5 pt for asking a question within a reasonable context

This example requires students to formulate an original problem, given a function.

Item Bank Submissions

Teachers are encouraged to submit test items of all types for consideration by the Nova Scotia Examination Advisory Group for Mathematics 12 and Advanced Mathematics 12.

Send materials to:

Lennie Comeau, Mathematics Evaluation Coordinator
Evaluation Services Division
Nova Scotia Department of Education
PO Box 578
Halifax, NS
B3J 2S9

or e-mail to comeaulj@gov.ns.ca

Security

Nova Scotia Examinations are secure. This means that all examination materials must be sent to your regional marking site as soon as possible as directed by your Board Assessment Coordinator. The materials include all Student Booklets, both used and unused. All examination materials are numbered and personalized, and each booklet sent to a school is tracked. No part of the examination, including student work, is to be reproduced in any form or by any means, electronic or mechanical, including photocopying, recording or by any other information storage or retrieval system. In addition, teachers must not make use of the exam questions in their teaching.

Securing the NSE is critical to ensuring that the evaluation of student achievement is valid and fair. Users of the examination results draw conclusions about the ability of students based on the scores the students achieve.

The evaluation of student achievement in relation to the selected learning outcomes on these examinations is premised on students' encountering the tasks for the first time. Any prior exposure compromises the validity of the conclusions drawn about student ability. Because the Department of Education will use assessment items from past (secured) examinations in future examinations, all involved must do their part to secure these examinations.

The use of particular examination questions on a subsequent examination is an important part of ensuring that different examinations render reliable and valid information about student achievement over time. Through the use of one or more anchor questions, two different Mathematics examinations can be equated, meaning that we can calculate the degree to which one examination is easier or harder than another, and then make appropriate adjustments to equate the two administrations. In this way, we can assert with greater confidence that changes in results over a period of time represent real changes in the standard of student performance and not variation in the examinations themselves.

Administration of Examination

The 2008–2009 administration dates for NSE Mathematics 12 and Advanced Mathematics 12 are January 26 and June 12, 2009. In addition to this Information Guide, the following materials relating to the administration of the examinations are distributed to schools along with the examinations in the week prior to the date of writing:

- Nova Scotia Examination (NSE) Packing Slip Mathematics 12/Advanced Mathematics 12 (to be used to verify the materials sent to the school and to account for materials returned to the regional marking site)
- Student lists with corresponding booklet numbers
- Quality Control Declaration (to be completed by the School Assessment Coordinator declaring that the examinations have been secured prior to the examination)
- Instructions to Teachers (invigilation directions)
- Instructions on clearing graphing calculator memory

Note: The School Assessment Coordinator should open the box(es) of examinations materials as soon as possible after receipt and check that booklets match the school list.

Pre-Administration

- Two months prior to the date of the examination, teachers and the School Assessment Coordinator must consult regarding the number of examinations required for each course as well as the types and numbers of examinations required in alternate formats (Braille, large print, audio CD).
- Teachers ensure that students have been informed of what they will need in advance of the examination: an HB pencil, a graphing calculator, and a ruler.
  NOTE: Schools should be able to provide graphing calculators to students who do not have their own.
- The School Assessment Coordinator ensures that exams are scheduled according to the dates in the provincial assessment schedule.
- The School Assessment Coordinator ensures that students with special needs will be accommodated.
- The School Assessment Coordinator verifies the correctness and number of materials sent by the department.
- The School Assessment Coordinator discusses exam protocol and specific instructions with invigilators, and distributes "Instructions to Teachers" sheet (see above).
- The School Assessment Coordinator ensures that the examination venue does not display material that might advantage students in writing the examination.
- The School Assessment Coordinator maintains security of the examinations and ensures that neither students nor teachers have access to the examinations until the morning of the administration date.

During Administration

- Teachers/invigilators ensure students are under supervision at all times.
- Teachers/invigilators ensure students work independently at all times.
- Teachers/invigilators allow up to three hours to write the examination.
- Students retain their examinations and stay in the examination room for at least one hour after the administration has begun (or longer, if so required by school examination procedures).
- Each student receives a personalized examination booklet and a personalized *Student Response Form* for recording responses to selected response questions. This form must be completed using an HB pencil.
- Other than selected response questions, students do all their work in the examination booklets.
- Teachers/invigilators collect all examination materials, including scrap paper, from students before students leave the examination room, all materials must be accounted for.
- Teachers/invigilators do not read to students or discuss examination questions with students.
- Students work at their own pace; however, they should take note of the suggested times given for each task, one hour twenty minutes for the selected response and 1 hour forty minutes for the constructed response.

Post-Administration

- In the case of a student for whom adaptations were made in his or her writing of the examination, the teacher fills out the Adaptations Box on the front cover of each student response booklet.
- Teachers write in and bubble each student's term mark on their corresponding *Student Response Form*. Teachers do this for all students currently registered in Mathematics 12 or Advanced Mathematics 12 even if the student did not write the examination.
- As soon as possible following the completion of the examination, teachers must return to the School Assessment Coordinator all student booklets (used and unused) and the student response forms (used and unused). The School Assessment Coordinator accounts for (and if necessary follows up on) all materials sent to the school, signs the Quality Control Declaration, and packages the required materials. Instructions for posting or pick-up will be sent to schools by the Board Assessment Coordinator.
- The Nova Scotia Examinations are secure. Therefore all booklets received by the school, including the student booklets (used and unused) must be accounted for and returned to the department. Under no circumstances is reproduction of any part of the examination, including student work, permitted.

Eligibility/Exemptions

Eligibility—Mathematics 12

All students registered in Mathematics 12 will write the NSE Mathematics 12 on the dates specified in the provincial assessment schedule. Students studying Mathematics 12 by correspondence will also write on the specified dates.

Students who are on Individual Program Plans relating to Mathematics and students enrolled in the International Baccalaureate Mathematics course(s) are working within outcome frameworks that may differ from those of the Atlantic Canada Curriculum. These students will be evaluated using other approved forms of assessment and will not write NSE Mathematics 12.

Eligibility—Advanced Mathematics 12

All students registered in Advanced Mathematics 12 will write the NSE Advanced Mathematics 12 on the dates specified in the provincial assessment schedule. Students studying Advanced Mathematics 12 by correspondence will also write on the specified dates.

Students who are on Individual Program Plans relating to Mathematics and students enrolled in the International Baccalaureate Mathematics course(s) are working within outcome frameworks that may differ from those of the Atlantic Canada Curriculum. These students will be evaluated using other approved forms of assessment and will not write NSE Advanced Mathematics 12.

Exemptions

The principal, in consultation with the student and/or parent/guardian, may grant an exemption to an individual student in the case of illness, bereavement, or other exceptional circumstances. In such cases the student’s mark will be determined by the Mathematics 12 or Advanced Mathematics 12 teacher in consultation with the principal. Exceptional circumstances are determined on a case-by-case basis as professional judgment and consultation are required.

Exemptions are not granted on the basis of how challenging the examination might be for a particular student. For example, an international student who is enrolled in Mathematics 12 and seeks a course credit in Mathematics 12 must write the examination even if the teacher believes the language competence of that student might not be sufficient to allow success on the examination. The examination assesses the learning outcomes of the course, and it is a requirement for course completion.

Adaptations

Certain students will require adaptations in order to allow them to demonstrate their abilities in relation to learning outcomes. These adaptations should in no way change or modify the learning outcomes of the course, but rather provide for the long- and short-term needs of the students by furnishing them with alternate ways to show that they have met the outcomes. Decisions regarding adaptations will be made at the school level and will reflect those adaptations documented in the student's cumulative record file as being needed during assessment periods, so long as these adaptations do not compromise or alter the validity of the examination.

Scoring and Reporting

Regional Level

As of January 2008, all exams are marked at regional marking sessions under the guidance of the professional staff of Evaluation Services Division. Results are returned to Board Assessment Coordinators for distribution to schools within a day or two of the completion of the marking sessions.

Regional sessions are led by experienced mathematics teachers that attend a two day training session at the Department of Education. All regional sites are connected by Internet using the Marratech system. Should any revisions be made to the marking guide, all sites are consulted. Professional staff from the evaluation services division are also onsite in each region to assist the session leaders.

Provincial Level

Given that all exams are marked regionally under the guidance of trained teachers and Evaluation Services Division professional staff there is no sampling for central marking. Results are reported based on the census marking that occurs regionally.

Exam Scoring Norms for Constructed Response

Solving problems by using mathematics is communicating your reasoning using a specialized language. Just as the English language has its grammatical conventions, the mathematics language has its own usage conventions. Grade 12 students should be proficient in the use of the mathematics language and the scoring norms reflect this necessary adherence to the conventions.

- Strict adherence to the marking guide is necessary.

- “No Response” is to be bubbled in as “n”.

- With the exception of answers in the probability section, all exact final answers must be simplified and reduced. (i.e. reduce fractions, express radicals in simplest radical form etc...)

- There shall be a deduction (0.5 pt) for:

  - rounding errors a maximum of two times on the exam.
  - failing to simplify or reduce, a maximum of two times on the exam.
  - for not clearly indicating a final answer, a maximum of two times on the exam.

  Indicate each rounding, simplification or box deduction with a checkmark in the appropriate box on the bubble sheet (bottom left).

- Deductions (0.5 pt) for computational and transcription errors shall not exceed half the value of the points awarded for conceptual understanding.

- Deduct only once if the same error is repeated within a question.

- If a student makes a conceptual error the student shall receive no point value associated with **that** concept. The student may still receive points for other concepts involved in the solving of the problem. Should the student make an error that completely changes the intent of the problem such that the intended concepts are now absent from the solution then no points will be awarded for work following that error.

- If a question answer box states that students must show proof of work then a final answer only in the box shall result in 0 pts.

Appendices

Appendix A: Outcomes as outlined in the Atlantic Canada Mathematics Curriculum—Mathematics 12/ Advanced Mathematics 12 [Grouped according to the general curriculum outcomes strands (GCOs)]

Specific Curriculum Outcomes for Nova Scotia Examinations in Mathematics 12 and Advanced Mathematics 12: Specific curriculum outcomes are statements that describe what students are expected to know and be able to do at each grade level. They are intended to help teachers design learning experiences and assessment tasks.

Note: Questions on the examination are based on the statements below and are arranged according to the tables of specifications (pages 3 and 4). Teachers have been provided with the *Atlantic Canada Mathematics Curriculum, Mathematics 12 and Advanced Mathematics 12* for their complete course of study.

GCO A: Students will demonstrate number sense and apply number theory concepts.

By the end of Mathematics 12 and Advanced Mathematics 12, students will be expected to

A3 demonstrate an understanding of the role of irrational numbers in applications
A4 demonstrate an understanding of the nature of the roots of quadratic equations
A5 demonstrate an understanding of the role of real numbers in exponential and logarithmic expressions and equations
A6 develop an understanding of factorial notation and apply it to calculating permutations and combinations
A7 describe and interpret domains and ranges using set notation
A9 represent non-real roots of quadratic equations as complex numbers

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

By the end of Mathematics 12 and Advanced Mathematics 12, students will be expected to

B1 demonstrate an understanding of the relationships that exist between arithmetic operations and the operations used when solving equations
B2 demonstrate an understanding of the recursive nature of exponential growth
B8 determine probabilities using permutations and combinations
B10 derive and apply the quadratic formula
B11Adv analyse the quadratic formula to connect its components to the graphs or quadratic functions
B12 apply real number exponents in expressions and equations
B13 demonstrate an understanding of the properties of logarithms and apply them

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

By the end of Mathematics 12 and Advanced Mathematics 12, students will be expected to

C1 model real-world phenomena using quadratic functions
C2 model real-world phenomena using exponential functions
C3 sketch graphs from descriptions, tables, and collected data
C4 demonstrate an understanding of patterns that are arithmetic, power, and geometric and relate them to corresponding functions
C8 describe and translate between graphical, tabular, written, and symbolic representations of quadratic relationships
C9 translate between different forms of quadratic equations
C10Adv determine the equation of a quadratic function using finite differences
C11 describe and translate between graphical, tabular, written, and symbolic representations of exponential and logarithmic relationships
C15 relate the nature of the roots of quadratic equations and the x-intercepts of the graphs of corresponding functions
C19 demonstrate an understanding, algebraically and graphically, that the inverse of an exponential function is a logarithmic function
C22 solve quadratic equations
C23 solve problems involving quadratic equations
C24 solve exponential and logarithmic equations
C25 solve problems involving exponential and logarithmic equations
C29 analyse tables and graphs to distinguish between linear, quadratic, and exponential relationships
C31 analyse and describe the characteristics of quadratic functions
C32 demonstrate an understanding of how the parameter changes affect the graphs of quadratic functions
C33 analyse and describe the characteristics of exponential and logarithmic functions
C34 demonstrate an understanding of how the parameter changes affect the graphs of exponential functions
C35Adv write exponential functions in transformational form, and as mapping rules to visualize and sketch graphs

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

By the end of Mathematics 12 and Advanced Mathematics 12, students will be expected to

D1 develop and apply formulas for distance and midpoint

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships

By the end of Mathematics 12 and Advanced Mathematics 12, students will be expected to

E3Adv write the equations of circles and ellipses in transformational form, and as mapping rules to visualize and sketch graphs
E4 apply properties of circles
E5 apply inductive reasoning to make conjectures in geometric situations
E7 investigate, make, and prove conjectures associated with chord properties of circles
E8Adv investigate, make, and prove conjectures associated with angle relationships in circles
E11 write proofs using various axiomatic systems and assess the validity of deductive arguments
E12 demonstrate an understanding of the concept of converse
E13Adv analyse and translate between symbolic, graphical, and written representations of circles and ellipses
E14Adv translate between different forms of equations of circles and ellipses
E15Adv solve problems involving the equations and characteristics of circles and ellipses
E16Adv demonstrate the transformational relationship between the circle and the ellipse

GCO F: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships

By the end of Mathematics 12 and Advanced Mathematics 12, students will be expected to

F1 analyse, determine and apply scatter plots and determine the equations for curves of best fit, using appropriate technology

GCO G: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships

By the end of Mathematics 12 and Advanced Mathematics 12, students will be expected to

G1 develop and apply simulations to solve problems
G2 demonstrate an understanding that determining probability requires the quantifying of outcomes
G3 demonstrate an understanding of the fundamental counting principle and apply it to calculate probabilities
G4 apply area diagrams and tree diagrams to interpret and determine probabilities of dependent and independent events
G5Adv determine conditional probabilities
G7 distinguish between situations that involve combinations and permutations
G8 develop and apply formulas to evaluate permutations and combinations
G9Adv demonstrate an understanding of binomial expansion and its connection to combinations
G10Adv connect Pascal’s Triangle with combinatorial coefficients

Appendix B: Outcomes to be assessed in the 2009/10 NSE—Mathematics 12 [Grouped according to units of study]

Specific curriculum outcomes are statements that describe what students are expected to know and be able to do at each grade level. They are intended to help teachers design learning experiences and assessment tasks.

Note: Only those outcomes that will be assessed on the NSE for Mathematics 12 are listed below. Please note that some curriculum outcomes are not assessed on the NSE for Mathematics 12.

Quadratic unit:

C4	demonstrate an understanding of patterns that are arithmetic, power, and geometric and relate them to corresponding functions
C29	analyse tables and graphs to distinguish between linear, quadratic, and exponential relationships
C3	sketch graphs from descriptions, tables, and collected data
C1	model real-world phenomena using quadratic functions
F1	analyse, determine and apply scatter plots and determine the equations for curves of best fit, using appropriate technology
C8	describe and translate between graphical, tabular, written, and symbolic representations of quadratic relationships
A7	describe and interpret domains and ranges using set notation
C31	analyse and describe the characteristics of quadratic functions
C23	solve problems involving quadratic equations
B1	demonstrate an understanding of the relationships that exist between arithmetic operations and the operations used when solving equations
C9	translate between different forms of quadratic equations
C32	demonstrate an understanding of how the parameter changes affect the graphs of quadratic functions
B10	derive and apply the quadratic formula
C22	solve quadratic equations
C15	relate the nature of the roots of quadratic equations and the x-intercepts of the graphs of corresponding functions
A9	represent non-real roots of quadratic equations as complex numbers
A3	demonstrate an understanding of the role of irrational numbers in applications
A4	demonstrate an understanding of the nature of the roots of quadratic equations

Exponential Growth unit:

C2	model real-world phenomena using exponential functions
C3	sketch graphs from descriptions, tables, and collected data
B2	demonstrate an understanding of the recursive nature of exponential growth

C4 demonstrate an understanding of patterns that are arithmetic, power, and geometric and relate them to corresponding functions
C29 analyse tables and graphs to distinguish between linear, quadratic, and exponential relationships
A7 describe and interpret domains and ranges using set notation
C33 analyse and describe the characteristics of exponential and logarithmic functions
A5 demonstrate an understanding of the role of real numbers in exponential and logarithmic expressions and equations
B12 apply real number exponents in expressions and equations
C11 describe and translate between graphical, tabular, written, and symbolic representations of exponential and logarithmic relationships
C34 demonstrate an understanding of how the parameter changes affect the graphs of exponential functions
F1 analyse, determine and apply scatter plots and determine the equations for curves of best fit, using appropriate technology
B1 demonstrate an understanding of the relationships that exist between arithmetic operations and the operations used when solving equations
C24 solve exponential and logarithmic equations
C25 solve problems involving exponential and logarithmic equations
C19 demonstrate an understanding, algebraically and graphically, that the inverse of an exponential function is a logarithmic function
B13 demonstrate an understanding of the properties of logarithms and apply them

Circle Geometry unit:

E5 apply inductive reasoning to make conjectures in geometric situations
E7 investigate, make, and prove conjectures associated with chord properties of circles
E12 demonstrate an understanding of the concept of converse
E4 apply properties of circles
D1 develop and apply formulas for distance and midpoint

Probability unit:

G2 demonstrate an understanding that determining probability requires the quantifying of outcomes
G3 demonstrate an understanding of the fundamental counting principle and apply it to calculate probabilities
A6 develop an understanding of factorial notation and apply it to calculating permutations and combinations
G7 distinguish between situations that involve combinations and permutations
G8 develop and apply formulas to evaluate permutations and combinations
B8 determine probabilities using permutations and combinations

Appendix C: Outcomes to be assessed in the 2009/10 NSE—Advanced Mathematics 12 [Grouped according to units of study]

Specific curriculum outcomes are statements that describe what students are expected to know and be able to do at each grade level. They are intended to help teachers design learning experiences and assessment tasks.

Note: Only those outcomes that will be assessed on the NSE for Advanced Mathematics 12 are listed below. Please note that some curriculum outcomes are not assessed on the NSE for Advanced Mathematics 12.

Quadratic unit:

C4	demonstrate an understanding of patterns that are arithmetic, power, and geometric and relate them to corresponding functions
C29	analyse tables and graphs to distinguish between linear, quadratic, and exponential relationships
C3	sketch graphs from descriptions, tables, and collected data
C10Adv	determine the equation of a quadratic function using finite differences
C1	model real-world phenomena using quadratic functions
F1	analyse, determine and apply scatter plots and determine the equations for curves of best fit, using appropriate technology
C8	describe and translate between graphical, tabular, written, and symbolic representations of quadratic relationships
A7	describe and interpret domains and ranges using set notation
C31	analyse and describe the characteristics of quadratic functions
C23	solve problems involving quadratic equations
B1	demonstrate an understanding of the relationships that exist between arithmetic operations and the operations used when solving equations
C9	translate between different forms of quadratic equations
C32	demonstrate an understanding of how the parameter changes affect the graphs of quadratic functions
B10	derive and apply the quadratic formula
B11Adv	analyse the quadratic formula to connect its components to the graphs or quadratic functions
C22	solve quadratic equations
C15	relate the nature of the roots of quadratic equations and the x-intercepts of the graphs of corresponding functions
A9	represent non-real roots of quadratic equations as complex numbers
A3	demonstrate an understanding of the role of irrational numbers in applications
A4	demonstrate an understanding of the nature of the roots of quadratic equations

Exponential Growth unit:

C2	model real-world phenomena using exponential functions
C3	sketch graphs from descriptions, tables, and collected data
B2	demonstrate an understanding of the recursive nature of exponential growth
C4	demonstrate an understanding of patterns that are arithmetic, power, and geometric and relate them to corresponding functions
C29	analyse tables and graphs to distinguish between linear, quadratic, and exponential relationships
A7	describe and interpret domains and ranges using set notation
C33	analyse and describe the characteristics of exponential and logarithmic functions
A5	demonstrate an understanding of the role of real numbers in exponential and logarithmic expressions and equations
B12	apply real number exponents in expressions and equations
C11	describe and translate between graphical, tabular, written, and symbolic representations of exponential and logarithmic relationships
C34	demonstrate an understanding of how the parameter changes affect the graphs of exponential functions
F1	analyse, determine and apply scatter plots and determine the equations for curves of best fit, using appropriate technology
C35Adv	write exponential functions in transformational form, and as mapping rules to visualize and sketch graphs
B1	demonstrate an understanding of the relationships that exist between arithmetic operations and the operations used when solving equations
C24	solve exponential and logarithmic equations
C25	solve problems involving exponential and logarithmic equations
C19	demonstrate an understanding, algebraically and graphically, that the inverse of an exponential function is a logarithmic function
B13	demonstrate an understanding of the properties of logarithms and apply them

Circle Geometry unit:

A7	describe and interpret domains and ranges using set notation
E5	apply inductive reasoning to make conjectures in geometric situations
E7	investigate, make, and prove conjectures associated with chord properties of circles
E12	demonstrate an understanding of the concept of converse
E4	apply properties of circles
E15Adv	solve problems involving the equations and characteristics of circles and ellipses
D1	develop and apply formulas for distance and midpoint
E8Adv	investigate, make, and prove conjectures associated with angle relationships in circles
E11	write proofs using various axiomatic systems and assess the validity of deductive arguments
E13Adv	analyse and translate between symbolic, graphical, and written representations of circles and ellipses
E14Adv	translate between different forms of equations of circles and ellipses

E3Adv write the equations of circles and ellipses in transformational form, and as mapping rules to visualize and sketch graphs

E16Adv demonstrate the transformational relationship between the circle and the ellipse

Probability unit:

G2 demonstrate an understanding that determining probability requires the quantifying of outcomes

G3 demonstrate an understanding of the fundamental counting principle and apply it to calculate probabilities

G5Adv determine conditional probabilities

A6 develop an understanding of factorial notation and apply it to calculating permutations and combinations

G7 distinguish between situations that involve combinations and permutations

G8 develop and apply formulas to evaluate permutations and combinations

B8 determine probabilities using permutations and combinations

Appendix D: Formula Sheet — Mathematics 12

Quadratics Unit

General form: $y = \mathrm{a}x^2 + \mathrm{b}x + \mathrm{c}$

Standard form: $y = \mathrm{a}(x - \mathrm{h})^2 + \mathrm{k}$

Transformational form: $\frac{1}{\mathrm{a}}(y - \mathrm{k}) = (x - \mathrm{h})^2$

If $\mathrm{a}x^2 + \mathrm{b}x + \mathrm{c} = 0$ then $x = \frac{-\mathrm{b} \pm \sqrt{\mathrm{b}^2 - 4\mathrm{ac}}}{2\mathrm{a}}$

Exponential Growth Unit

$y = \mathrm{ab}^x$

$\log_a(xy) = \log_a x + \log_a y$

$\log_a(x \div y) = \log_a x - \log_a y$ or $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

$\log_a x^b = b(\log_a x)$

Circle Geometry Unit

$\mathrm{d} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

The coordinates of M are: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$m = \frac{\Delta y}{\Delta x}$

Probability Unit

$$\mathrm{P(A\ and\ B)} = \mathrm{P(A)} \times \mathrm{P(B)}$$

$$\mathrm{P(A\ or\ B)} = \mathrm{P(A)} + \mathrm{P(B)} - \mathrm{P(A\ and\ B)}$$

$${}_nP_r = \frac{n!}{(n-r)!} \qquad {}_nC_r = \frac{n!}{r!(n-r)!}$$

Appendix E: Formula Sheet — Advanced Mathematics 12

Quadratic Unit

General form: $y = \mathrm{a}x^2 + \mathrm{b}x + \mathrm{c}$

Standard form: $y = \mathrm{a}(x - \mathrm{h})^2 + \mathrm{k}$

Transformational form: $\frac{1}{\mathrm{a}}(y - \mathrm{k}) = (x - \mathrm{h})^2$

If $\mathrm{a}x^2 + \mathrm{b}x + \mathrm{c} = 0$ then $x = \frac{-\mathrm{b} \pm \sqrt{\mathrm{b}^2 - 4\mathrm{ac}}}{2\mathrm{a}}$

Exponential Growth Unit

$y = \mathrm{ab}^x$

$\mathrm{A}(y - \mathrm{C}) = \mathrm{b}^{\mathrm{B}(x - \mathrm{D})}$

$\log_a(xy) = \log_a x + \log_a y$

$\log_a(x \div y) = \log_a x - \log_a y$ or $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

$\log_a x^b = b(\log_a x)$

Circle Geometry Unit

$$\mathrm{d}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

The coordinates of M are: $\left(\dfrac{x_1+x_2}{2},\dfrac{y_1+y_2}{2}\right)$

General form: $\mathrm{A}x^2+\mathrm{A}y^2+\mathrm{D}x+\mathrm{E}y+\mathrm{F}=0$

$$\mathrm{A}x^2+\mathrm{B}y^2+\mathrm{D}x+\mathrm{E}y+\mathrm{F}=0$$

Standard form: $(x-\mathrm{h})^2+(y-\mathrm{k})^2=\mathrm{r}^2$

Transformational form: $\left[\dfrac{1}{\mathrm{r}}(x-\mathrm{h})\right]^2+\left[\dfrac{1}{\mathrm{r}}(y-\mathrm{k})\right]^2=1$

$$\left[\frac{1}{\mathrm{a}}(x-\mathrm{h})\right]^2+\left[\frac{1}{\mathrm{b}}(y-\mathrm{k})\right]^2=1$$

$$m=\frac{\Delta y}{\Delta x}$$

Probability Unit

$$\mathrm{P(A\ and\ B)}=\mathrm{P(A)}\times\mathrm{P(B)}$$

$$\mathrm{P(A\ or\ B)}=\mathrm{P(A)}+\mathrm{P(B)}-\mathrm{P(A\ and\ B)}$$

$$\mathrm{P(A|B)}=\frac{\mathrm{P(A\ and\ B)}}{\mathrm{P(B)}}$$

$${}_nP_r=\frac{n!}{(n-r)!} \qquad {}_nC_r=\frac{n!}{r!(n-r)!}$$

Appendix F: List of Mathematical Terms and Concepts

Students should be familiar with the mathematical terms and concepts below. The terms pertinent only to Advanced Mathematics 12 students are marked with an asterisk.

Quadratic Unit

- 1^{st}, 2^{nd} … degree function
- 1^{st}, 2^{nd} … level of differences
- altitude
- appropriate scale
- arithmetic and power sequence
- axis of symmetry
- coefficient, integral coefficient
- common difference
- completing the square
- complex numbers
- cubic, quartic
- curve of best fit
- difference of squares
- discrete
- discriminant
- domain
- factors
- finite differences*
- finite sequence
- general form
- horizontal distance
- horizontal translation
- imaginary number
- inadmissible root
- infinite sequence
- intersect
- labeling axis
- mapping rule
- maximize
- maximum value
- minimum value
- model
- n^{th} term of a sequence
- non-linear
- number systems (I, R, N, W)
- parabola
- parabolic
- perfect square
- projectile
- quadratic function
- quadratic sequence
- quadratic formula
- radicals
- range
- reflection
- regression
- roots
- scatter plot
- sequence of differences
- set notation (interval notation)
- standard form
- transformational form
- transformations
- vertex
- vertical distance
- vertical stretch
- vertical translation
- units of measure
- x-intercept(s)
- y-intercept
- zeroes

Exponential Unit

- argument
- asymptote
- base
- common difference
- common ratio
- decay curve
- decreasing function
- dependent
- domain
- exponent
- exponential sequence
- extraneous root
- focal point
- geometric sequence
- growth curve
- increasing function
- independent
- half-life
- laws of exponents
- laws of logarithms
- logarithmic scales*
- logarithms
- power
- range

Circle Geometry Unit

- acute
- altitude
- angles
- bisect
- central angle*
- centre
- chord
- circle
- circumcenter
- circumference
- collinear
- complementary angle
- concentric
- congruent
- conjecture
- converse statement
- corresponding angles
- cyclic quadrilateral*
- diagonals
- diameter
- distance
- ellipse*
- equiangular
- equidistant
- equilateral triangle
- Euclidian geometry
- exterior angle
- iff statements
- inscribed*
- intercepted arc*
- interior angle
- intersect
- isosceles triangle
- line segment
- major axis*
- midpoint
- minor axis*
- obtuse
- parallel
- perpendicular
- perpendicular bisector
- Pythagorean theorem
- radius
- scalene triangle
- semicircle
- similar
- subtended*
- supplementary angle
- transversal
- unit circle
- vertically opposite angles
- vertices

Probability Unit

- area diagram
- combinations
- complement of event
- conditional probability*
- dependent events
- event
- expected value
- experimental probability
- factorial
- fundamental counting principle
- independent events
- multiplication principle
- mutually exclusive events
- outcomes
- permutations
- probability
- random sample
- random variable
- sample space
- simulation
- success
- theoretical probability
- tree diagram
- Venn diagram

Appendix G: Using a Graphing Calculator

In order for students to show their understanding when solving a problem using a graphing calculator, some demonstration of that use will be required to obtain full value on any given question.

As a teacher said at a previous session:

> "I would expect students to show graphs when solving graphically just like I would expect students to show algebra when solving algebraically."
>
> Beth Calabrese, J.L. Ilsley, Halifax, October 2003.

Please note:

- A graphing calculator (TI-82, TI-83, TI-83PLUS, TI-84, TI-84PLUS) should be available to students when writing the NSE in Mathematics 12 or Advanced Mathematics 12.
- The memory of all graphing calculators will be cleared (instructions provided in Appendix H). If students choose to use their own graphing calculators, they should be advised that the memory will be cleared by the teacher before writing the exam.
- If a question instructs students to solve:
  - without the use of the graphing calculator…;
  - algebraically…;
  - without using regression…;

  no points will be awarded if solved by using the graphing calculator other than for basic calculations.

Examples are provided on the following pages that indicate the expectations for writing solutions when using a graphing calculator to solve a given problem.

EXAMPLE - QUESTION #8, page 17 (selected from *Mathematical Modeling Book 3*)

The rides Superman the Escape and Tower of Terror accelerate riders to 160 km/h in 7 s, then raise them to a vertical height of 40 storeys. On the reverse route, riders travel backward up to 160 km/h again before magnetic brakes bring them to a complete stop.

The table shows the horizontal distance traveled in each second of acceleration.

Time (s)	1	2	3	4	5	6	7
Distance (m)	3.17	12.7	28.57	50.79	79.36	114.27	155.54

(a) Find the equation of the curve of best fit using technology, and justify why your equation best models the situation. (2 points) {F1, C1, C4, C8, C29 - Level 2}

(b) How many seconds, correct to two decimal places, would it take to reach a horizontal distance of 200 m? (2 points) {C22, C23 - Level 2}

<u>Solutions for (a)</u>

Points awarded:
- 1.5 pts for concluding model should be quadratic with supporting work
- 0.5 pt for final answer

(a)

t	1	2	3	4	5	6	7
d	3.17	12.7	28.57	50.79	79.36	114.27	155.54

9.53 15.87 22.22 28.57 34.91 41.27

6.34 6.35 6.35 6.34 6.36 ← constant 2^{nd} level difference ∴ quadratic

Quadreg ⇒ $y = 3.17x^2 + 0.0038x - 0.0057$

OR

(a) Quadreg $R^2 = 0.9999$

Expreg $R^2 = 0.9268$

} ∴ Quadratic

$y = 3.17x^2 + 0.0038x - 0.0057$

Solutions for (b)

Points awarded:

- 0.5 pt for identifying graphed functions
- 1 pt for graph identifying intersection point or *x*-intercept (or using tables)
- 0.5 pt for final answer

(b)

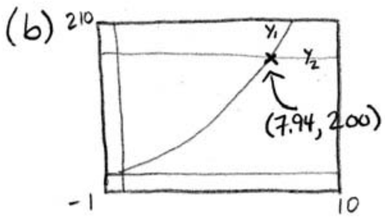
7.92	199.106
7.93	199.609
7.94	200.113 ←
7.95	200.617

It will take 7.94 sec.

Note: since table illustrated uses the function established in (a) it does not need to be re-stated. However, it would be good practice to do so.

OR

(b)



$Y_1 = 3.17x^2 + 0.0038x - 0.0057$

$Y_2 = 200$

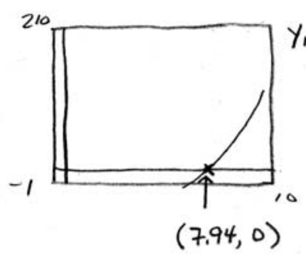
It will take 7.94 sec.

OR

(b) $y = 3.17x^2 + 0.0038x - 0.0057$

$200 = 3.17x^2 + 0.0038x - 0.0057$

$0 = 3.17x^2 + 0.0038x - 200.0057$



$Y_1 = 3.17x^2 + 0.0038 - 200.0057$

It will take 7.94 sec.

OR

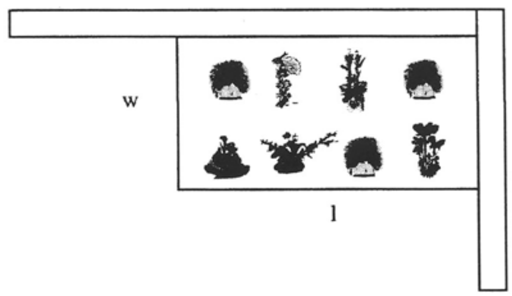
(b) $y = 3.17x^2 + 0.0038 - 200.0057$

x	y
7.93	−0.6568
7.94	−0.1537
7.95	0.3500

⇒ It will take 7.94 sec.

EXAMPLE - QUESTION #21, page 21 (selected from *Mathematical Modeling Book 3*)

Suppose the gardener used two adjacent sides of the wall and 50 m of bordering to form the garden



(a) Express the area of the garden as a function of its width. (2 points) {C1, C8 - Level 2}

(b) Find the maximum area of the garden. (2 points) {C1, C23, C31 - Level 1}

<u>Solution for (a)</u>

Points awarded:

- 1 pt for calculating the length as a function of the width
- 1 pt for calculating the area as a function of the width

(a) $l + w = 50$

$l = 50 - w$

$A = l \times w$

$= (50 - w)w$

OR

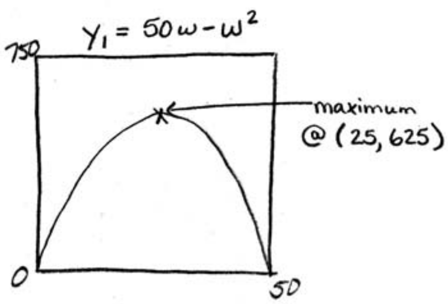
$= 50w - w^2$

<u>Solution for (b)</u>

Points awarded:

- 1 pt for calculating the length as a function of the width
- 1 pt for calculating the area as a function of the width

(b)



The maximum area of the garden is 625 m².

EXAMPLE - QUESTION #29, page 35 (selected from *Mathematical Modeling Book 3*)

Find two numbers whose difference is 13 and whose squares when added together yield a minimum. (3.5 points) {C8, C9, C23, C31 - Level 2}

Solution

Points awarded:

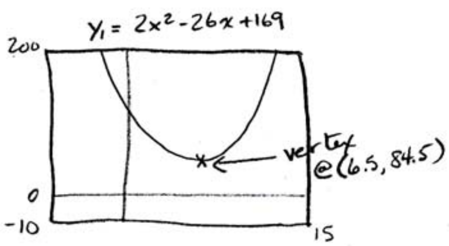
- 1 pt for setting up quadratic equation
- 0.5 pt for identifying graphed function
- 1 pt for graph with vertex clearly indicated
- 1 pt for answers

Let x represent a number.

$x^2 + (x-13)^2 = m$

$x^2 + x^2 - 26x + 169 = m$

$m = 2x^2 - 26x + 169$



Therefore one of the numbers is 6.5.

Second number $= 6.5 - 13$

$= -6.5$

The two numbers are 6.5 and −6.5.

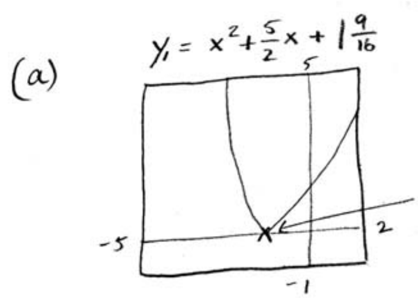
QUESTION page 58, Procedure A (selected from *Mathematical Modeling Book 3*)

Find the roots to each quadratic equation. (2.5 points each) {C8, C22 - Level 1}

Solution

(a) $x^2 + \frac{5}{2}x + 1\frac{9}{16} = 0$

Points awarded:
- 0.5 pt for indicating graphed function
- 1 pt for graph showing *x*-intercept(s)
- 1 pt for final answer(s)

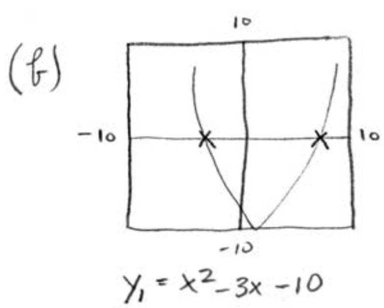


2 equal roots

$x = -1,25$

(b) $x^2 - 3x - 10 = 0$

Points awarded:
- 0.5 pt for indicating graphed function
- 1 pt for graph showing *x*-intercept(s)
- 1 pt for final answer(s)



2 roots

$x = -2$ and $x = 5$

Appendix H: How to clear the memory on graphing calculators
(TI-82, TI-83, TI-83PLUS, TI-84, TI-84PLUS)

All memory should be cleared from the calculators before the students write the examination. Teachers must follow the following steps:

- To clear the memory, turn the calculator 'ON'.
- Press 2nd +
- Select reset.
- Press ENTER twice.
- Select reset.
- Press ENTER

Note, the screen should display DONE, RAM Cleared, or MEMORY Cleared.

<u>After resetting the graphing calculator:</u>

⇒ The values of 'R' (correlation coefficient) or 'R^2' will not appear when performing a regression.

To set diagnostic on, press 2nd 0 x^{-1} scroll down using ▼ and select diagnostic on.

Press ENTER twice.

⇒ The display contrast will also be reset. If the screen display is too light press 2nd and press and hold ▲ until you reach the desired contrast.

⇒ You should also have extra batteries on hand.

Note: This procedure does not erase archived memory, if you wish to delete such memory - note you are not required to - please refer to the Texas Instruments website (http://education.ti.com/educationportal/sites/US/productDetail/us_testguard_20.html) and follow their instructions.